CIRCLE DEFINITIONS AND THEOREMS

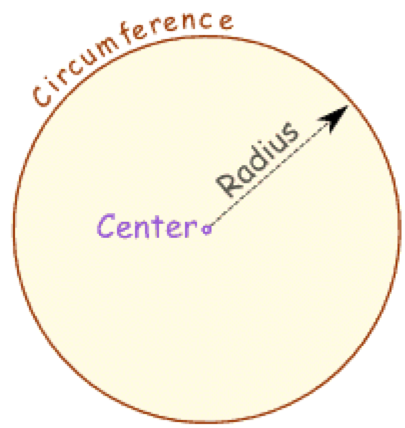
DEFINITIONS

Circle- The set of points in a plane equidistant from a given point(the **center** of the circle).

Radius- A segment from the center of the circle to a point on the circle(the distance from the center to a point on the circle.)

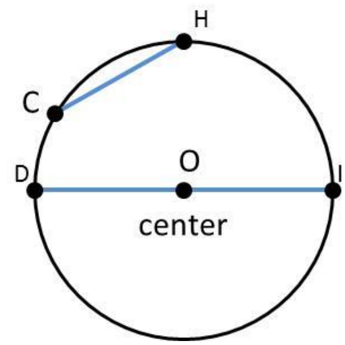
Circumference – distance around the edge of the circle

Congruent Circles- two circles with the same **radius.**



Diameter – A segment that goes through the center of the circle, with both endpoints on the edge of the circle.

Chord - A line segment that goes from one point to another on the circle's **circumference.**



$\overline{DI}$ is the diameter.

$\overline{CH}$ is a chord.

DO and OI are both radii.

$$Diameter = 2 * radius.$$

$$Radius = \frac{1}{2} * diameter$$

Tangent – a line that intersects a circle at only one point. The radius at the point of tangency is **perpendicular** to the tangent line.

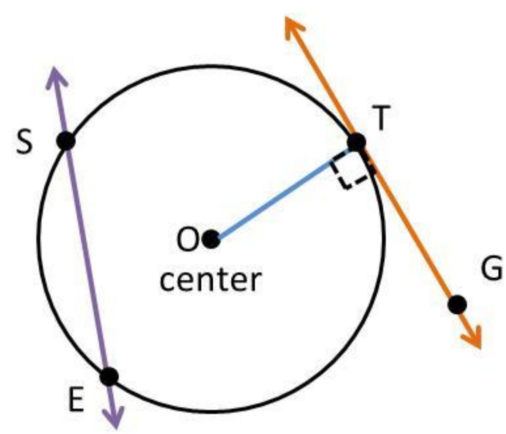
$\overleftrightarrow{TG}$ is a tangent line.

The point of tangency is the point T.

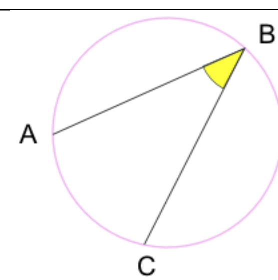
$\overline{OT}$ is a radius and $\overleftrightarrow{TG} \perp \overline{OT}$

Secant – a line that intersects a circle at **two points.**

$\overleftrightarrow{SE}$ is a secant line.



Inscribed Angle - an angle made from points sitting on the circle's edge.



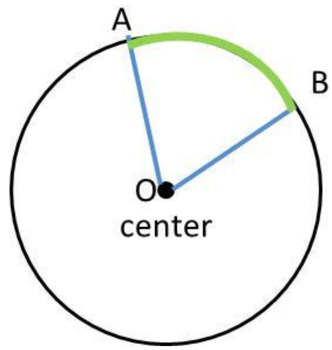
A and C are "end points"
B is the "apex point"

Central angle- an angle with vertex at the center of the circle

Arc – part of the circumference (edge) of the circle.

The measure of an arc is equal to the measure of

the central angle formed by its endponts.



$\overset{\frown}{AB}$ ***is an arc***

$\angle AOB$ ***is a central angle.***

$m\angle AOB = m\overset{\frown}{AB}$

Arcs are named by their endpoints. The dashed arc to the right would be called "arc AB". or "arc BA", the order of the endpoints does not matter. As a shorthand this can be written as the letters AB with a curving line above them

Example: $\overset{\frown}{AB}$ which is read "arc AB".

Notice that this naming can be ambiguous. For example it may mean the major arc AB, where you go the long way around the bottom of the circle. Unless stated otherwise, it always means the minor arc - the shortest of the two.

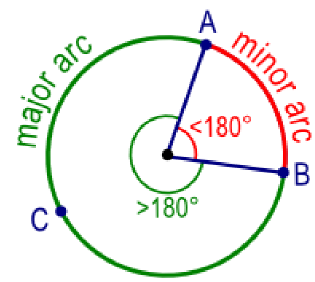
If you want to indicate the major arc, add an extra point and use three letters in the name. For example in the

diagram on the right the major arc is indicated by $\overset{\frown}{ACB}$ which is the long arc from A to B going around the bottom via C.

There are two measures of an arc

1. The length of the arc
2. The angle of the arc

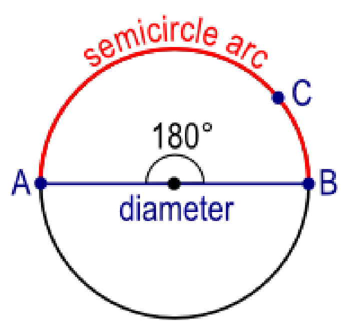
Here is a semicircle arc with a central angle of 180° it covers exactly half of the circumference.The endpoints A and B lie on the diameter of the circle.When naming this arc, we use an extra point C, now we have arc ACB. The third point tells us which half of the circumference the semicircle arc covers. If there are just two points we presume that the named arc is the smallest one on the circumference, the minor arc (as long as the arc is not a semicircle where both arcs are the same size).



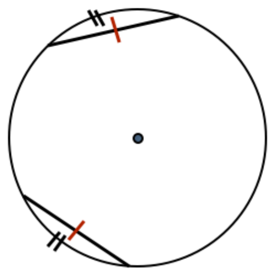
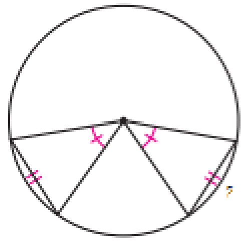
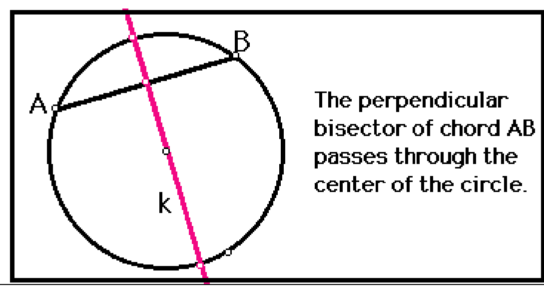
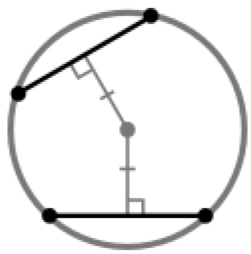
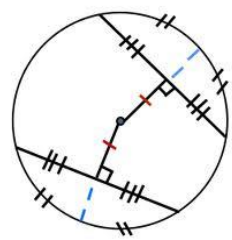
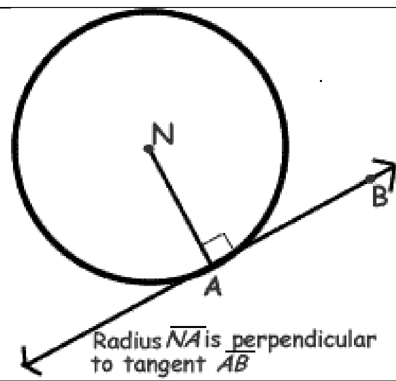
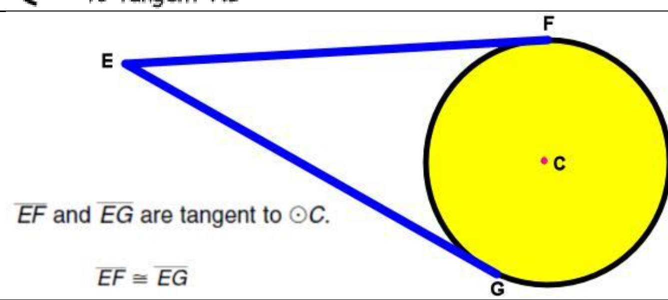
Minor arc – arc whose measure is less that 180 degrees.

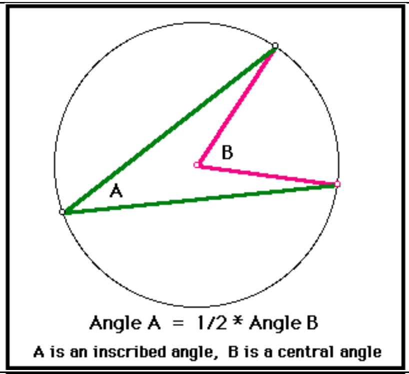
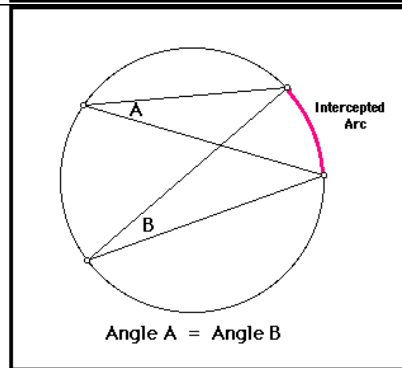
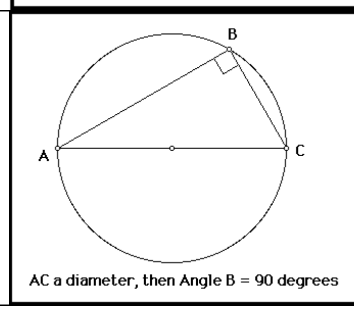
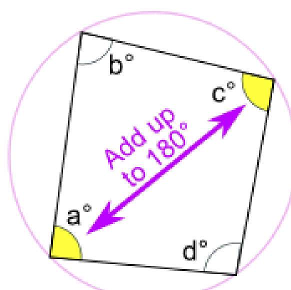
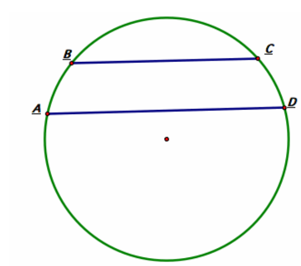
Major arc – arc whose measure is greater than 180 degrees.

Semicircle arc - arc whose measure = 180 degrees.



THEOREMS

Chord Central Angles Theorem If two chords in a circle are congruent, then they determine two central angles that are **congruent.**	
Chord Arcs Theorem If two chords in a circle are congruent, then their intercepted arcs are **congruent.**	
Perpendicular to a Chord Theorem The perpendicular from the center of a circle to a chord is the **bisector** of the chord.	
Chord Distance to Center Theorem Two congruent chords in a circle are **equidistant** from the center of the circle.	 
Perpendicular Bisector of a Chord Theorem The perpendicular bisector of a chord passes through the **center** of the circle.	
Tangent Theorem A tangent to a circle is **perpendicular** to the radius drawn to the point of tangency.	
Tangent Segments Theorem Tangent segments to a circle from a point outside the circle are **congruent.**	

Inscribed Angle Theorem The measure of an angle inscribed in a circle is **one-half** the measure of the central angle.	
Inscribed Angles Intercepting Arcs Theorem Inscribed angles that intercept the same arc are **congruent.**	
Angles Inscribed in a Semicircle Theorem Angles inscribed in a semicircle are **right angles**.	
Cyclic Quadrilateral Theorem The opposite angles of a cyclic quadrilateral are **supplementary.**	
Parallel Lines Intercepted Arcs Theorem Parallel lines intercept **congruent arcs** on a circle.	 If $\overline{BC} \parallel \overline{AD}$, $arcAB \cong arcCD$

MORE CIRCLE THEOREMS!!!

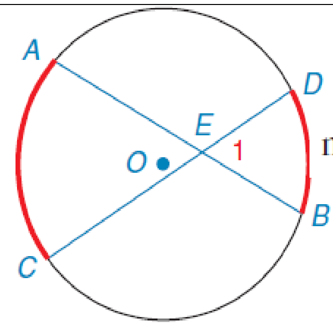
TABLE 6.1

Methods for Measuring Angles Related to a Circle

Location of the Vertex of the Angle	Rule for Measuring the Angle
Center of the circle	The *measure* of the intercepted arc
In the *interior* of the circle	*One-half the sum* of the measures of the intercepted arcs
On the circle	*One-half the measure* of the intercepted arc
In the *exterior* of the circle	*One-half the difference* of the measures of the two intercepted arcs

Intersecting Chords Theorem

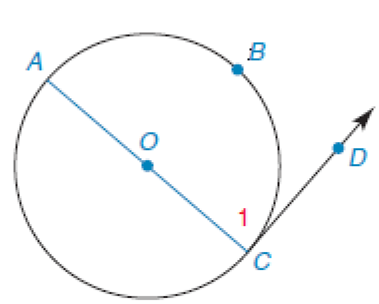
The measure of an angle formed by two chords that intersect within a circle is one-halfthe sum of the measures of the arcs intercepted by the angle and its vertical angle.



$$m\angle 1 = \tfrac{1}{2}(m\widehat{AC} + m\widehat{DB})$$

Tangent-Chord Angle Theorem

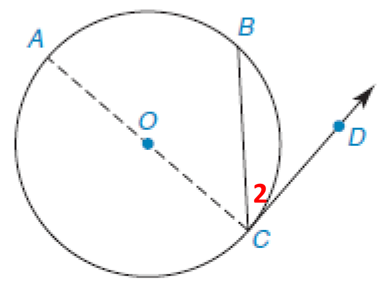
The measure of an angle formed by a tangent and a chord drawn to the point of tangency is one-half the measure of the intercepted arc.



(a) Case 1
The chord is a diameter.

$\overline{AC}$ is the chord, $\overrightarrow{CD}$ is the tangent

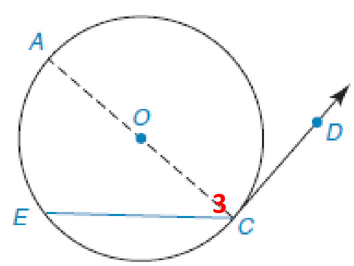
$$m\angle 1 = \frac{1}{2} m\widehat{ABC}$$



(b) Case 2
The diameter is in the exterior of the angle.

$\overline{BC}$ is the chord, $\overrightarrow{CD}$ is the tangent

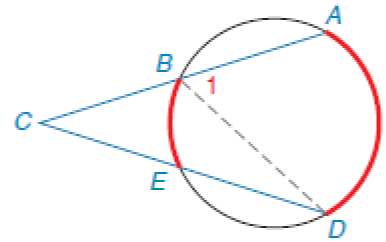
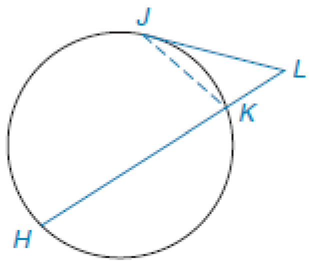
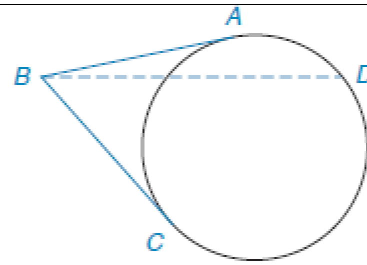
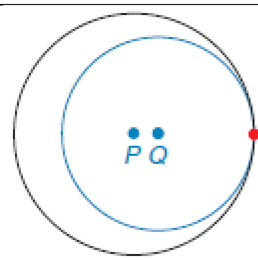
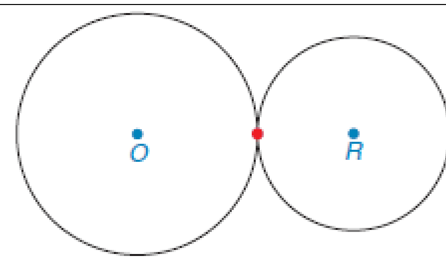
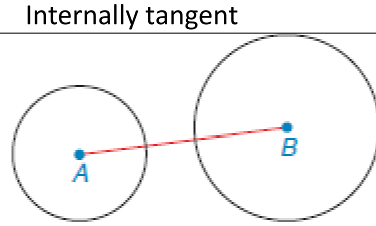
$$m\angle 2 = \frac{1}{2} m\widehat{BC}$$



(c) Case 3
The diameter lies in the interior of the angle.

$\overline{EC}$ is the chord, $\overrightarrow{CD}$ is the tangent

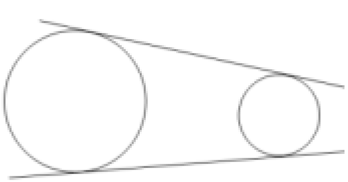
$$m\angle 3 = \frac{1}{2} m\widehat{AEC}$$

Two Secant Angle Theorem The measure of an angle formed when two secants intersect at a point outside the circle is one-half the difference of the measures of the two intercepted arcs.	 $m\angle C = \frac{1}{2}(m\widehat{AD} - m\widehat{BE})$
Tangent-Secant Angle Theorem If an angle is formed by a secant and a tangent that intersect in the exterior of a circle, then the measure of the angle is one-half the difference of the measures of its intercepted arcs.	 $m\angle L = \frac{1}{2}(m\widehat{HJ} - m\widehat{JK})$
Tangent-Tangent AngleTheorem If an angle is formed by two intersecting tangents, then the measure of the angle is onehalf the difference of the measures of the intercepted arcs (the major arc minus the minor arc)	 $m\angle ABC = \frac{1}{2}(m\widehat{ADC} - m\widehat{AC})$
<u>DEFINITIONS</u> **Tangent Circles** – circles that intersect at only one point.	 Internally tangent  Externally tangent
Line of Centers - the line (or line segment) containing the centers of both circles.	

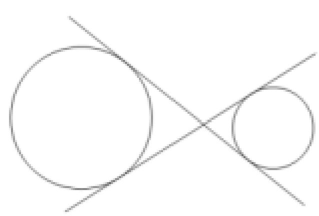
Tangents to two circles

Given two circles, there are lines that are tangents to both of them at the same time.

If the circles are separate (do not intersect), there are four possible common tangents:

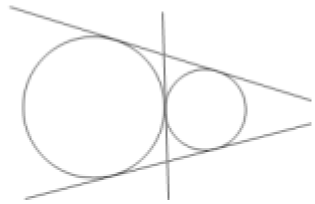


Two external tangents

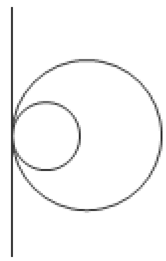


Two internal tangents

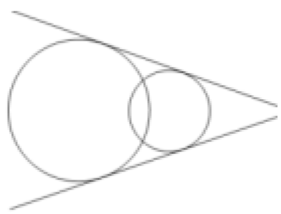
If the two circles touch at just one point, there are three possible tangent lines that are common to both:



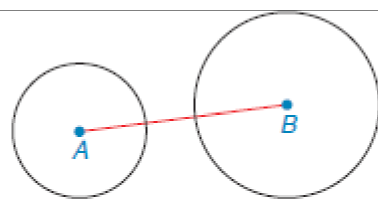
If the two circles touch at just one point, with one inside the other, there is just one line that is a tangent to both:



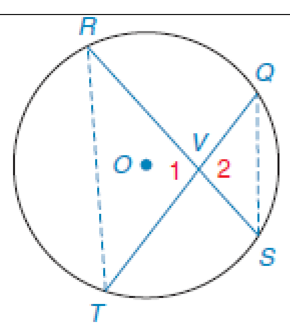
If the circles overlap - i.e. intersect at two points, there are two tangents that are common to both:



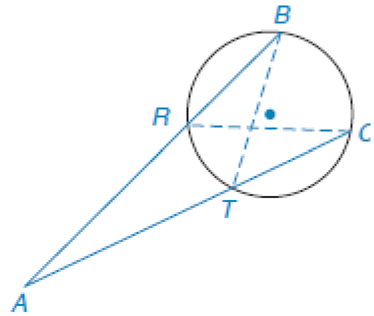
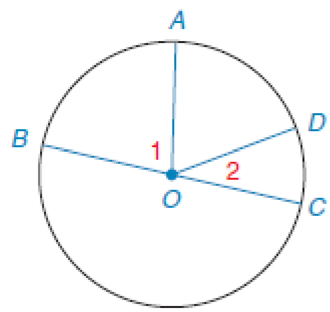
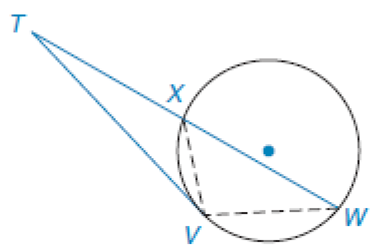
Line of Centers
- the line (or line segment) containing the centers of both circles.



Intersecting Chords Lengths Thoerem
If two chords intersect within a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$$RV \cdot VS = TV \cdot VQ$$

Secant Segments from an External Point Theorem If two secant segments are drawn to a circle from an external point, then the products of the lengths of each secant with its external segment are equal.	 $AB \cdot RA = AC \cdot TA$
Central Angle /Intercepting Arc Correlation Theorem In a circle (or in congruent circles) containing two unequal central angles, the larger angle corresponds to the larger intercepted arc (and vice versa!)	 $m\angle 1 > m\angle 2 \Leftrightarrow m\overset{\frown}{AB} > m\overset{\frown}{CD}$
Tangent-Secant Segment Length Theorem If a tangent segment and a secant segment are drawn to a circle from an external point, then the square of the length of the tangent equals the product of the length of the secant with the length of its external segment.	 $(TV)^2 = TW \cdot TX$