POLYNOMIALS REVIEW

Math Vocabulary for Chapter 8 Factoring

polynomial – is a term or sum of terms in which all variables have whole number exponents.
Example: \(3x, \text{ or } x^2 + 1, \text{ or } -3x^2 + 3x + 1\)

monomial – a number, a variable, or a product of numbers and variables.
Example: 3, 2x, \(-4x^2\) are all monomials.

binomial – the sum of two monomials that are unlike terms.

trinomial – the sum of three monomials that are unlike terms.

like terms – terms of a variable expression that have the same variable and the same exponent.
Example: 3x and 3x^2 are unlike terms, but 3x and 2x are like terms.

factor – (in multiplication) a number being multiplied.
Example: What are the factors of 121? 1, 11, and 121.
\[121 = 11 \times 11, \quad 121 = 1 \times 121\]

to factor a polynomial – to write a polynomial as a product of other polynomials

to factor a trinomial of the form \(ax^2 + bx + c\) – to express the trinomial as the product of two binomials.
Example: \(x^2 + 5x + 6 = (x+2)(x+3)\)

to factor by grouping – to group and factor terms in a polynomial in such a way that a common binomial factor is found.
Example: \(2x(x+1) – 3(x+1) = (x + 1)(2x – 3)\)

factor completely – to write a polynomial as a product of factors that are nonfactorable over the integers.

FOIL method – A method of finding the product of two binomials in which the sum of the products of the First terms, of the Outer terms, of the Inner terms, and of the Last terms is found.
Example: \((x+2)(x+3) = x^2 + 3x + 2x + 2\times3 = x^2 + 5x + 6\)

common factor – a factor that is common to two or more numbers.
Example: What are the common factors of 12 and 16x^2?
The factors of 12x are 1,2,3,4,6,12, and x
The factors of 16x^2 are 1,2,4,8,16, x, x
The common factors of 12x and 16x^2 are 1x, 2x, and 4x
The Greatest Common Factor of 12x and 16x^2 is 4x.
### POLYNOMIALS REVIEW

#### EXPRESSIONS

<table>
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<tr>
<th>Examples: 3+2(1-4), -3x+x, (3x+2)^2</th>
<th>Can be simplified</th>
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<tbody>
<tr>
<td>Can1: Don’t forget order of operations! 3+2(1-4)^2 can be simplified to 3+2(-3)^2 which becomes 3+2(9) which becomes 3+18 which becomes 21. Ex2: -3x + x can be simplified to -2x</td>
<td></td>
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#### EQUATIONS

<table>
<thead>
<tr>
<th>Examples: 3x + 2 = 5, 5x + 5y=10, x(x-5) = -6</th>
<th>Can be solved:</th>
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<tr>
<td>Can be evaluated: (3x+2)^2 can be evaluated at x=-1. (3(-1)+2)^2 is (-3+2)^2 which is (-1)^2 which is 1.</td>
<td>Can be solved:</td>
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#### Systems of Linear equations

| Systems of Linear equations have either one solution (independent), no solutions (inconsistent), or infinitely many solutions (dependent). An independent system is the case when the equations represent two intersecting lines. The solution is the intersection point. An inconsistent system is the case when the equations represent two parallel lines. They never intersect. A dependent system is the case when the equations represent the same line. Therefore they intersect everywhere on the line. |

#### Rules for Variable Expressions:

Only like terms can be **added**, and when **adding** like terms, do **not change** the exponent of the variable.

\[ 5x^2 + 3x^2 = 8x^2 \]

When multiplying variable expressions, add exponents of like variables

\[ (5xy^3)(2y^2) = 10xy^{3+2} = 10xy^5 \]

When taking powers of variable expression that is a monomial (one term), multiply exponents of EVERY term inside the parentheses.

\[ (2x^3y^4)^3 = 2^3x^{3*3}y^{4*3} = 8x^9y^{12} \]

When taking powers of a variable expression that is a binomial, trinomial or some other polynomial, use the rules of polynomial multiplication.

For example:

\[ (x+2)^2 = x^2+2x+4 \]

**Example 2:**

\[ (x^2+3x+5)^2 = (x^2+3x+5)(x^2+3x+5) = (x^2+3x+5)x^2 + (x^2+3x+5)3x + (x^2+3x+5)5 \] (DISTRIBUTIVE PROPERTY)
POLYNOMIALS REVIEW

A General Strategy for Factoring a Polynomial

1. Do all the terms in the polynomial have a common factor? If so, factor out the Greatest Common Factor. Make sure that you don’t forget it in your final answer.

Example: $24x^4 - 6x^2 = 6x^2(4x^2 - 1)$. Also look to see if the other polynomial factor and be factored more. $(4x^2-1)=(2x-1)(2x+1)$, so the final answer is $24x^4 - 6x^2 = 6x^2(2x-1)(2x+1)$.

2. Count the number of terms in the polynomial.

Two terms: Is it a difference of squares? Factor by using: $a^2-b^2=(a+b)(a-b)$

Example: $36x^2 - 49 = (6x)^2 - 7^2 = (6x-7)(6x+7)$

If the polynomial can’t be factored, it is PRIME.

Three terms: Is it a perfect square trinomial?

If it is it would be in the form $a^2x^2 + 2abx + b^2$, which is factored as $(a+b)^2$

or $a^2x^2 + 2abx + b^2$ which is factored as $(a-b)^2$.

Example: $4x^2 + 12x + 9 = (2x)^2 + 2(2)(3)x + 3^2 = (2x + 3)^2$

Is it of the form $x^2 + bx + c$?

Factor by finding two numbers that multiply to $c$ and add to $b$.

Example: $x^2 - 3x - 4 = (x+1)(x-4)$ because $1*-4 = -4$ and $1 + -4 = -3$

Can’t find the numbers? Maybe the polynomial is PRIME.

Is it of the form $ax^2 + bx + c$?

Try factoring by the Grouping Method (or $a*c$ Method) or Trial and Error.

Example: $2x^2 + 13x + 15$ (the $a*c$ method means multiply $2*15$ which is 30. Find factors of 30 that add up to the middle term’s coefficient, which in this case is 13. $3*10=30$ and $3+10 = 13$. Split the middle term into two parts: $2x^2 + 10x + 3x + 15$ and then factor by grouping.

$2x(x+5)+3(x+5) = (2x+3)(x+5)$

Those methods don’t work? Maybe the polynomial is PRIME.

Four terms: Try Factoring by Grouping. Group the 1st two terms and the last two terms. Factor out the Greatest Common Factor from each grouping. Then factor out the common binomial term.

3. Always factor completely. Double check that each of your factors can not be factored more.

4. Check your work by multiplying the factors together. Does it result in the original polynomial?