Equations:

An equation is two expressions set equal to one another.

Numerical Equations:

\[ 8 = 3 + 5 \]
\[ 12 = 3(4) \]
\[ 3 = \frac{6}{2} \]

Algebraic Equations:

\[ x = 3(4) \]
\[ y - 3 = 4 \]

Equations can be true or false.

True: \[ 3 = 2+1 \]  
False: \[ 2+3 = 2(3) \]

Algebraic Equations can be true or false depending on the value of the variable.  
y – 3 = 4 is true if y = 7 and false if y equals anything else.  
Some algebraic equations are false no matter what: \( y = y + 2 \) can never be true no matter what y is.

Algebraic Equations can be solved. A solution of an equation is the value of the variable that makes the equation true.

Example 1 p. 106

Is -3 a solution of the equation \( 4x + 16 = x^2 - 5 \)?

Let’s substitute -3 for x in the equation (Don’t forget to surround it with parentheses!) and see if the equation is true.

\[ 4(-3) + 16 = (-3)^2 - 5 \]

\[-12 + 16 = 9 - 5 \]
\[ 4 = 4 \quad \text{TRUE} \]

Now you try this one. (Go around and check how they do. The answer is NO)

Is 5 a solution of \( 10x - x^2 = 3x - 10 \)?

Two equations are equivalent equations when they have the same solution.  
x + 1 = 4 is equivalent to \( x + 2 = 5 \)

In both cases, x must be 3 for the equation to be true.

How do you solve equations?

Get x by itself one side of the equation (isolate x) so you are left with \( x = \_\_\_\_\_\_\_ \). The right side of the equation will just have a numerical expression that can be simplified.

SOLUTIONS OF EQUATIONS CAN AND SHOULD ALWAYS BE CHECKED! Just substitute the solution in the original equation and see if one side equals the other.
Solving Equations

**Subtraction Property of Equality** –
Subtracting the same thing from both sides of an equation does not change the solution.

\[ x + 2 = 5 \]

Subtract 2 from both sides:

\[ x + 2 - 2 = 5 - 2 = 3 \]

This new equation will have the same solution as \( x + 2 = 5 \).

\[ x + 0 = 3 \]
\[ x = 3 \]

*Hey! We just “isolated” \( x \)!*
Check your answer by substituting \( x = 3 \) in the original equation.

\[ 3 + 2 = 5 \]? Yes!

Soooo…. **if you have an equation with a number added to a variable on one side, subtract that number from both sides to get the variable by itself, or to “isolate the variable.”**

But that example was too easy. Try this one:

\[ x + 15 = 67 \]

**Addition Property of Equality** –
Adding the same thing to both sides of an equation does not change the solution.

\[ x - 17 = 34 \]
Add 17 to both sides.
\[ x - 17 + 17 = 34 + 17 \]
This new equation will have the same solution as \( x - 17 = 34 \).

\[ x + 0 = 51 \]
\[ x = 51 \]
Check your answer by substituting \( x = 51 \) in the original equation.

\[ 51 - 17 = 34 \]? Yes!

Soooo…. **if you have an equation with a number subtracted from a variable on one side, add that number to both sides to get the variable by itself.**

**Example**

*Last year a hairdresser lost 17 customers who moved away. If he now has 73 customers, how many did he have originally?*

What are we being asked to find?
The number of customers the hairdresser had before he lost some. Let this equal \( x \).

What is given? He has 73 customers now. He lost 17.
Translation: “Lost” means subtraction.
What was 17 subtracted from? \( \rightarrow \) the original number of customers, or \( x \). The current number of customers should equal the original number minus the amount lost because of moving away.

\[ \text{current number} = \text{original number} - \text{amount lost} \]
\[ 73 = x - 17 \]

Now solve for \( x \). Add 17 to both sides to isolate \( x \).
\[ 73 + 17 = x - 17 + 17 \] This results in \( 90 = x \)  
*Check: Does 90 - 17 = 73? Yes!*
Division Property of Equality –
Dividing the same thing on both sides of an equation does not change the solution.

\[ 5x = 15 \]

\[ \frac{5x}{5} = \frac{15}{5} \]

\[ x = 3 \]

Check : \( 5(3) = 15 \)? Yes!

If you have an equation with a number multiplied by a variable, divide both sides of the equation by that number to get the variable by itself.

Examples of English words for multiplication: multiplied by 15 (15x), increased by a factor of six (6x), doubled (2x), tripled (3x), quadrupled (4x), increased fivefold (5x).

Multiplication Property of Equality –
Multiplying the same thing on both sides of an equation does not change the solution.

If your equation has a variable divided by a number, multiply both sides by that number to isolate the variable.

Examples of English words for division: halved (x/2), a third of (x/3), split in four (x/4)

Now you try p.72 #58

\[ \frac{x}{3} = 13 \]

\[ 3 \left( \frac{x}{3} \right) = 3(13) \]

\[ x = 39 \]

Check : \( \frac{39}{3} = 13 \)? Yes!

Notice that \( x \) divided by 3 is written as a fraction, \( \frac{x}{3} \).
When multiplying a whole number by a fraction, think of the whole number as a fraction with denominator 1. For example 3 = 3/1, so

\[ 3 \left( \frac{x}{3} \right) = \left( \frac{3}{1} \right) \left( \frac{x}{3} \right) = \frac{x}{1} = x \]
Solving equations of the form $ax + b = c$

In order to isolate the variable $x$, we must first combine like terms before isolating the variable.

Example 2: Solve:

$$5 = 9 - 2x$$

$5$ and $9$ are both constants. They can be combined. To get $9$ to the other side, subtract it from

$$5 - 9 = 9 - 9 - 2x$$

$-4 = -2x$ The minus sign to the left of $2x$ does not disappear when the $9$ disappears.

Divide both sides by $-2$ to solve for $x$.

$$\frac{-4}{-2} = \frac{-2x}{-2}$$

$$2 = x$$

CHECK THE RESULT! $5 = 9 - 2(2)$?

$5 = 9 - 4 = 5$ YES!

You try this one: Solve: $3x - 6 = -5$.

\[ \frac{2}{5}x - 3 = -7 \]

$3$ and $-7$ are both constants, so we should combine them. To get $3$ to the other side, use the addition property of equality. Since $3$ is subtracted on the left side, add $3$ to both sides to remove it from the left side and move to the right side.

\[ \frac{2}{5}x - 3 + 3 = -7 + 3 \]

\[ \frac{2}{5}x = -4 \]

Now we use the division property of equality to isolate $x$.

Dividing both sides by $2/5$ is the same as MULTIPLYING BY THE RECIPROCAL.

\[ \left(\frac{5}{2}\right)\frac{2}{5}x = -4\left(\frac{5}{2}\right) \]

\[ x = -10 \]

CHECK:

\[ \frac{2}{5}(-10) - 3 = -7 \]

\[-4 - 3 = -7 \]

\[-7 = -7 \quad \text{YES!} \]
The Trick with Fractions!

Fractions are messy to deal with. When solving equations with fractions in them we can take advantage of the multiplication property of equality to get rid of them while keeping an equivalent equation.

What we do is multiply BOTH SIDES of the equation (that is everything on each side) by the LCD of all the fractions.

\[
\frac{2}{3} + \frac{1}{4}x = -\frac{1}{3}
\]

The denominators are 3 and 4. The LCD is 12

\[
(12)\frac{2}{3} + (12)\frac{1}{4}x = -\frac{1}{3}(12)
\]

\[
8 + 3x = -4
\]

Now we can combine like terms.

8 and -4 are like terms.

Combine them by subtracting 8 from both sides.

\[
8 - 8 + 3x = -4 - 8
\]

\[
3x = -12
\]

Divide both sides by 3.

\[
\frac{3x}{3} = \frac{-12}{3}
\]

\[
x = -4
\]

CHECK!

\[
\frac{2}{3} + \frac{1}{4}(-4) = -\frac{1}{3}
\]

\[
\frac{2}{3} + (-1) = -\frac{1}{3}
\]

Change -1 so that it has a common denominator

\[
\frac{2}{3} + \left(-\frac{3}{3}\right) = -\frac{1}{3}
\]

\[
\frac{2}{3} - \frac{3}{3} = -\frac{1}{3}
\]

\[
-\frac{1}{3} = -\frac{1}{3}
\]

Yes!
Example 4

Solve

\[4x - 3 = 8x - 7\]

4x and 8x are like terms that can be combined.
- 3 and -7 are like terms that can be combined.

Let's combine the constants first. Usually we like to put the variables on the left and the constants on the right, but either way works.

We'll add 3 to both sides to get the constants on the right.

\[4x - 3 + 3 = 8x - 7 + 3\]

\[4x = 8x - 4\]

Now we'll combine the variable terms.

Subtract 8x from both sides to get the variable terms on the left.

\[4x - 8x = 8x - 8x - 4\]

\[-4x = -4\]

Now we can divide both sides by -4.

\[x = 1\]

CHECK!

\[4(1) - 3 = 8(1) - 7\]

1 = 1 YES!
Strategy for Solving Algebraic Equations:

1. Use the distributive property to remove parentheses:
   \[3(x - 3) + 3 = 18 - 5x\]
   becomes \[3x - 9 + 3 = 18 - 5x\]

2. Combine like terms on either side of the equation.
   -9 and 3 can be added to get -6.
   \[3x - 6 = 18 - 5x\]

3. Use the addition or subtraction properties of equality to get the variables on one side of the = symbol and the constant terms on the other.
   3x and 5x are like terms. Add 5x to each side to get the variable terms on the left.
   \[3x + 5x - 6 = 18 - 5x + 5x\]
   \[8x - 6 = 18\]

4. Continue to combine like terms whenever possible.
   6 and 21 are like terms. Since 6 is subtracted from 8x, add 6 to both sides to move it to the other side.
   \[8x - 6 + 6 = 18 + 6\]
   \[8x = 24\]

5. Undo the operations of multiplication and division to isolate the variable.
   Divide both sides by 8 to get x by itself.
   \[8x/8 = 24/8\]
   \[x = 3\]

6. Check the results by substituting your found value for x into the original equation.
   \[3(x - 2) + 5x = 18\]
   \[3(3-2) + 5(3) = 18 ?\]
   \[3(1) + 5(3) = 3 + 15 = 18 ?\] Yes.
   So \[x = 3\] is the solution to the equation.
Sometimes application problems give you the formula to use, and you just have to know where to substitute the values of the variables.

Example 8 p.135

A principle of physics states that when a lever system balances,

\[ F_1 x = F_2 (d-x) \]

Where \( F_1 \) is the force on Side 1 of the lever, and \( F_2 \) is the force on Side 2 of the lever.
\( x = \) the distance from the end of Side 1 to the “fulcrum” or point of support in which the lever turns.
\( d = \) total length of the lever.

A lever is 10 ft long. A force of 100 lbs is applied to one end of the lever, and a force of 400 lbs is applied to the other end. Then the system balances, how far is the fulcrum from the 100 lb force?

Step 1) What are we asked to find? How far (distance) is the fulcrum from the 100 lb force. (This is \( x \), if we let the 100 lb force be \( F_1 \))
Given info: Lever is 10 ft long, so \( d=10 \) ft
\( F_1 = 100 \) lbs, \( F_2 = 400 \) lbs

Step 3) Form an equation.

\[ F_1 x = F_2 (d-x) \]
100\( x \) = 400(10-x) Now use the strategy for solving equations.