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Preface

Preparatory Mathematics was written with the community college student in mind. More specifically, it was written from the point of view of a college student who needs to understand key concepts of arithmetic without having to delve too deeply into concepts that are not essential for his or her career path. This doesn’t mean that this book is a watered-down presentation of arithmetic or that the author is not interested in helping students acquire a solid foundation of basic mathematics. What this book presents are the most essential topics of arithmetic, and they are presented in a way that should allow students to mainly learn arithmetic on their own and at their own pace.

As a former high school instructor, and now as a community college instructor, I have come to realize that students need a textbook that is as accessible as possible. I’ve read and used basic math textbooks that are excellent tools for instructors because they are highly detailed, cover all topics of arithmetic, have a lot of practice exercises per section, have many ancillaries and overall are visually pleasing. However, textbooks that may be considered by educators to be well-written may be intimidating to a student because they are too detailed. Often, these highly-regarded textbooks have a key defect: most pages are crowded with too many ideas, and they are written with a tiny font size. Understandably, students may not be able to use these books as a self-teaching tool. Most of the time, the students will simply have to rely on their instructor to attain mastery of arithmetic and the course textbook serves only as an expensive source of practice exercises.

By writing this preparatory mathematics book, my hope is that students will feel much less intimidated to read it and will motivate them to take learning into their own hands. The key aspects of this book are these:

- It is self-contained. The only prerequisite is that the student knows the times tables. All the important concepts are covered and build upon previously-presented topics. Concepts or facts that are not essential or have no connection whatsoever with real-life applications are not presented. This includes certain vocabulary and mathematical properties that I believe can be safely skipped. In the future, as students continue to make progress in their mathematics learning path they will be ready to learn more abstract concepts.
- A large font size and a limited number of ideas per page make the book less dense, and consequently, more accessible to a wider spectrum of students.
- Plenty of worked-out examples and practice exercises with solutions are provided. Some educators believe that this approach may not allow students to think and become independent problem-solvers. What I believe is that students who are learning basic math at the college level need to see a lot of examples and need easy access to solutions to check their work until they “get it.” They may be adults, but often they have had a terrible experience with previous math courses. That is why a need exists for writing a basic mathematics textbook that can help them change their mindset and their predisposition about learning math.
• This book makes the learning process as interactive as possible. It contains links to online applets and instructional videos for students who need additional instruction and/or prefer a more hands-on approach to learning math. In a 6-week course where the class meets four times per week, these video links are useful for students who need to see examples worked out and explained at a slower pace. In a 12-week course where the class meets only twice a week, the videos can serve as a way for students to review certain concepts during the block of days that the class does not meet.

• Each chapter includes a detailed overview of the main concepts that are covered, as well as a chapter test. There are also cumulative reviews to ensure that students retain mastery of concepts that were covered in previous chapters.

• The book will be improved as it continues to be piloted in Preparatory Math courses here at Cerritos Community College in the near future.

The author welcomes feedback from instructors who would be interested in adopting this textbook and, of course, from students themselves who were the source of inspiration for creating this work in the first place.

Luis Soto-Ortiz
Cerritos Community College
Chapter 1

Working with Whole Numbers
# Chapter 1 Overview

By the end of this chapter, you will achieve mastery of the following concepts:

- **Writing Whole Numbers**
  - The number 391,460 is written as: “Three hundred ninety-one thousand, four hundred sixty.”

<table>
<thead>
<tr>
<th>Place Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>hundred thousands</td>
<td>300,000</td>
</tr>
<tr>
<td>thousands</td>
<td>400</td>
</tr>
<tr>
<td>hundreds</td>
<td>90,000</td>
</tr>
<tr>
<td>tens</td>
<td>60</td>
</tr>
<tr>
<td>ones</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

- **Place and Place Value**
  - 3 has place value 300,000
  - 9 has place value 90,000
  - 1 has place value 1,000
  - 4 has place value 400
  - 6 has place value 60
  - 0 has place value 0

- **Rounding Numbers**
  - Round 6,835,294 to the nearest:
    - Thousand: 6,835,000
    - Ten: 6,835,290
    - Hundred thousand: 6,800,000
    - Ten thousand: 6,840,000
    - Million: 7,000,000

- **Adding Whole Numbers**
  - \[ \begin{array}{c}
    2 \quad 1 \\
    7 \quad 8 \quad 1 \quad 5 \\
    9 \quad 3 \quad 2 \quad 0 \\
    6 \quad 2 \quad 1 \quad 5 \\
    + \quad 7 \quad 4 \quad 3 \\
    \hline
    2 \quad 4 \quad 0 \quad 9 \quad 3
  \end{array} \]

- **Subtracting Whole Numbers**
  - \[ \begin{array}{c}
    7 \quad 13 \quad 3 \quad 17 \\
    8 \quad 3 \quad 6 \quad 5 \quad 4 \quad 7 \\
    - \quad 1 \quad 5 \quad 4 \quad 2 \quad 3 \quad 9 \\
    \hline
    6 \quad 8 \quad 2 \quad 3 \quad 0 \quad 8
  \end{array} \]

- **Multiplying Whole Numbers**
  - We begin by multiplying the number in the second row by the number in the first row, starting with the digit in the ones place (9), and working our way to the left.

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 x 8</td>
<td>24</td>
</tr>
<tr>
<td>7 x 3</td>
<td>21</td>
</tr>
<tr>
<td>6 x 9</td>
<td>54</td>
</tr>
<tr>
<td>4 x 2</td>
<td>8</td>
</tr>
<tr>
<td>1 x 7</td>
<td>7</td>
</tr>
<tr>
<td>0 x 8</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Division</th>
<th>Quotient</th>
<th>Remainder</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 ÷ 2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4 ÷ 2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>0 ÷ 3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3 ÷ 7</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>5 ÷ 2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>7 ÷ 2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Division is a process that involves determining the maximum number of times that the divisor fits into the dividend.
Section 1.1 – Place Value

Whole numbers appear in everyday situations. We encounter whole numbers in ATM machines when we withdraw money, conduct an inventory, carry out a census count, and when counting the number of votes a politician received in an election campaign. Hence, it is important to develop number sense. Whole numbers consist of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 and each digit is given a place value depending on its place or position. The following example serves to illustrate the concept of place and place value.

Example 1.1.1

The amount $1,845,027 is written as

“One million, eight hundred forty-five thousand, twenty-seven dollars.”

The digit 1 is in the millions place, and so its place value is 1,000,000.
The digit 8 is in the hundred thousands place, and so its place value is 800,000.
The digit 4 is in the ten thousands place, and so its place value is 40,000.
The digit 5 is in the thousands place, and its place value is 5,000.
The digit 0 is in the hundreds place, and its place value is 0.
The digit 2 is in the tens place, and its place value is 20.
The digit 7 is in the ones place and its place value is 7.
This means that

$1,845,027 = $1,000,000 + $800,000 + $40,000 + $5,000 + $0 + $20 + $7$
The amount $29,632 is written as

“Twenty-nine thousand, six hundred thirty-two dollars.”

The 2 on the left is in the *ten thousands place*, and so its place value is 20,000.
The 9 is in the *thousands place*, and its place value is 9,000.
The 6 is in the *hundreds place*, and its place value is 600.
The 3 is in the *tens place*, and its place value is 30.
The 2 on the right is in the *ones place* and it represents 2.

This means that

$29,632 = $20,000 + $9,000 + $600 + $30 + $2$

**Note:** We do not use the word “and” when we talk about whole numbers. For example, for the amount $1,845,027 we do not say “one million, eight hundred forty-five thousand, *and* twenty-seven dollars.” We use the word “and” when we have decimal numbers. The correct way to say the amount $78.35 is “seventy-eight dollars and thirty-five cents.” We use the word “and” to represent the decimal point. We will discuss decimals in greater detail in Chapter 5.

You should memorize the names of the various places (tens, hundreds, thousands, etc.) in which each digit of a whole number is located. Knowing the place name will allow you to determine the corresponding place value of each digit. Knowing the place names will be helpful later on when you are asked to round a whole number to a specific place.
1. For the number 25,963,407 determine which digit is in the
   a. Hundreds place 4
   b. Ones place 7
   c. Ten thousands place 6
   d. Millions place 5
   e. Tens place 0

2. For the number 9,148 determine which digit is in the
   a. Thousands place 9
   b. Tens place 4
   c. Ones place 8
   d. Hundreds place 1

3. For the number 638,941 determine the place of each digit
   a. 3 ten thousands
   b. 8 thousands
   c. 1 ones
   d. 9 hundreds
   e. 4 tens
   f. 6 hundred thousands

4. For the number 86,430,127 determine the place value of each digit
   a. 1 100
   b. 3 30,000
   c. 6 6,000,000
   d. 7 7
   e. 0 0
   f. 4 400,000
   g. 2 20
**Homework 1.1**

1. For the number 53,784 determine which digit is in the
   a. Tens place
   b. Thousands place
   c. Ones place
   d. Ten thousands place
   e. Hundreds place

   a. 53
   b. 8
   c. 3
   d. 4
   e. 7

2. For the number 380,157 determine which digit is in the
   a. Thousands place
   b. Hundreds place
   c. Hundred thousands place
   d. Tens place

   a. 0
   b. 1
   c. 3
   d. 5

3. For the number 9,207,586 determine the place of each digit
   a. 5
   b. 0
   c. 7
   d. 6
   e. 2
   f. 9

   a. hundreds
   b. ten thousands
   c. thousands
   d. ones
   e. hundred thousands
   f. millions

4. For the number 76,192,835 determine the place value of each digit
   a. 9
   b. 3
   c. 7
   d. 2
   e. 5
   f. 6
   g. 8

   a. 90,000
   b. 30
   c. 70,000,000
   d. 2,000
   e. 5
   f. 6,000,000
   g. 800
Section 1.2 – Rounding Whole Numbers

Rounding a whole number is often useful because it gives us an estimate of cost, expense, or earnings in a particular situation. Suppose that last year you had your patio remodeled for $5,647. Last year you also bought a new car for $36,825. You may want to write down a quick estimate of your expenses from last year. For example, you may decide to write the cost of remodeling the patio as $5,650 and the cost of the new car as $36,800. This is called rounding.

To “round” a whole number in a particular way, you first need to decide to which place you want to round the amount. Do you want to round the amount to the nearest ten, to the nearest hundred, to the nearest thousand, etc.?

Before continuing reading, please review Section 1.1 which explains “place” and “place value” if you are not familiar with these terms.

Method to Round Whole Numbers:

Step 1: Determine to which place (tens, hundreds, thousands, etc.) you want to round a given whole number.

Step 2: Find the digit that is located in that place. We call this digit the place digit. Then draw an “imaginary wall” to the right of the place digit.

Step 3: Look at the digit immediately to the right of the wall you drew in step 2. If this digit is 5 or more (5, 6, 7, 8 or 9), increase the place digit in step 2 by 1 and change all the digits to the right of the wall to zero. Otherwise, if this digit is less than 5 (0, 1, 2, 3, 4), leave the place digit in step 2 as it is, and make all the digits to the right of the wall zero.

We will now illustrate this method to round whole numbers.
**Example 1.2.1**

Suppose you want to round $34,781 to the nearest ten. Since 8 is in the tens place, we call 8 the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is 1 (it is less than 5), we leave the place digit 8 as is and make the 1 into a zero. Therefore, $34,781 rounded to the nearest ten is $34,780. This means that $34,781 is closer to $34,780 than to $34,790 when we count by tens.

Original number: $34,781
Rounded to the nearest ten: $34,780

**Example 1.2.2**

Suppose you want to round $49,750,269 to the nearest hundred thousand. Since 7 is in the hundred thousands place, we call 7 the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is 5 (it is 5 or greater), we add 1 to the 7, which now becomes an 8, and make all the digits to the right of the wall zero. Therefore, $49,750,269 rounded to the nearest hundred thousand is $49,800,000. This means that $49,750,269 is closer to $49,800,000 than to $49,700,000 when we count by hundred thousands.

Original number: $49,750,269
Rounded to the nearest hundred thousand: $49,800,000
Example 1.2.3

If we want to round $637 to the nearest hundred, we first notice that the digit 6 is in the hundreds place. Thus, we call 6 the place digit and draw a wall to the right of the place digit. Since the digit to the right of the wall is 3 (it is less than 5), we leave the 6 as is and make all the digits to the right of the wall zero. Therefore, $637 rounded to the nearest hundred is $600. This means that $637 is closer to $600 than to $700 when we count by hundreds.

Original number: $637
Rounded to the nearest hundred: $600

Example 1.2.4

Let’s round $5,930 to the nearest thousand. We notice that the digit 5 is in the thousands place. Thus, we call 5 the place digit and draw a wall to the right of the place digit. Since the digit to the right of the wall is 9 (it is 5 or greater), we add 1 to the 5, which now becomes 6, and make all the digits to the right of the wall zero. Therefore, $5,930 rounded to the nearest thousand is $6,000. This means that $5,930 is closer to $6,000 than to $5,000 when we count by thousands.

Original number: $5,930
Rounded to the nearest thousand: $6,000
Example 1.2.5

Let’s round $398,724 to the nearest ten thousand. We notice that in the ten thousands place there is a 9. Thus, we call 9 the place digit and draw a wall to the right of the place digit. Since the digit to the right of the wall is an 8 (it is 5 or greater), we add 1 to the 9, which now becomes 10, but there is no way to fit a 10 into a single place. So we add 1 to the number 39, which now becomes 40. We then turn all the digits to the right of the wall into zeros. Therefore, $398,724 rounded to the nearest ten thousand is $400,000. This means that $399,724 is closer to $400,000 than to $390,000 when we count by ten thousands.

Original number: $398,724
Rounded to the nearest ten thousand: $400,000

Example 1.2.6

Let’s round $2,399,954 to the nearest hundred. We notice that in the hundreds place there is a 9. Thus, we call 9 the place digit and draw a wall to the right of the place digit. Since the digit to the right of the wall is a 5 (it is 5 or greater), we add 1 to the 9, which now becomes 10, but there is no way to fit a 10 in a single place. So we add 1 to 23999 which now becomes 24000. We then turn all the digits to the right of the wall into zeros. Therefore, $2,399,954 rounded to the nearest hundred is $2,400,000. This means that $2,399,954 is closer to $2,400,000 than to $2,399,900 when we count by hundreds.

Original number: $2,399,954
Rounded to the nearest hundred: $2,400,000
Let’s round $38$ to the nearest hundred. We notice that in the hundreds place there is no digit. When this happens, we assume there is a zero in the hundreds place. Thus, we call 0 the place digit and draw a wall to the right of the place digit. Since the digit to the right of the wall is a 3 (it is less than 5), we leave the zero in the hundreds place as it is. We then turn all the digits to the right of the wall into zeros. Therefore, $38$ rounded to the nearest hundred is $0$. This means that $38$ is closer to $0$ than to $100$ when we count by hundreds.

Original number: $\$38 = \$038$

Rounded to the nearest hundred: $\$000 = \$0$

Example 1.2.8

Let’s round $53,414$ to the nearest hundred thousand. We notice that in the hundred thousands place there is no digit. When this happens, we assume there is a zero in the hundred thousands place. Thus, we call 0 the place digit and draw a wall to the right of the place digit. Since the digit to the right of the wall is a 5 (it is 5 or greater), we add 1 to the zero in the hundred thousands place, which now becomes 1. We then turn all the digits to the right of the wall into zeros. Therefore, $53,414$ rounded to the nearest hundred thousand is $100,000$. This means that $53,414$ is closer to $100,000$ than to $0$ when we count by hundred thousands.

Original number: $\$53,414 = \$053,414$

Rounded to the nearest hundred thousand: $\$100,000$
1. Round the number 38,946,018 to the nearest
   a. thousand 38,946,000
   b. hundred thousand 38,900,000
   c. ten 38,946,020
   d. million 39,000,000
   e. hundred 38,946,000
   f. ten thousand 38,950,000

2. Round the number 457,195 to the nearest
   a. hundred 457,200
   b. ten 457,200
   c. hundred thousand 500,000
   d. thousand 457,000
   e. million 0

3. Round the number 865,974,096 to the nearest
   a. hundred million 900,000,000
   b. hundred thousand 866,000,000
   c. hundred 865,974,100
   d. million 866,000,000
   e. ten thousand 865,970,000
   f. billion 1,000,000,000

4. Classwork 1.2

5. Homework 1.2

   1. Round the number 68,905,745 to the nearest
      a. thousand 68,906,000
      b. hundred thousand 68,900,000
      c. ten 68,905,750
      d. million 69,000,000
      e. hundred 68,905,700
      f. ten thousand 68,910,000
2. Round the number 8,997,095 to the nearest  
   a. hundred 8,997,100  
   b. ten 8,997,100  
   c. hundred thousand 9,000,000  
   d. thousand 8,997,000  
   e. million 9,000,000  

3. Round the number 502,915,746 to the nearest  
   a. hundred million 500,000,000  
   b. hundred thousand 502,900,000  
   c. hundred 502,915,700  
   d. million 503,000,000  
   e. ten thousand 502,920,000  
   f. ten 502,915,750
Section 1.3 – Adding Whole Numbers

Edgar will jog around the local park this Saturday morning. He would like to know the total distance that he will jog. To compute it, Edgar needs to determine the perimeter of the park. The perimeter of a closed figure is the total distance around it. Assuming that we know the length of each side of the rectangular park, we must add all the lengths to compute the perimeter.

To perform any addition (or subtraction) we must line up all the digits vertically so that digits in the same column have the same place (ones, tens, hundreds, etc.).

The park has a perimeter of 6,352 feet. This is the distance that Edgar will jog when he completes his exercise routine. Again, it is important to understand that when adding or subtracting whole numbers, the digits must be arranged vertically in a way that each column contains digits that are located in the same place.
**Note:** When adding whole numbers, you may add them in any order.

\[
12 + 5 + 7 = 5 + 12 + 7 = 7 + 12 + 5 = 24
\]

This is *not true* for subtraction.

---

**Example 1.3.1**

Find the perimeter of a park that has the following shape and dimensions:

The perimeter of the park is 4,313 yards.
**Example 1.3.2**

Compute the following sum: $423,560 + 74,288 + 1,430$.

Remember that the order in which you add the numbers is not important, but it is very important to line up their digits correctly according to their place.

\[
\begin{array}{ccccccc}
& 4 & 2 & 3 & 5 & 6 & 0 \\
& 7 & 4 & 2 & 8 & 8 & \\
+ & & 1 & 4 & 3 & 0 & \\
\hline
4 & 9 & 9 & 2 & 7 & 8 & \\
\end{array}
\]

Answer: $499,278$

**Example 1.3.3**

Compute the following sum: $9,989 + 87 + 7096$.

\[
\begin{array}{ccccccc}
& 9 & 9 & 8 & 9 \\
& 7 & 0 & 9 & 6 & \\
+ & & & & 8 & 7 & \\
\hline
1 & 7 & 1 & 7 & 2 & \\
\end{array}
\]

Answer: $17,172$
Perform each addition.

1. 
   \[ \begin{array}{c}
   9 \ 3 \ 7 \ 0 \\
   + \ 1 \ 5 \ 3 \ 8 \\
   \end{array} \]
   \[ \text{10,908} \]

2. 
   \[ \begin{array}{c}
   6 \ 9 \ 5 \ 1 \\
   1 \ 2 \ 0 \ 7 \\
   4 \ 9 \ 2 \ 3 \\
   + \ 8 \ 9 \ 0 \\
   \end{array} \]
   \[ \text{13,971} \]

3. 
   \[ \begin{array}{c}
   7 \ 1 \ 9 \ 0 \\
   8 \ 5 \ 4 \ 8 \\
   + \ 1 \ 5 \ 9 \ 6 \\
   \end{array} \]
   \[ \text{17,334} \]

4. 
   \[ \begin{array}{c}
   7 \ 8 \ 0 \ 1 \\
   2 \ 6 \ 5 \\
   7 \ 9 \ 5 \\
   + \ 2 \ 6 \\
   \end{array} \]
   \[ \text{8,887} \]

5. Perform the addition: $36,995 + $845 + $183,005 + $26 \quad \text{$220,871$}
Homework 1.3

Perform each addition.

1. 
   \[
   \begin{array}{ccccc}
   7 & 6 & 2 & 0 \\
   \hline
   & & & & \\
   9 & 2 & 9 & 6 \\
   \end{array}
   \]

   \[16,916\]

2. 
   \[
   \begin{array}{cccc}
   5 & 0 & 7 & 9 \\
   6 & 0 & 2 & 5 \\
   7 & 1 & 3 & 5 \\
   \hline
   & & & \\
   3 & 6 & 7 \\
   \end{array}
   \]

   \[18,606\]

3. 
   \[
   \begin{array}{cccc}
   6 & 9 & 9 & 2 \\
   4 & 5 & 3 & 4 \\
   \hline
   & & & \\
   1 & 7 \\
   \end{array}
   \]

   \[11,543\]

4. 
   \[
   \begin{array}{cccc}
   1 & 6 & 2 & 2 \\
   9 & 2 & 6 & 9 \\
   8 & 3 & 2 \\
   \hline
   & & & \\
   8 & 2 & 6 \\
   \end{array}
   \]

   \[12,549\]

5. Perform the addition: $341 + $562,092 + $32,910 + $488 \[595,831\]
In the previous section you learned how to add whole numbers. In this section, we will go over subtraction of whole numbers. When we subtract, we find the difference between two quantities. Suppose one house sells for $397,800 and another sells for $316,400. If we want to determine the difference in price between the two houses, we must subtract the larger amount minus the smaller amount. In this section, we will review the process of subtraction. Although we still have to line up the digits according to their place just like we did for addition, there are instances when we have to “borrow” to be able to complete the subtraction process.

**Example 1.4.1**

A house sells for $397,800 and another sells for $316,400. Find the difference in price between these houses.

The order in which you subtract the two numbers is important. In this context, you must subtract (take away) the smaller amount from the larger amount. Another way to say this is “larger amount minus smaller amount.”

You begin by lining the digits according to their place. Then, we subtract the digits column by column starting with the column on the far right.

\[
\begin{array}{cccccc}
3 & 9 & 7 & 8 & 0 & 0 \\
- & 3 & 1 & 6 & 4 & 0 \\
\hline
0 & 8 & 1 & 4 & 0 & 0
\end{array}
\]

Answer: $81,400

The more expensive house costs $81,400 more. Notice that to perform this subtraction, we did not have to borrow because each digit in the top row is equal to or greater than the digit below it.
Example 1.4.2
Sally received 2,876 votes in the school election for treasurer, while Tim received 1,741 votes. How many more votes did Sally receive than Tim?

We must subtract (take away) the smaller amount from the larger amount.

After lining up the digits according to their place, we subtract the digits column by column, starting with the column on the far right.

\[
\begin{array}{cccc}
2 & 8 & 7 & 6 \\
- & 1 & 7 & 4 & 1 \\
\hline
1 & 1 & 3 & 5 \\
\end{array}
\]

Answer: Sally received 1,135 more votes than Tim.

Again, notice that we did not have to borrow because in each column, the top digit was always equal or greater than the digit in the bottom.

Example 1.4.3
Robert’s initial annual salary was $35,167? He now earns $47,380 per year. How much more does Robert earn per year now that he did initially.

We must subtract (take away) the smaller amount from the larger amount. This is the same as “larger minus smaller.”

After lining up the digits according to their place, we subtract the digits column by column, starting with the column on the far right. However, you will notice that we will have to borrow because zero is less than seven (0 < 7).

\[
\begin{array}{cccc}
4 & 7 & 3 & 8 & 0 \\
- & 3 & 5 & 1 & 6 & 7 \\
\hline
\end{array}
\]

The 0 needs to “borrow” from the 8 because 0 is less than 7.
The 0 borrows one ten from the tens place. That is, 0 borrows a 1 from the 8 because 8 is in the tens place and represents 8 ten dollar bills. The 8 now becomes 7. We then write 10 ones and ignore the zero, and write 7 tens and ignore the 8 (see the red numbers below). After rewriting the subtraction problem as shown below, we then proceed with the subtraction:

\[
\begin{array}{cccccc}
7 & 10 \\
4 & 7 & 3 & 8 & 0 \\
- & 3 & 5 & 1 & 6 & 7 \\
1 & 2 & 2 & 1 & 3 \\
\end{array}
\]

Answer: Robert now earns $12,213 more per year than what he initially earned per year.

**Example 1.4.4**

Perform the following subtraction: $563,871 − $402,736

After lining up the digits according to their place, you will notice that we have to borrow because 1 is less than 6 (1 < 6).

The 0 borrows one ten from the tens place. That is, the 1 borrows a 1 from the 7 because 7 is in the tens place and represents 7 ten dollar bills. The 7 now becomes a 6. What we actually write is 11 one dollar bills and we ignore the 1, and write 6 ten dollar bills and ignore the 7 (see the red numbers below). After rewriting the subtraction problem as shown below, we then proceed with the subtraction process:
Subtract $25,368 from $73,459

This subtraction statement means that we must place the $73,459 in the top row and $25,368 in the bottom row. After lining up the digits according to their place, you will notice that we have to borrow more than once because some of the digits in the top row are smaller than the corresponding digits in the bottom row.

\[
\begin{array}{c}
\text{3} & \text{15} \\
\text{7} & \text{3} & \text{4} & \text{5} & \text{9} \\
- & \text{2} & \text{5} & \text{3} & \text{6} & \text{8} \\
\hline
\text{0} & \text{9} & \text{1}
\end{array}
\]

Answer: $161,135

Example 1.4.5

Subtract $25,368 from $73,459

Example 1.4.6

Subtract $391,872 from $2,173,009

\[
\begin{array}{c}
\text{2} & \text{9} & \text{10} \\
\text{2} & \text{1} & \text{7} & \text{3} & \text{0} & \text{0} & \text{9} \\
- & \text{3} & \text{9} & \text{1} & \text{8} & \text{7} & \text{2} \\
\hline
\text{1} & \text{1} & \text{3} & \text{7}
\end{array}
\]

Since 0 < 7, the 0 “borrows” from the 0 to its left, but only after that zero has borrowed a 1 from the 3. That way, the zero in the hundreds place will have something to give to the zero in the tens place.
Let’s do one more example that involves multiple borrowing.

Example 1.4.7

The distance from Los Angeles to Chicago is approximately 1,739 miles. The distance from Los Angeles to Washington D.C. is approximately 2,668 miles. How many miles is Washington D.C. farther from Los Angeles than Chicago?

Answer: $1,781,137
Perform each subtraction.

1. \[
\begin{array}{c}
8 & 9 & 2 & 5 \\
- & 7 & 3 & 1 & 0 \\
\end{array}
\]

2. \[
\begin{array}{c}
7 & 3 & 6 & 4 \\
- & 5 & 3 & 8 \\
\end{array}
\]

3. \[
\begin{array}{c}
7 & 1 & 9 & 0 & 1 \\
- & 1 & 5 & 9 & 6 & 2 \\
\end{array}
\]

4. \[
\begin{array}{c}
9 & 0 & 0 & 5 & 7 & 1 \\
- & 7 & 2 & 6 & 8 & 8 \\
\end{array}
\]

Since 6 < 7, the 6 “borrows” a 1 from the 2 to its left. The 6 becomes 16 and the 2 becomes 1.

Answer: Washington D.C. is 929 miles farther from Los Angeles compared to the distance from Los Angeles to Chicago.
5. Subtract $73,860 from $107,500. $33,640
6. Compute $375,236 minus $186,509. $188,727

**Homework 1.4**

Perform each subtraction.

1. 
   
   \[
   \begin{array}{r}
   6 5 9 9 \\
   \hline
   4 2 9 6
   \end{array}
   \]
   
   2,303

2. 
   
   \[
   \begin{array}{r}
   8 4 0 5 \\
   \hline
   5 1 4 7
   \end{array}
   \]
   
   3,258

3. 
   
   \[
   \begin{array}{r}
   4 4 1 9 5 \\
   \hline
   1 7 5 7 5
   \end{array}
   \]
   
   26,620

4. 
   
   \[
   \begin{array}{r}
   1 3 4 5 8 \\
   \hline
   9 6 7 9
   \end{array}
   \]
   
   3,779

5. Subtract $3,658 from $25,000. $21,342
6. Compute $854,071 minus $594,263. $259,808
Section 1.5 – Multiplying Whole Numbers

A real-life application of the statement $2 \times 9 = 18$ could be that there are two boxes of nine pencils per box, which means there is a total of 18 pencils. This makes sense because $9 + 9 = 18$. Moreover, the statement $2 \times 9 = 18$ could also mean that there are nine boxes of 2 pencils per box. Again, there is a total of 18 pencils because

$$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 18$$

Similarly, the statement $15 \times 20 = 300$ can be interpreted as saying that there are fifteen groups of twenty items in each group, or 20 groups consisting of 15 items each. In both cases there is a total of 300 items because adding 20 fifteen times is equivalent to adding 15 twenty times. That is, $15 \times 20 = 20 \times 15 = 300$. This means that we can multiply any numbers (whole numbers, fractions and decimals) in any order we prefer because the result, or product, will be the same.

Multiplication is equivalent to repeated addition. You can construct the times tables by counting by ones, then counting by twos, then counting by threes, and so on. This gives the times table which you are familiar with, and that is shown below. You should memorize the times table below to obtain computations involving multiplication and division faster, and to avoid making simple multiplication mistakes.

![Times Table](https://source.com)
The website above has expanded times tables, in case you need to see the times table for 13, 14, etc.

Let’s look at some examples involving multiplication of whole numbers.

**Example 1.5.1**

Richard earns $15 per hour. He worked 36 hours this week. How much did Richard earn this week?

When you multiply numbers, the order in which you multiply them does not matter. The long way to answer this question is to add $15 thirty-six times, since multiplication represents repeated addition. That is, $15 + $15 + $15 + $15 + ... but that would be too much work. Instead, we can obtain the answer much faster by multiplying 15 by 36 through the process described below.

When multiplying whole numbers, it is a good idea to line up the digits according to their place, but this is not required. This is done to make the multiplication process look more organized. The objective is to multiply all the digits in the bottom row by all the digits in the top row by following the process shown below.

\[
\begin{array}{c}
3 \\
15 \\
\times \\
36 \\
\hline \\
90 \\
115 \\
\times \\
3 \\
\hline \\
450 \\
540 \\
\end{array}
\]

We begin with the right-most digit on the second row (6) and multiply 6 times each digit on the top row starting with 5, then the 1. Since the product of 6 and 5 has two digits (30), we place the 0 below the 6 and the 3 above the 1 in the tens place. We then multiply 6 times 1 and add the 3 to the answer. 6 times 1 equals 6, and plus 3 gives 9.

We now multiply the next digit in the second row (3) times 5 and then times 1. Since 3 times 5 equals 15, we place the 5 below the 9 and the 1 above the 3. We then multiply 3 times 1 and then we add the 1 that we placed above (in red). Finally, we add the last two rows. If you want, you may add zeros in the empty spots to make the addition look clearer.

**Answer:** Richard earned $540 this week.
If Cindy pays $1,758 per month on her combined car and house expenses, how much will she pay in the next 24 months?

\[
\begin{array}{c}
323 \\
1758 \\
\times \\
24 \\
\hline
7032 \\
\end{array}
\]

First, we write the number with the most digits in the top row. Then, starting with the right-most digit on the second row (4), we multiply 4 times each digit on the top row starting with 8, then the 5 and so on. Since the product of 4 and 8 has two digits (32), we place the 2 below the 4 and the 3 above the 5. We then multiply 4 times 5 and add the 3 to the answer. 4 times 5 equals 20, and plus 3 gives 23. We place the 3 below the 2 and the 2 above the 7. We then follow the same process when we multiply 4 times 7 and 4 times 1.

\[
\begin{array}{c}
111 \\
1758 \\
\times \\
24 \\
\hline
7032 \\
\end{array}
\]

We now multiply the 2 in the second row times each digit in the top row. Finally, we add the last two rows.

Answer: Cindy will spend a total of $42,192 on car and house expenses in the next 24 months (2 years).

Find the product of 4,987 and 301.

\[
\begin{array}{c}
4987 \\
\times \\
301 \\
\hline
4987 \\
\end{array}
\]

First, we write the number with the most digits in the top row. Then, starting with the right-most digit on the second row (1), we multiply the 1 times each digit in the top row.
Students who have memorized the times table tend to answer questions involving multiplication faster. Also, knowing the times tables by memory makes it easier to perform long division, as you will see in the next section.

There are online tools that can help you to memorize the times table. One of them is the following applet:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=155

Please visit this website, or seek help from your instructor, if you need help memorizing the times table.
Perform each multiplication.

1. \[
\begin{array}{c}
185 \\
\times 29 \\
\end{array}
\]
   \[5,365\]

2. \[
\begin{array}{c}
7039 \\
\times 612 \\
\end{array}
\]
   \[4,307,868\]

3. \[
\begin{array}{c}
94 \\
\times 78 \\
\end{array}
\]
   \[7,332\]

4. \[
\begin{array}{c}
827 \\
\times 68 \\
\end{array}
\]
   \[56,236\]

5. \[
\begin{array}{c}
4397 \\
\times 158 \\
\end{array}
\]
   \[694,726\]

6. Multiply \(65 \times 398\).
   \[25,870\]

7. Multiply \(997 \times 34\).
   \[33,898\]

8. Multiply \(152 \times 706\).
   \[107,312\]
Perform each multiplication.

1. 
   \[
   \begin{array}{c}
   5 \ 7 \\
   \times \ 2 \ 6 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   1,482 \\
   \end{array}
   \]

2. 
   \[
   \begin{array}{c}
   1 \ 3 \ 9 \\
   \times \ 7 \ 6 \ 0 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   105,640 \\
   \end{array}
   \]

3. 
   \[
   \begin{array}{c}
   7 \ 9 \ 9 \\
   \times \ 3 \ 1 \ 4 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   250,886 \\
   \end{array}
   \]

4. 
   \[
   \begin{array}{c}
   6 \ 8 \ 0 \ 4 \\
   \times \ 3 \ 7 \ 4 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   2,544,696 \\
   \end{array}
   \]

5. 
   \[
   \begin{array}{c}
   9 \ 4 \ 2 \ 6 \\
   \times \ 8 \ 7 \ 5 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   8,247,750 \\
   \end{array}
   \]

6. Multiply $687 \times 42$.
   \[
   28,854
   \]

7. Multiply $45,200 \times 67$.
   \[
   3,028,400
   \]

8. Multiply $91 \times 3,172$.
   \[
   288,652
   \]
Section 1.6 – Dividing Whole Numbers

Let’s begin by looking at an example that involves division of whole numbers. Dale works at a Farmer’s Market. There are 245 apples that he needs to put in boxes of 8 apples each. How many full boxes can he make, and how many apples will be left over?

To answer this question, we begin by imagining that we have already placed 8 apples in the first box. This means we now have $245 - 8 = 237$ apples. If we place 8 apples in the second box, we will now have only $237 - 8 = 229$ apples. By subtracting this way we will be able to determine the number of full boxes that can be made, and the number that will be left over. However, this process would take too long. Instead, we will use division to answer the question because division represents repeated subtraction. What we need to do is to divide the 245 apples by groups of 8. This is written as

\[
245 \div 8 \quad \text{“245 divided by 8”}
\]

| \[ \begin{array}{c}
245 \\
8 \\
\end{array} \] | \[ \begin{array}{c}
\text{“245 divided by 8”} \\
\text{“245 divided by 8”} \\
\text{“245 divided by 8”} \\
\end{array} \] |
|---|---|

Note the location in which the 245 and the 8 were placed. If you are given the expression $245 \div 8$, the 245 must be placed inside the division box and the 8 outside. This placement has nothing to do with the fact that 245 is greater than 8.

To perform the division, we look at the leftmost digit of 245 (it’s 2) and notice that the maximum number of full boxes of 8 apples that can be made with 2 apples is zero. We indicate this as follows:

\[
0 \\
8 \big| 245
\]

We then consider the two leftmost digits of 245 (together they form a 24). We then determine that the number of full boxes containing 8 apples each, that can be made with 24 apples, is 3 since $8 \times 3 = 24$. We express this as
We bring down the last digit of 245 (it’s a 5) and notice that the number of full boxes of 8 apples that can be made with 5 apples is zero. We express this as follows

\[
\begin{array}{c}
&0 & 3 \\
8 & \underline{2} & 4 & 5 \\
- & \underline{2} & 4 \\
0 & 0
\end{array}
\]

Notice that 030 = 30. We define 30 to be the quotient, 8 is the divisor, 245 is the dividend and 5 is the remainder.

Answer: Dale can make 30 full boxes of 8 apples each, and there will be 5 apples left over.

**Example 1.6.1**

An employer will distribute $7,344 evenly among his 6 workers. How much will each worker receive?

To “distribute evenly” means to divide $7,344 by 6.
We notice that if there were only $7 and 6 workers, each worker would get $1 and there would be $1 left over. In other words, 6 fits a maximum of 1 time into 7, since 6x1=6 but 6x2=12 is bigger than 7.

We bring down the next digit (3). We notice that 6 fits a maximum of 2 times into 13, since 6x2=12 but 6x3=18 is bigger than 13.

We bring down the next digit (4). We notice that 6 fits a maximum of 2 times into 14, since 6x2=12 but 6x3=18 is bigger than 14.

We bring down the last digit (4). We notice that 6 fits a maximum of 4 times into 24, since 6x4=24 but 6x5=30 is bigger than 24. The zero remainder means that the money was distributed evenly among the 6 workers and there was no money left over.

Answer: By distributing $7,344 evenly among the 6 workers, each worker will receive $1,224. The zero remainder means that $7,344 can be divided equally among 6 people as a full dollar amount. That is, everyone will get an amount in full dollars and no cents.
Example 1.6.2

Perform the following division: \( \frac{4065}{15} \)

\[
\begin{array}{c|ccccc}
  & 0 & 2 & 7 & 1 \\
\hline
1 & 5 & 4 & 0 & 6 & 5 \\
- & 3 & 0 & 1 & 0 & 6 \\
\hline
  & 1 & 0 & 5 \\
- & 1 & 5 & 0 \\
\hline
  & 1 & 5 \\
- & 1 & 5 & 0 \\
\hline
  & 0 \\
\end{array}
\]

We notice that 15 does not fit into 4 at all, so we consider the first two digits (40) instead. We know that 15 fits a maximum of 2 times into 40, since \(15 \times 2 = 30\) but \(15 \times 3 = 45\) is bigger than 40.

Next, we bring down the 6. Counting by 15's or from a multiplication table you should notice that 15 fits a maximum of 7 times into 106, since \(15 \times 7 = 105\) but \(15 \times 8 = 120\) which is bigger than 106.

Finally, bring down the last digit (5). We notice that 15 fits a maximum of 1 time into 15, since \(15 \times 1 = 15\) but \(15 \times 2 = 30\). After subtracting, we get a zero remainder. This means that 15 fits exactly 271 times into 4,065 because \(15 \times 271 = 4,065\).

Answer: \( \frac{4065}{15} = 271 \)

Remember that 0271 = 271.
Example 1.6.3

Perform the following division: \[ \frac{734,904}{8} \]

This expression means \( 734,904 \div 8 \) as well as \[ 8 \overline{7 3 4 9 0 4} \]

We notice that 8 does not fit into 7 at all, so we consider the first two digits (73) instead. We know that 8 fits a maximum of 9 times into 73 because \( 8 \times 9 = 72 \). After subtracting, we bring down the 4 and notice that 8 fits a maximum of 1 time into 14, since \( 8 \times 1 = 8 \) but \( 8 \times 2 = 16 \) which is greater than 14.

We bring down the 9 and notice that 8 fits a maximum of 8 times into 69, since \( 8 \times 8 = 64 \) but \( 8 \times 9 = 72 \) which is greater than 69. We then subtract 64 from 69.

We bring down the 0 and notice that 8 fits a maximum of 6 times into 50, since \( 8 \times 6 = 48 \) but \( 8 \times 7 = 56 \) which is greater than 50. We then subtract 48 from 50.
At a fundraiser ever, a total of $3,360 were raised. If 32 people attended the event and contributed the same amount, how much did each person contribute?

Finally, we bring down the last digit (4) and notice that 8 fits a maximum of 3 times into 24, since 8x3=24 but 8x4=32 which is greater than 24. We then subtract. We are done. The dividend is 734,904, the divisor is 8 the quotient is 91,863 and the remainder is 0.

Answer: \( \frac{734,904}{8} = 91,863 \)

Example 1.6.4

At a fundraiser ever, a total of $3,360 were raised. If 32 people attended the event and contributed the same amount, how much did each person contribute?

Since 32 does not fit into 3, we consider the first two digits (33) instead. We know that 32 fits a maximum of 1 time into 33 because 32x1=32 but 32x2=64 which is bigger than 33. We then subtract 32 from 33.
After going over these questions involving division, we hope that the student understands the importance of knowing the times table by memory. Knowing the times table allowed us to perform division without getting stuck on the multiplication steps.

Next, we bring down the 6 but notice that 32 does not fit into 16 at all. We must indicate this by writing a 0 in the quotient. We bring down the next digit (0) and consider 160. Counting by 32’s or by multiplication, we determine that 32 fits a maximum of 5 times into 160. We are done. The dividend is 3,360, the divisor is 32 the quotient is 105 and the remainder is 0.

Answer: Each person who attended the fundraiser contributed $105.

After going over these questions involving division, we hope that the student understands the importance of knowing the times table by memory. Knowing the times table allowed us to perform division without getting stuck on the multiplication steps.

**Classwork 1.6**

Perform each division. Identify the dividend, divisor, quotient and remainder.

1. \[2 \overline{9 6 4 3 7}\] 48218 R1

2. \[7 \overline{5 6 4 9}\] 807 R0

3. \[1 3 \overline{7 5 1 0}\] 577 R9

4. \[2 7 \overline{9 0 7 6 8}\] 3361 R21
5. Divide: $378,056 \div 19$  
   $19,897$ R13

6. Divide: $71,575 \div 45$  
   $1,590$ R25

7. Divide: $\frac{85,932}{3}$  
   $28,644$ R0

8. Divide: $\frac{792,580}{20}$  
   $39,629$ R0

**Homework 1.6**

Perform each division. Identify the dividend, divisor, quotient and remainder.

1. $15721$ R0

2. $536$ R3

3. $348$ R10

4. $2111$ R14

5. Divide: $49,108 \div 27$  
   $1,818$ R22

6. Divide: $95,624 \div 6$  
   $15,937$ R2

7. Divide: $\frac{3,528}{8}$  
   $441$ R0

8. Divide: $\frac{25,172}{7}$  
   $3,596$ R0
Section 1.7 – Applications Involving Whole Numbers

In this section, we will discuss applications involving addition, subtraction, multiplication, and division of whole numbers. The challenge in solving word problems often lies in determining what operation is most suitable to use to answer a particular question. A strategy to solve word problems involves identifying certain keywords that can give you an idea what operation (+, −, × or ÷) to use. Below is a list of keywords that often appear in word problems, and the corresponding operation that should be used.

<table>
<thead>
<tr>
<th>Keyword / Phrase</th>
<th>Operation to Perform</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>addition</td>
</tr>
<tr>
<td>how much</td>
<td>addition</td>
</tr>
<tr>
<td>sum</td>
<td>addition</td>
</tr>
<tr>
<td>cumulative</td>
<td>addition</td>
</tr>
<tr>
<td>together/altogether</td>
<td>addition</td>
</tr>
<tr>
<td>combined</td>
<td>addition</td>
</tr>
<tr>
<td>perimeter</td>
<td>addition</td>
</tr>
<tr>
<td>in all</td>
<td>addition</td>
</tr>
<tr>
<td>difference</td>
<td>subtraction</td>
</tr>
<tr>
<td>how much more</td>
<td>subtraction</td>
</tr>
<tr>
<td>how many more</td>
<td>subtraction</td>
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<tr>
<td>withdrew</td>
<td>subtraction</td>
</tr>
<tr>
<td>by how much</td>
<td>subtraction</td>
</tr>
<tr>
<td>deduct</td>
<td>subtraction</td>
</tr>
<tr>
<td>decrease by</td>
<td>subtraction</td>
</tr>
<tr>
<td>more than / less than</td>
<td>subtraction</td>
</tr>
<tr>
<td>of</td>
<td>multiplication</td>
</tr>
<tr>
<td>times</td>
<td>multiplication</td>
</tr>
<tr>
<td>product</td>
<td>multiplication</td>
</tr>
</tbody>
</table>
We will now look at some real-life applications of the four operations mentioned above. We will discuss how to go about determining the most suitable operation to use, as well as show all the important steps in the calculations.

**Example 1.7.1**

Eddie earned $2,356 in January, $2,108 in February, $2,625 in March and $2,276 in April. How much did Eddie earn in these four months?

The keyword “combined” indicates that we must find a total amount. Therefore, the appropriate operation to use is addition. To add the whole numbers, we must line up the digits vertically according to their place.

<table>
<thead>
<tr>
<th>find the area</th>
<th>multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>at this rate</td>
<td>multiplication</td>
</tr>
<tr>
<td>per</td>
<td>multiplication</td>
</tr>
<tr>
<td>(a unit rate is given)</td>
<td>multiplication</td>
</tr>
<tr>
<td>each</td>
<td>multiplication</td>
</tr>
<tr>
<td>(a unit rate is given)</td>
<td>multiplication</td>
</tr>
<tr>
<td>distribute</td>
<td>division</td>
</tr>
<tr>
<td>divide</td>
<td>division</td>
</tr>
<tr>
<td>cut into</td>
<td>division</td>
</tr>
<tr>
<td>separate/group into</td>
<td>division</td>
</tr>
<tr>
<td>per</td>
<td>division</td>
</tr>
<tr>
<td>(asked to find a unit rate)</td>
<td>division</td>
</tr>
<tr>
<td>each</td>
<td>division</td>
</tr>
<tr>
<td>(asked to find a unit rate)</td>
<td>division</td>
</tr>
</tbody>
</table>

Answer:

Eddie earned $9,365 in the four months combined.
A high school basketball team is holding a fundraiser to buy new uniforms and shoes. So far they have been able to raise $974. If they need to raise $1,803 how much more do they need to raise to reach their target?

The keywords here are “how much more,” which indicate that we must subtract the larger amount from the smaller amount. By doing so, we will be able to determine the difference between the two amounts, which will tell us how much more money needs to be raised to reach the target of $1,803.

Example 1.7.2

<table>
<thead>
<tr>
<th>1</th>
<th>8</th>
<th>0</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

Answer:

Having raised $974 so far, the basketball team needs to raise $829 more to reach their target of $1,803.

Example 1.7.3

A movie theater sold $6,552 in tickets last night. If each ticket costs $18, how many people bought a ticket?

The keyword here is “each” and it gives us a hint that we must divide the total amount of $6,552 by $18 to determine the number of moviegoers last night. A longer process to answer the question would be to subtract $18 from $6,552 repeatedly to see how many times $18 “fits” into $6,552. We will perform the division $6,552 ÷ $18 which allows us to obtain the answer more expeditiously.
David works as a teaching assistant. His salary is $17 per hour. If he works 16 hours per week, how much will he earn in 20 weeks?

The keywork here is “per,” which is used to represent a rate. In this case it is an hourly rate. Since we know the number of hours that David works per week, we can determine the total amount he earns in one week by multiplying $17 \times 16 = 272$. Thus, he earns $272 per week. To determine how much he will earn if he works 20 weeks, we multiply $272 \times 20 = 5,440$. David will earn $5,440 working 20 weeks as a teaching assistant.

Example 1.7.5

Suppose that we need to cut a rope that is 624 inches long into 13 pieces of equal length. How long should each piece of rope be?

Answer:
A total of 364 tickets were sold, which also represents the number of moviegoers last night.
The keywords here are “cut … into … pieces of equal length.” To cut something into pieces of equal length means to distribute or divide evenly. Therefore, the appropriate operation to use is division.

\[
\begin{array}{c}
4 \\
3 \\
- 104 \\
- 104 \\
\hline
0
\end{array}
\]

**Answer:**

Each piece of rope should measure 48 inches.

---

**Classwork 1.7**

1. Mr. Jones planted 12,856 square meters of wheat and 4,953 square meters of corn. How much more wheat did he plant than corn.

   *Mr. Jones planted 7,903 m² more of wheat than of corn.*

2. If a rocket is moving at a rate of 17,894 miles per hour, how far will it travel in 5 hours?

   *The rocket will travel 89,470 miles in 5 hours.*

3. If Steven had $12,000 in his checking account but withdrew $3,714 to pay off a debt, how much does he have left in his checking account?

   *Steven has $8,286 left in his checking account.*

4. Eric is a member of the track and field team in his school. If he ran a total of 240 miles in 60 days, how many miles did he average per day?

   *Eric averaged 4 miles per day.*

5. Find the perimeter of a triangle whose sides measure 3,267 inches, 5,431 inches and 4,299 inches.

   *The perimeter of the triangle is 12,997 inches.*

6. If the average attendance to a Dodgers game each of the last 5 days was 41,387 what was the total attendance the last 5 days?

   *The total attendance the last five days was approximately 206,935.*

7. Buying airtime in the radio station KNWS costs $5,372 per minute and buying airtime in KSND costs $4,721 per minute. If a company buys 20 minutes of airtime in each station, how much more will the company have to pay KNWS than KSND?

   *The company will pay $13,020 more to KNWS for the same amount of airtime.*
8. Grace is ordering new office equipment. If she ordered a new computer for $899, a printer for $432 and a toner cartridge for $79, what is the total cost of the new office equipment?

The total cost of the new office equipment is $1,410.

**Homework 1.7**

1. Ted can jog 36 miles in 4 minutes. At that rate, how many miles can he jog in one minute?

Ted can jog 9 miles in 1 minute.

2. John currently has $8,567 left in his checking account after spending $2,715. How much did John had in his checking account originally?

John originally had $11,282 in his checking account.

3. Natalie has 372 apples which she needs to place in boxes of 14 apples each. How many full boxes can she make, and how many apples will be left over?

Natalie can make 26 boxes of 14 apples each. There will be 8 apples left over.

4. Find the perimeter of a square whose sides measure 3,716 km each.

The perimeter of the square is 14,864 kilometers.

5. The Eiffel Tower is 1,063 feet high, while the height of the Petronas Towers is 1,483 feet high. By how much are the Petronas Towers higher than the Eiffel Tower?

The Petronas Towers are 420 feet higher than the Eiffel Tower.

6. Robert needs to reach at least 30,000 flight miles to receive a discount on plane tickets. If Robert has flown 23,769 miles so far, how many more flight miles does he need to accumulate to reach 30,000 miles?

Robert needs to fly at least 6,231 more miles to reach 30,000 flight miles.

7. A new LED TV currently costs $1,300 when paid in full. There is also a payment plan available where the customer must pay a monthly payment of $72 for 2 years. How much more will a customer who pays in monthly installments end up paying for the LED TV than a customer who pays the $1,300 in full?

The customer who pays in monthly installments will pay $428 more for the LED TV.

8. Nancy will receive a $25 commission for each new washer or dryer she sells. How many washers or dryers must she sell to reach $1,000 in commission?

Nancy must sell 40 washers or dryer to reach $1,000 in commission.


Chapter 1 Test
Answer the following questions for the number 372,019:

1. Determine which digit is in the following place:
   a. Hundreds place 0
   b. Ones place 9
   c. Ten thousands place 7
   d. Hundred thousands place 3
   e. Tens place 1

2. Determine the place value of the following digits:
   a. 2 2,000
   b. 9 9
   c. 3 300,000
   d. 7 70,000
   e. 0 0
   f. 1 10

3. Round the number 7,425,399 to the specified place:
   a. Hundred thousands 7,400,000
   b. Tens 7,425,400
   c. Thousands 7,425,000
   d. Millions 7,000,000
   e. Ten millions 10,000,000
   f. Ten thousands 7,430,000

4. Round the number 247,995 to the specified place:
   a. Tens 248,000
   b. Thousands 248,000
   c. Millions 0
   d. Hundreds 248,000
   e. Ten thousands 250,000
Perform each operation.

5. 
\[
\begin{array}{c}
7 & 4 & 2 & 1 \\
9 & 3 & 5 & 5 \\
1 & 2 & 0 & 8 \\
\hline
+ & 6 & 3 \\
\end{array}
\]
\[
18,047
\]

6. 
\[
\begin{array}{c}
2 & 7 & 7 & 6 & 6 \\
8 & 9 & 1 & 1 & 4 \\
5 & 3 & 8 & 9 \\
\hline
+ & 5 & 3 & 2 \\
\end{array}
\]
\[
122,801
\]

7. 
\[
\begin{array}{c}
4 & 2 & 0 & 1 & 3 \\
2 & 5 & 1 & 6 & 4 \\
8 & 2 & 4 & 7 \\
\hline
+ & 7 & 5 & 4 & 9 \\
\end{array}
\]
\[
82,973
\]

8. Add: \(1,367,533 + 7,084 + 32,490\) \\
\[
1,407,107
\]

9. 
\[
\begin{array}{c}
2 & 7 & 6 & 8 \\
- & 1 & 3 & 4 & 8 \\
\end{array}
\]
\[
1,420
\]

10. 
\[
\begin{array}{c}
9 & 3 & 6 & 0 & 3 \\
- & 4 & 6 & 5 & 7 & 8 \\
\end{array}
\]
\[
47,025
\]

11. 
\[
\begin{array}{c}
3 & 0 & 4 & 7 & 0 \\
- & 8 & 5 & 3 & 2 \\
\end{array}
\]
\[
21,938
\]

12. Subtract 34,579 from 50,000. \\
\[
15,421
\]

13. 
\[
\begin{array}{c}
5 & 2 & 3 \\
\times & 4 & 7 \\
\end{array}
\]
\[
24,581
\]

14. 
\[
\begin{array}{c}
6 & 7 & 9 & 0 & 1 \\
\times & 3 & 5 \\
\end{array}
\]
\[
2,376,535
\]

15. 
\[
\begin{array}{c}
4 & 9 & 6 \\
\times & 7 & 8 & 3 \\
\end{array}
\]
\[
388,368
\]

16. Multiply: \(375 \times 1000\) \\
\[
375,000
\]

17. 
\[
8 \longdiv{5231}
\]
\[
653 \text{ R7}
\]

18. 
\[
23 \longdiv{67834}
\]
\[
2949 \text{ R7}
\]

19. \(37,569 \div 7\) \\
\[
5367 \text{ R0}
\]

20. Divide: \(\frac{632322}{9}\) \\
\[
70258 \text{ R0}
\]
21. On average, Dave can type 48 words per minute. How many words can he type in 12 minutes?

22. Thelma needs to place 325 chocolates in boxes of 12 each. How many full boxes of chocolates can Thelma make? How many chocolates will be left over?

Thelma can make 27 full boxes of 12 chocolates per box. There will be 1 chocolate left.

23. Anthony’s earnings the last four months were as follows:

M1: $3,075  M2: $2,925  M3: $3,682  M4: $2,994

How much did Anthony earn in total the last four months?

Anthony earned $12,676 in the last four months.

24. Ben has paid $137,825 of a $150,000 loan. What is the remaining balance on this loan?

The remaining balance on the loan is $12,175.

25. Celine had $7,500 in her checking account. She then wrote a check for $2,938. How much money is left in her checking account?

Celine has $4,562 left in her checking account.
Chapter 2

Factors, Exponents and Square Roots
Chapter 2 Overview

By the end of this chapter, you will achieve mastery of the following concepts:

- **Factors**
- **Multiples of a Number**
  
  Since \(5 \times 14 = 70\), we call 5 and 14 factors of 70. This means that 5 and 14 divide 70 evenly because the remainder is zero.

  Since 5 and 14 divide 70 evenly, 70 is divisible by 5 and 14. Therefore, 70 is a multiple of 5 and 14.

- **Rules of Divisibility**
  
<table>
<thead>
<tr>
<th>Divisible by</th>
<th>Condition</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>All whole numbers are divisible by 1.</td>
<td>0,1,2,3,4,5,6,7,8,9,10,11,12,13,...</td>
</tr>
<tr>
<td>2</td>
<td>If the number is even.</td>
<td>0,2,4,6,8,10,12,14,16,18,20,22,...</td>
</tr>
<tr>
<td>3</td>
<td>If the sum of the digits is divisible by 3.</td>
<td>0,3,6,9,12,15,18,21,24,27,30,33,...</td>
</tr>
<tr>
<td>4</td>
<td>If the 2 rightmost digits are divisible by 4.</td>
<td>0,4,8,12,16,20,24,28,32,36,40,44,...</td>
</tr>
<tr>
<td>5</td>
<td>If the number ends with a 5 or 0.</td>
<td>0,5,10,15,20,25,30,35,40,45,50,...</td>
</tr>
<tr>
<td>6</td>
<td>If the number is divisible by 2 and 3.</td>
<td>0,6,12,18,24,30,36,42,48,54,60,...</td>
</tr>
<tr>
<td>9</td>
<td>If the sum of the digits is divisible by 9.</td>
<td>0,9,18,27,36,45,54,63,72,81,90,...</td>
</tr>
<tr>
<td>10</td>
<td>If the number ends with 0</td>
<td>0,10,20,30,40,50,60,70,80,90,100,...</td>
</tr>
</tbody>
</table>

- **Prime Factorization**
  
  \(120 = 2 \times 2 \times 2 \times 3 \times 5\)

  The prime factorization of a number is a representation of the number using prime factors that multiplied together give the original number.

- **GCF**
- **LCM**
  
  The GCF of a set of numbers is the largest number that divides all of the numbers evenly. 8 is the GCF of 24 and 40.

  The LCM is the smallest multiple that a set of numbers have in common. 120 is the LCM of 40 and 60.

- **The Order of Operations**
  
<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Please</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simplify parentheses, brackets, and square roots.</td>
<td>Please</td>
</tr>
<tr>
<td>2</td>
<td>Simplify exponents.</td>
<td>Excuse</td>
</tr>
<tr>
<td>3</td>
<td>Do multiplications and divisions from left to right.</td>
<td>My Dear</td>
</tr>
<tr>
<td>4</td>
<td>Do additions and subtractions from left to right.</td>
<td>Aunt Sally</td>
</tr>
</tbody>
</table>

  Example
  
  \(40 - 3^2 + 84 ÷ 2\)
  
  \(40 - 9 + 84 ÷ 2\)
  
  \(40 - 9 + 42\)
  
  \(31 + 42 = 73\)

- **Area**
  
  Area is the amount of “land” or territory inside of a closed geometric figure.
Section 2.1 – Factors and Multiples

When you want to prepare a salad, you select certain ingredients (lettuce, tomatoes, broccoli, celery, olives, etc.) to give the salad a specific taste and consistency. You can think of the salad as the final result, or product, of combining the ingredients. In general, the order in which you combine these ingredients does not affect the taste and consistency of the salad.

We saw in the previous chapter that whenever we multiply two or more whole numbers, we get another whole number as the final result. For example,

\[17 \times 3 \times 11 = 561\]

We can think of the numbers 17, 3 and 11 as the “ingredients” that multiplied together make up the “salad,” or final result of 561. In mathematics, instead of talking about ingredients we call them factors and instead of a salad we call it a product or multiple. Thus, we say that the whole numbers 17, 3 and 11 are the factors that multiplied together give the product 561. The number 561 is a multiple of 17, 3, and 11, which means that 561 is divisible by 17, 3, and 11.

**Example 2.1.1**

In the following equation, name the factors and the multiple.

\[13 \times 47 \times 5 \times 241 = 736,255\]

Answer: The numbers 13, 47, 5 and 241 are factors of the product 736,255, whereas the product 736,255 is a multiple of 13, 47, 5 and 241. This means that 736,255 is divisible by 13, 47, 5 and 241. You can think of 736,255 as being the “salad” and the numbers 13, 47, 5 and 241 its “ingredients.”
In the following equation, name the factors and the product.

\[ 6 \times 4 \times 23 \times 11 = 6,072 \]

Answer: The numbers 6, 4, 23 and 11 are factors of 6,072, whereas the number 6,072 is a multiple of 6, 4, 23 and 11.

**Note:** The order in which you multiply the factors doesn’t affect the final product. For example:

\[ 8 \times 5 \times 2 = 5 \times 8 \times 2 = 2 \times 8 \times 5 = 80 \]

Whenever we can write a whole number as the product of a set of whole numbers (its factors), we say that the product is divisible by those numbers. That is, any whole number is divisible by its factors.

**Example 2.1.2**

Since \( 7 \times 8 = 56 \), we know that 56 is divisible by 7 and that 56 is divisible by 8. Therefore, 56 is a multiple of 7 and 8, while 7 and 8 are factors of 56. It should be clear that 56 has other factors besides 7 and 8, namely 1, 2, 4, 14, 28 and 56.

**Example 2.1.3**

Since 7x8=56, then 56 is the product and 7 and 8 are factors of 56. Notice that whenever we divide a product by one of its factors, the quotient is also a factor and the remainder is always zero.
Since \(2 \times 28 = 56\), we know that 56 is divisible by 2 and that 56 is divisible by 28. Therefore, 56 is a multiple of 2 and 28, while 2 and 28 are factors of 56.

**Example 2.1.4**

Since \(2 \times 28 = 56\), then 56 is the product and 2 and 28 its factors. Notice that whenever we divide a product by one of its factors, the quotient is also a factor and the remainder is always zero.

**Note:** 1 is a factor of any whole number \(N\) because \(1 \times N = N\). This also means that any whole number \(N\) is divisible by 1.

For example,

1 is a factor of 17 since \(1 \times 17 = 17\). Therefore, 17 is divisible by 1.

1 is a factor of 5,788 since \(1 \times 5,788 = 5,788\). Therefore, 5,788 is divisible by 1.

Let’s check that the remainder is zero:
The following are special whole numbers that you should become familiar with:

<table>
<thead>
<tr>
<th>Number</th>
<th>Definition</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Even</strong></td>
<td>Any number that is divisible by 2. It has 2 as a factor.</td>
<td>0, 2, 4, 6, 8, 10, 12, 14, 16, …</td>
</tr>
<tr>
<td><strong>Odd</strong></td>
<td>Any number that is not divisible by 2. It does not have 2 as a factor.</td>
<td>1, 3, 5, 7, 9, 11, 13, 15, 17,…</td>
</tr>
<tr>
<td><strong>Prime</strong></td>
<td>Any whole number that has exactly 2 different factors: 1 and the number itself.</td>
<td>2, 3, 5, 7, 11, 13, 17, 19, 23,…</td>
</tr>
<tr>
<td><strong>Composite</strong></td>
<td>Any whole number greater than 1 that is not a prime number.</td>
<td>4, 6, 8, 9, 10, 12, 14, 15, 16,…</td>
</tr>
</tbody>
</table>

**Note:** The number 1 is not prime because it has only one factor: itself  \(1 \times 1 = 1\).

| Table of Prime Numbers Less Than 1,000 |
|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 |
| 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 |
| 71 | 73 | 79 | 83 | 89 | 97 | 101 | 103 | 107 | 109 |
| 113 | 127 | 131 | 137 | 139 | 149 | 151 | 157 | 163 | 167 |
| 173 | 179 | 181 | 191 | 193 | 197 | 199 | 211 | 223 | 227 |
| 229 | 233 | 239 | 241 | 251 | 257 | 263 | 269 | 271 | 277 |
| 281 | 283 | 293 | 307 | 311 | 313 | 317 | 331 | 337 | 347 |
| 349 | 353 | 359 | 367 | 373 | 379 | 383 | 389 | 397 | 401 |
| 409 | 419 | 421 | 431 | 433 | 439 | 443 | 449 | 457 | 461 |
| 463 | 467 | 479 | 487 | 491 | 499 | 503 | 509 | 521 | 523 |
| 541 | 547 | 557 | 563 | 569 | 571 | 577 | 587 | 593 | 599 |
| 601 | 607 | 613 | 617 | 619 | 631 | 641 | 643 | 647 | 653 |
| 659 | 661 | 673 | 677 | 683 | 691 | 701 | 709 | 719 | 727 |
| 733 | 739 | 743 | 751 | 757 | 761 | 769 | 773 | 787 | 797 |
| 809 | 811 | 821 | 823 | 827 | 829 | 839 | 853 | 857 | 859 |
| 863 | 877 | 881 | 883 | 887 | 907 | 911 | 919 | 929 | 937 |
| 941 | 947 | 953 | 967 | 971 | 977 | 983 | 991 | 997 | 1007 |
Example 2.1.5

Make a table of all the factors of 24.

Answer:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

The number at the top of the table shown in blue (24) is a multiple of all the numbers in green. Conversely, the numbers in green are factors of 24.

In the table above, we see that 24 is a factor and multiple of itself. In fact, any whole number $N$ is a factor and multiple of itself because we can always write $1 \times N = N$.

Example 2.1.6

Make a table of all the factors of 120.

Answer:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60 and 120 are factors of 120 because

1 x 120 = 120
2 x 60 = 120
3 x 40 = 120
4 x 30 = 120
5 x 24 = 120
6 x 20 = 120
8 x 15 = 120
10 x 12 = 120
The number at the top of the table in blue (120) is a multiple of all the numbers in green. Similarly, the numbers in green are factors of 120.

Please seek help from your instructor if you have difficulty understanding the difference between a multiple and a factor.

Instructional video on what a multiple of a number is:

http://www.youtube.com/watch?v=vbNeXLvqM90

**Classwork 2.1**

1. Is 4 a factor of 216? Explain why or why not.

   4 is a factor of 216 because 4 \( \times 54 = 216 \) (dividing 216 by 4 gives a zero remainder).

2. Is 17 a factor of 91? Explain why or why not.

   17 is not a factor of 91 because dividing 91 by 17 gives a remainder that is not zero.

3. Given that \( 5 \times 13 \times 17 = 1105 \) state whether the following statements are true or false:

   A. 5 is a factor of 1105. \( \text{True} \)
   
   B. 17 is a factor of 1105. \( \text{True} \)
   
   C. 1105 is a factor of 13. \( \text{False} \)
   
   D. 1105 is a multiple of 17. \( \text{True} \)
   
   E. 1105 is a multiple of 5. \( \text{True} \)
   
   F. 1105 is divisible by 13. \( \text{True} \)
   
   G. 5 is a factor of 13. \( \text{False} \)
4. Is 200 a multiple of 10?  Yes
5. Is 512 a multiple of 16?  Yes
6. Is 720 a multiple of 11?  No
7. Is 77 a multiple of 11?  Yes
8. Is 11 a multiple of 77?  No

9. Write a table of all the factors of 90.
   Answer:
   
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

10. Write a table of all the factors of 64.
    Answer:
    
    |    | 1   | 64  |
    |----|-----|-----|
    | 2   | 32  |
    | 4   | 16  |
    | 8   | 8   |
1. Is 9 a factor of 325? Explain why or why not.

9 is not a factor of 325 because dividing 325 by 9 gives a remainder that is not zero.

2. Is 12 a factor of 240? Explain why or why not.

12 is a factor of 240 because $12 \times 20 = 240$ (dividing 240 by 12 gives a zero remainder).

3. Given that $8 \times 19 = 152$ state whether the following statements are true or false:

   A. 8 is a factor of 152.  True
   B. 19 is a factor of 152.  True
   C. 152 is a multiple of 8. True
   D. 152 is divisible by 19. True
   E. 152 is a multiple of 19. True
   F. 152 is a factor of 8. False
   G. 8 is a factor of 19. False

4. Is 34,590 a multiple of 5?  Yes

5. Is 400 a multiple of 1,200?  No

6. Is 6 a factor of 30?  Yes

7. Is 144 divisible by 8?  Yes

8. Is 341 divisible by 3?  No

9. Write a table of all the factors of 44.

   Answer:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>
10. Write a table of all the factors of 150.

Answer:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>
Section 2.2 – Rules of Divisibility

In the previous section, we learned what it means for a number to be divisible by another number. In particular, if we have \( a \times b = c \), this means that \( a \) and \( b \) are both factors of \( c \). This also means if we divide \( c \) by \( a \) or by \( b \), the remainder will be zero. Therefore, \( c \) is divisible by \( a \) and by \( b \). For example, since \( 15 \times 2 = 30 \), both 15 and 2 are factors of 30. This means that 30 is divisible by 15 and by 2. Consequently, we get a zero remainder when we divide 30 by 2 and when we divide 30 by 15.

One way to determine if a whole number is divisible by a second number is to divide the first number by the second number, and check that we get a zero remainder. However, this method might be time consuming in some instances. Therefore, it is advantageous to memorize the following divisibility rules and apply them as appropriate. There are many divisibility rules, but only the most basic and easy to remember are presented in this table.

<table>
<thead>
<tr>
<th>Divisible by</th>
<th>Condition</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>All whole numbers are divisible by 1.</td>
<td>0,1,2,3,4,5,6,7,8,9,10,11,12,13,…</td>
</tr>
<tr>
<td>2</td>
<td>If the number is even.</td>
<td>0,2,4,6,8,10,12,14,16,18,20,22,…</td>
</tr>
<tr>
<td>3</td>
<td>If the sum of the digits is divisible by 3.</td>
<td>0,3,6,9,12,15,18,21,24,27,30,33,…</td>
</tr>
<tr>
<td>4</td>
<td>If the 2 rightmost digits are divisible by 4.</td>
<td>0,4,8,12,16,20,24,28,32,36,40,44,…</td>
</tr>
<tr>
<td>5</td>
<td>If the number ends with a 5 or 0.</td>
<td>0,5,10,15,20,25,30,35,40,45,50,…</td>
</tr>
<tr>
<td>6</td>
<td>If the number is divisible by 2 and by 3.</td>
<td>0,6,12,18,24,30,36,42,48,54,60,…</td>
</tr>
<tr>
<td>9</td>
<td>If the sum of the digits is divisible by 9.</td>
<td>0,9,18,27,36,45,54,63,72,81,90,…</td>
</tr>
<tr>
<td>10</td>
<td>If the number ends with 0</td>
<td>0,10,20,30,40,50,60,70,80,90,100,…</td>
</tr>
</tbody>
</table>
Example 2.2.1
Determine whether the number 345,726 is divisible by 1, 2, 3, 5 or 6.

Answer:
345,726 is divisible by 1 because all whole numbers are divisible by 1.
345,726 is divisible by 2 because 345,726 is an even number.
345,726 is divisible by 3 because the sum of the digits 3+4+5+7+2+6 = 27 and 27 is divisible by 3.
345,726 is not divisible by 5 because the rightmost digit is not 5 or 0.
345,726 is divisible by 6 because 345,726 is divisible by 2 and by 3.

Example 2.2.2
Determine whether the number 68,970 is divisible by 1, 2, 3, 4, 9 or 10.

Answer:
68,970 is divisible by 1 because all whole numbers are divisible by 1.
68,970 is divisible by 2 because 68,970 is an even number.
68,970 is divisible by 3 because the sum of the digits 6+8+9+7= 30 and 30 is divisible by 3.
68,970 is not divisible by 4 because the number formed by the 2 rightmost digits is 70, but 70 is not divisible by 4.
68,970 is not divisible by 9 because the sum of the digits 6+8+9+7= 30 and 30 is not divisible by 9.
68,970 is divisible by 10 because the rightmost digit is a zero.
Instructional videos on the application of the Rules of Divisibility can be found in the following sites:

http://www.youtube.com/watch?v=AXlz_dHmye4
http://www.youtube.com/watch?v=kBhbv4AVDII

Classwork 2.2
The following questions ask you to determine whether a number is a factor of the given number. You may use any method to determine this, including the rules of divisibility that were presented in this section.

1. Is 2 a factor of 7,986?   Yes
2. Is 8 a factor of 6039?   No
3. Is 5 a factor of 34,780?   Yes
4. Is 8 a factor of 7,432?   Yes
5. Is 10 a factor of 7,901?   No
6. Is 7 a factor of 7,910?   Yes
7. Is 9 a factor of 666?   Yes
8. Is 538 divisible by 2?   Yes
9. Is 7,872 divisible by 3?   Yes
10. Is 345 divisible by 5?   Yes
The following questions ask you to determine whether a number is a factor of the given number. You may use any method to determine this, including the rules of divisibility that were presented in this section.

1. Is 9 a factor of 504?  
   Yes

2. Is 6 a factor of 530?  
   No

3. Is 2 a factor of 687,421?  
   No

4. Is 5 a factor of 120?  
   Yes

5. Is 7 a factor of 821?  
   No

6. Is 10 a factor of 16,785?  
   No

7. Is 9 a factor of 440?  
   No

8. Is 470 divisible by 2?  
   Yes

9. Is 16,002 divisible by 3?  
   Yes

10. Is 120 divisible by 3?  
    Yes
Section 2.3 – Prime Factorization

Recall that a prime number has exactly two different factors: 1 and the number itself. For your convenience, here again is a list of all the prime numbers that are less than 1000:

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>19</th>
<th>23</th>
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<tbody>
<tr>
<td>29</td>
<td>31</td>
<td>37</td>
<td>41</td>
<td>43</td>
<td>47</td>
<td>53</td>
<td>59</td>
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<td>71</td>
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<td>83</td>
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<td>97</td>
<td>101</td>
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<td>163</td>
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<td>197</td>
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<tr>
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<td>449</td>
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<td>463</td>
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<td>479</td>
<td>487</td>
<td>491</td>
<td>499</td>
<td>503</td>
<td>509</td>
<td>521</td>
<td>523</td>
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<td>541</td>
<td>547</td>
<td>557</td>
<td>563</td>
<td>569</td>
<td>571</td>
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<td>617</td>
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<td>631</td>
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</tr>
<tr>
<td>659</td>
<td>661</td>
<td>673</td>
<td>677</td>
<td>683</td>
<td>691</td>
<td>701</td>
<td>709</td>
<td>719</td>
<td>727</td>
</tr>
<tr>
<td>733</td>
<td>739</td>
<td>743</td>
<td>751</td>
<td>757</td>
<td>761</td>
<td>769</td>
<td>773</td>
<td>787</td>
<td>797</td>
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<tr>
<td>809</td>
<td>811</td>
<td>821</td>
<td>823</td>
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<td>829</td>
<td>839</td>
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<td>857</td>
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<tr>
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<td>941</td>
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<td>967</td>
<td>971</td>
<td>977</td>
<td>983</td>
<td>991</td>
<td>997</td>
<td></td>
</tr>
</tbody>
</table>

We have learned that whole numbers have factors, and thus can be written in factorized form. For example, some factorizations of the number 360 are

360 = 6 \times 5 \times 12
360 = 1 \times 9 \times 4 \times 10
360 = 18 \times 5 \times 4
360 = 1 \times 360
A factorization of a number shows factors that multiplied together give the original number. Note that although 4, 6 and 5 are factors of 360, the expression $4 \times 6 \times 5$ is not a factorization of 360 because $4 \times 6 \times 5 \neq 360$. Recall that the symbol $\neq$ means “not equal to.”

**Example 2.3.1**

Write 5 different factorizations of 2,000.

<table>
<thead>
<tr>
<th>Answer</th>
<th>2,000 = 1 × 100 × 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000 = 10 × 4 × 50</td>
<td></td>
</tr>
<tr>
<td>2,000 = 1 × 2 × 4 × 5 × 10 × 5</td>
<td></td>
</tr>
<tr>
<td>2,000 = 1 × 2000</td>
<td></td>
</tr>
<tr>
<td>2,000 = 50 × 40</td>
<td></td>
</tr>
</tbody>
</table>

In some applications, it is useful to factorize a whole number using only prime factors. That is, we would like to find the *prime factorization* of a number such that the factors are prime numbers, and we get the original number when we multiply the prime factors.

For example, the prime factorization of 2,000 is $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$ because all these factors of 2,000 are prime numbers, and when we multiply them we get 2,000.

The prime factorization of 90 is $2 \times 3 \times 3 \times 5$ because $2 \times 3 \times 3 \times 5 = 90$ and the numbers 2, 3 and 5 are prime.

**Note:** 1 is not a prime number because it has only one factor: itself $1 \times 1 = 1$. 
There is an easy way to find the prime factorization of any whole number. This method involves constructing a **tree of factors**. The only requirement that must be followed is that each factor appearing in the tree must be either a prime number or a composite number. Hence, 1 will never appear in a tree of factors because 1 is neither prime nor composite. The approach to construct a tree of factors is to split, or factor, the original number into a product of prime and/or composite factors, and then continue splitting these factors until we are left with prime factors at the end of the branches. If the tree of factors was constructed correctly, the product of all the prime factors located at the end of all the branches should be equal to the original number we started with.

For example, to find the prime factorization of 12, we begin by factoring 12 in any way we choose, as long as the factors are prime or composite. At the end of the branches, we will be left with only prime numbers that multiplied together give the original number we started with (12).

![Tree of Factors for 12]

The numbers in red are the prime factors of 12, and so the prime factorization of 12 is $2 \times 2 \times 3$. If you are familiar with exponents, you can write the prime factorization in compact form as $= 2^2 \times 3$. You will learn more about exponential notation in Section 2.5.

**Example 2.3.2**

Write the prime factorization of 45 in expanded form.

![Tree of Factors for 45]

Answer: $45 = 5 \times 3 \times 3$
Using exponents, the prime factorization of 45 is \(45 = 5 \times 3^2\).
Remember that the order in which you multiply any numbers does not matter.

**Example 2.3.3**

Write the prime factorization of 120 in expanded form.

\[
\begin{array}{c}
120 \\
10 \times 12 \\
5 \times 2 \times 3 \\
2 \times 2 \\
\end{array}
\]

Answer: \(120 = 5 \times 2 \times 2 \times 2 \times 3\)

Using exponents, the prime factorization of 120 is \(120 = 5 \times 2^3 \times 3\).

**Example 2.3.4**

Write the prime factorization of 350 in expanded form.

\[
\begin{array}{c}
350 \\
10 \times 35 \\
5 \times 2 \times 7 \times 5 \\
\end{array}
\]

Answer: \(350 = 2 \times 5 \times 5 \times 7\)

In exponential form, the answer is \(350 = 2 \times 5^2 \times 7\).
Example 2.3.5

Write the prime factorization of 504 in expanded form.

\[
\begin{align*}
504 & \quad 2 \times 252 \\
252 & \quad 2 \times 126 \\
126 & \quad 2 \times 63 \\
63 & \quad 3 \times 21 \\
21 & \quad 3 \times 7
\end{align*}
\]

Answer: \(504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7\)

In exponential form, the answer is \(504 = 2^3 \times 3^2 \times 7\)

Instructional video on finding the prime factorization of a whole number:
http://www.youtube.com/watch?v=YKXE2rMKPYA

The following website has an interactive tool to help you construct a tree of factors to find the prime factorization of any whole number:
http://www.softschools.com/math/factors/factor_tree/

Classwork 2.3
Write the prime factorization of each number.

1. 70
   \(70 = 2 \times 5 \times 7\)
2. 100
   \[100 = 2 \times 2 \times 5 \times 5\]

3. 231
   \[231 = 3 \times 7 \times 11\]

4. 441
   \[441 = 3 \times 3 \times 7 \times 7\]

5. 420
   \[420 = 2 \times 2 \times 3 \times 5 \times 7\]

6. 800
   \[800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5\]

7. 3600
   \[3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5\]

8. 26
   \[26 = 2 \times 13\]

9. 98
   \[98 = 2 \times 7 \times 7\]

10. 1000
    \[1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5\]

**Homework 2.3**

Write the prime factorization of each number.

1. 735
   \[735 = 3 \times 5 \times 7 \times 7\]

2. 180
   \[180 = 2 \times 2 \times 3 \times 3 \times 5\]

3. 924
   \[924 = 2 \times 2 \times 3 \times 7 \times 11\]

4. 60
   \[60 = 2 \times 2 \times 3 \times 5\]

5. 2300
   \[2300 = 2 \times 2 \times 5 \times 5 \times 23\]

6. 64
   \[64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2\]

7. 80
   \[80 = 2 \times 2 \times 2 \times 2 \times 5\]

8. 4620
   \[4620 = 2 \times 2 \times 3 \times 5 \times 7 \times 11\]

9. 81
   \[81 = 3 \times 3 \times 3 \times 3\]

10. 690
    \[690 = 2 \times 3 \times 5 \times 23\]
Section 2.4 – The Greatest Common Factor (GCF) and the Least Common Multiple (LCM)

In Section 2.1 you learned how to construct a *table of factors* to find all of the factors of a whole number. For example, the table of all the factors of 108 is shown below. The factors of 108 are in green, and 108 is a multiple of all its factors.

Moreover, in Section 2.3 you learned that the prime factorization of 108 can be obtained by first constructing a *tree of factors*, and then multiplying the prime factors that we get at the end of the branches.

Therefore, the prime factorization of 108 is $108 = 2 \times 2 \times 3 \times 3 \times 3$, which can also be written in exponential form as $108 = 2^2 \times 3^3$. 
Sometimes, it happens that two or more whole numbers share a common factor. For example, the numbers 60, 120 and 500 have 10 as a common factor because

\[
\begin{align*}
10 \times 6 &= 60 \\
10 \times 12 &= 120 \\
10 \times 50 &= 500
\end{align*}
\]

Since 10 is a factor of 60, 120 and 500, this also means that 60, 120 and 500 are multiples of 10. Don’t forget that 1 is a factor of all whole numbers, and so all whole numbers have the number 1 as a common factor. Similarly, all whole numbers are multiples of 1.

*If you need help understanding the difference between a factor and a multiple, please review section 2.1 before continuing reading this section.*

**Example 2.4.1**

Is 3 a common factor of 12 and 48? Explain.

Yes, because \(3 \times 4 = 12\) and \(3 \times 16 = 48\). Therefore, 3 is a common factor of 12 and 48.

**Example 2.4.2**

Is 15 a common factor of 15, 60 and 72? Explain.

No. Although \(15 \times 1 = 15\) and \(15 \times 4 = 60\), there is no whole number that multiplied times 15 will give 72. Therefore, 15 is not a factor of 72. Another way to see that 15 is not a factor of 72 is to realize that when we divide 72 by 15 (72 \(\div 15\)) the remainder is 12 instead of 0. We conclude that 15 is not a common factor of 15, 60 and 72.
Sometimes, we are interested in finding the **greatest common factor (GCF)** that two or more numbers share in common. For example, the numbers 20, 90 and 150 have 1, 2, 5 and 10 as common factors. Thus, the GCF of 20, 90 and 150 is 10. One way to find the GCF is to construct a table of factors for each numbers and then see which is the largest factor they all share in common (shown in bold green).

<table>
<thead>
<tr>
<th>20</th>
<th>90</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Unfortunately, sometimes this method to find the GCF takes too much time. A more efficient method to find the GCF is outlined below:

**Method to Find the Greatest Common Factor (GCF) of Two or More Numbers:**

**Step 1:** Find the prime factorization of each number using any method.

**Step 2:** Determine the number of times that each prime factor appears in each factorization and write down each prime factor the **least** number of times it appears in the factorizations. If a prime factor appears in some but not all the prime factorizations, do not write it down.

**Step 3:** Multiply the prime numbers you wrote down. The product is the GCF of the original set of numbers.
Example 2.4.3

Find the GCF of 504 and 108.

**Step 1:**

![Prime factorization diagrams for 504 and 108]

Prime factorization of 504:

\[ 504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \]

Prime factorization of 108:

\[ 108 = 2 \times 2 \times 3 \times 3 \times 3 \]

**Step 2:** We notice that 2 appears a minimum of 2 times, 3 appears a minimum of 2 times and 7 appears a minimum of zero times because it doesn’t appear in the prime factorization of 108.

**Step 3:** We write down \( 2 \times 2 \times 3 \times 3 \) and multiply to get 36, which is the GCF of 504 and 108.
Example 2.4.4

Find the GCF of 300 and 700.

Step 1:

Prime factorization of 300:

\[ 300 = 2 \times 2 \times 5 \times 5 \times 3 \]

Prime factorization of 700:

\[ 700 = 7 \times 2 \times 2 \times 5 \times 5 \]

Step 2: We notice that 2 appears a minimum of 2 times, 5 appears a minimum of 2 times, 3 appears a minimum of zero times because it doesn’t appear in the prime factorization of 300, and 3 appears a minimum of zero times because it doesn’t appear in the prime factorization of 700.

Step 3: We write down \( 2 \times 2 \times 5 \times 5 \) and multiply to get 100, which is the GCF of 300 and 700.
Example 2.4.5

Find the GCF of 200 and 231.

**Step 1:**

- Prime factorization of 200: 
  \[200 = 2 \times 2 \times 2 \times 5 \times 5\]
- Prime factorization of 231: 
  \[231 = 3 \times 7 \times 11\]

**Step 2:** We notice that 2, 3, 5, 7 and 11 appear a minimum of zero times in one of the factorizations.

**Step 3:** We do not multiply any numbers. The only factor 200 and 231 have in common is 1, so 1 is the GCF of 200 and 231. We say that 200 and 231 are relatively prime.

**Note:** A set of whole numbers that have 1 as the GCF are called relatively prime. This means that the only factor these numbers have in common is 1.
Method to Find the Least Common Multiple (LCM) of Two or More Numbers:

**Step 1:** Find the prime factorization of each number using any method.

**Step 2:** Determine the number of times that each prime factor appears in each factorization and write down each prime factor the *most* number of times it appears in the factorizations.

**Step 3:** Multiply the prime numbers you wrote down. The product is the LCM of the original numbers.

**Example 2.4.6**

Find the LCM of 20 and 54.

**Step 1:**

Prime factorization of 20:
20 = 2 × 2 × 5

Prime factorization of 54:
54 = 2 × 3 × 3 × 3

**Step 2:** We notice that 2 appears a maximum of 2 times, 5 appears a maximum of 1 time and 3 appears a maximum of 3 times.

**Step 3:** We write down 2 × 2 × 5 × 3 × 3 × 3 and multiply to get 540, which is the LCM of 20 and 54.
**Example 2.4.7**

Find the LCM of 504 and 108.

**Step 1:**

Prime factorization of 504:

\[ 504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \]

Prime factorization of 108:

\[ 108 = 2 \times 2 \times 3 \times 3 \times 3 \]

**Step 2:** We notice that 2 appears a maximum of 3 times, 3 appears a maximum of 3 times and 7 appears a maximum of 1 time.

**Step 3:** We write down \( 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \) and multiply to get **1512**, which is the LCM of 504 and 108.
Example 2.4.8

Find the LCM of 300 and 700.

Step 1:

Prime factorization of 300:
300 = 2 × 2 × 5 × 5 × 3

Prime factorization of 700:
700 = 7 × 2 × 2 × 5 × 5

Step 2: We notice that 2 appears a maximum of 2 times, 5 appears a maximum of 2 times, 3 appears a maximum of 1 time, and 7 appears a maximum of 1 time.

Step 3: We write down 2 × 2 × 5 × 5 × 3 × 7 and multiply to get 2100, which is the LCM of 300 and 700.
Example 2.4.9

Find the LCM of 200 and 231.

Step 1:

Prime factorization of 200:
$200 = 2 \times 2 \times 2 \times 5 \times 5$

Prime factorization of 231:
$231 = 3 \times 7 \times 11$

Step 2: We notice that 2 appears a maximum of 3 times, 5 appears a maximum of 2 times, while 3, 7 and 11 appear a maximum of 1 time in the factorizations.

Step 3: We write down $2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 7 \times 11$ and multiply to get 46,200 which is the LCM of 200 and 231.

Instructional video on how to find the GCF and LCM of a set of numbers:
http://www.youtube.com/watch?v=OT6q60O-w0Y

GCF calculator to check your work:

LCM calculator to check your work:
http://www.calculatorsoup.com/calculators/math/lcm.php
### Classwork 2.4
Find the greatest common factor (GCF) and the lowest common multiple (LCM) of each set of numbers.

1. 98 and 56  
   **GCF:** 14  
   **LCM:** 392

2. 100, 200 and 500  
   **GCF:** 100  
   **LCM:** 1,000

3. 30 and 80  
   **GCF:** 10  
   **LCM:** 240

4. 100 and 35  
   **GCF:** 5  
   **LCM:** 700

5. 9 and 20  
   **GCF:** 1  
   **LCM:** 180

6. 50 and 70  
   **GCF:** 10  
   **LCM:** 350

7. 20, 32 and 40  
   **GCF:** 4  
   **LCM:** 160

8. 64, 8 and 121  
   **GCF:** 1  
   **LCM:** 7,744

9. 25, 20 and 40  
   **GCF:** 5  
   **LCM:** 200

10. 90 and 210  
    **GCF:** 30  
    **LCM:** 630
Find the greatest common factor (GCF) and the lowest common multiple (LCM) of each set of numbers.

1. 12 and 30    \[\text{GCF: 6} \quad \text{LCM: 60}\]
2. 72, 216 and 18    \[\text{GCF: 18} \quad \text{LCM: 216}\]
3. 150 and 80    \[\text{GCF: 10} \quad \text{LCM: 1,200}\]
4. 63 and 98    \[\text{GCF: 7} \quad \text{LCM: 882}\]
5. 90 and 49    \[\text{GCF: 1} \quad \text{LCM: 4,410}\]
6. 150, 300 and 450    \[\text{GCF: 150} \quad \text{LCM: 900}\]
7. 16, 25 and 8    \[\text{GCF: 1} \quad \text{LCM: 400}\]
8. 6, 9 and 12    \[\text{GCF: 3} \quad \text{LCM: 36}\]
9. 14, 36 and 12    \[\text{GCF: 2} \quad \text{LCM: 252}\]
10. 70 and 90    \[\text{GCF: 10} \quad \text{LCM: 630}\]

For additional practice questions with solutions on finding the GCF, please see [http://cnx.org/content/m34874/latest/?collection=col10615/latest](http://cnx.org/content/m34874/latest/?collection=col10615/latest)

For additional practice questions with solutions on finding the LCM, please see [http://cnx.org/content/m34876/latest/?collection=col10615/latest](http://cnx.org/content/m34876/latest/?collection=col10615/latest)
Section 2.5 – Exponents, Square Roots and the Order of Operations

In the previous section, you learned how to prime factorize whole numbers by constructing a tree of factors that has prime numbers at the end of each branch. For example, the prime factorization of 720 is $2 \times 2 \times 2 \times 3 \times 3 \times 5$. We can write a more compact answer by using exponents: $720 = 2^4 \times 3^2 \times 5^1$.

**Exponents** are superscript numbers that indicate how many times a number must be multiplied by itself.

For example,

\[
7^4 = 7 \times 7 \times 7 \times 7 = 2401
\]

We must multiply 7 four times by itself. The number 7 is called the *base* and 4 is the *exponent*.

\[
2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64
\]

We must multiply 2 six times by itself. The number 2 is the *base* and 4 is the *exponent*.

\[
1^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1
\]

We must multiply 1 five times by itself. The number 1 is the *base* and 5 is the *exponent*.

We can write the prime factorization of a number using exponents, although this is not required. For example, here are some prime factorizations in expanded form and in exponential form:

\[
24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1
\]

\[
7000 = 5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 7 = 5^3 \times 2^3 \times 7^1
\]

\[
162 = 2 \times 3 \times 3 \times 3 \times 3 = 2^1 \times 3^4
\]
Warning:  \(2^3 \neq 6\)  
\(9^2 \neq 18\)  
\(1^{10} \neq 10\)  
\(5^3 \neq 15\)  
(\(2^3\) equals 8)  
(\(9^2\) equals 81)  
(\(1^{10}\) equals 1)  
(\(5^3\) equals 125)  
Remember that the exponent specifies the number of times that the base must be multiplied by itself.

The square root of a number is a number that multiplied by itself twice gives you the original number. We use the symbol \(\sqrt{\phantom{0}}\) to represent the positive square root of a number. For example, \(\sqrt{25} = 5\) because \(5 \times 5 = 25\). The equation \(\sqrt{25} = 5\) is read “The square root of twenty five is five.”

\[
\begin{align*}
\sqrt{1} &= 1 & \text{because} & & 1 \times 1 &= 1 \\
\sqrt{4} &= 2 & \text{because} & & 2 \times 2 &= 4 \\
\sqrt{9} &= 3 & \text{because} & & 3 \times 3 &= 9 \\
\sqrt{16} &= 4 & \text{because} & & 4 \times 4 &= 16 \\
\sqrt{400} &= 20 & \text{because} & & 20 \times 20 &= 400 \\
\sqrt{225} &= 15 & \text{because} & & 15 \times 15 &= 225 \\
\sqrt{1600} &= 40 & \text{because} & & 40 \times 40 &= 1600
\end{align*}
\]

The numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, ... are called perfect squares because

\[
1 = 1^2, \quad 4 = 2^2, \quad 9 = 3^2, \quad 16 = 4^2, \quad 25 = 5^2, \quad 36 = 6^2, \quad \text{etc.}
\]

Warning: \(\sqrt{49} \neq \sqrt{7}\) . Do not leave the square root symbol after you have found the square root of a number. The correct answer is \(\sqrt{49} = 7\) .
For example, \(\sqrt{100} \neq \sqrt{10}\). The correct answer is \(\sqrt{100} = 10\).
Similarly, \(\sqrt{16} \neq \sqrt{4}\). The correct answer is \(\sqrt{16} = 4\).
**The Order of Operations**

Let’s simplify the following expression: \( 100 - 12 \times 3 + 5 \)

Suppose that three students try different ways to simplify this expression and get the following results:

- **Tina**
  - \( 100 - 12 \times 3 + 5 \)
  - \( 88 \times 3 + 5 \)
  - \( 264 + 5 \)
  - \( 269 \)

- **Andrew**
  - \( 100 - 12 \times 3 + 5 \)
  - \( 100 - 36 + 5 \)
  - \( 64 + 5 \)
  - \( 69 \)

- **Mark**
  - \( 100 - 12 \times 3 + 5 \)
  - \( 100 - 12 \times 8 \)
  - \( 88 \times 8 \)
  - \( 704 \)

Who is correct? Well, without any rules as to how to simplify the expression no one is right or wrong. But notice the answers are completely different! To ensure that everyone obtains the same value, mathematicians have established a set of steps that must be followed to ensure that everyone gets the same answer. This set of steps is called the **Order of Operations**.

<table>
<thead>
<tr>
<th>The Order of Operations</th>
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<tbody>
<tr>
<td><strong>Step 1</strong></td>
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<td><strong>Step 2</strong></td>
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<tr>
<td><strong>Step 3</strong></td>
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<td><strong>Step 4</strong></td>
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</tbody>
</table>

Based on the order of operations, which student got the correct answer? **Andrew**.
Example 2.5.1

Write $7 \times 2 \times 2 \times 13 \times 5 \times 7 \times 5 \times 5$ in exponential notation.

Answer: $7^2 \times 2^2 \times 13^1 \times 5^3$

Example 2.5.2

Simplify by using the order of operations: $300 - 97 + 25 \times 2$.

Answer: $300 - 97 + 25 \times 2$

\[300 - 97 + 50\]
\[203 + 50\]
\[253\]

Example 2.5.3

Simplify by using the order of operations: $32 \div 4 + \sqrt{121} - 7$.

Answer: $32 \div 4 + \sqrt{121} - 7$

\[32 \div 4 + 11 - 7\]
\[8 + 11 - 7\]
\[19 - 7\]
\[12\]
**Example 2.5.4**

Simplify by using the order of operations: \(4^3 - 10 + 3^2 \times 10 \div 2\).

Answer:
\[
\begin{align*}
4^3 - 10 + 3^2 \times 10 \div 2 \\
64 - 10 + 9 \times 10 \div 2 \\
64 - 10 + 90 \div 2 \\
64 - 10 + 45 \\
54 + 45 \\
99
\end{align*}
\]

**Example 2.5.5**

Simplify by using the order of operations: \((50 - 40)^2 \times 3 + 72 \div 8\).

Answer:
\[
\begin{align*}
(50 - 40)^2 \times 3 + 72 \div 8 \\
10^2 \times 3 + 72 \div 8 \\
100 \times 3 + 72 \div 8 \\
300 + 72 \div 8 \\
300 + 9 \\
309
\end{align*}
\]
Example 2.5.6

Simplify by using the order of operations: \( \sqrt{49} + \frac{100}{5} - (19 - 18)^4 \).

Answer: 

\[
\begin{align*}
7 + \frac{100}{5} - 1^4 \\
7 + \frac{100}{5} - 1 \\
7 + 20 - 1 \\
27 - 1 \\
26
\end{align*}
\]

Example 2.5.7

Simplify by using the order of operations: \( \frac{42 ÷ 3 + 2^4}{2 + 48 ÷ 6} \).

Answer: 

\[
\begin{align*}
42 ÷ 3 + 16 \\
2 + 48 ÷ 6 \\
14 + 16 \\
2 + 8 \\
30 \\
10 = 3
\end{align*}
\]
Instructional videos on the order of operations:

http://www.youtube.com/watch?v=ikE1UmseIYM

http://www.youtube.com/watch?v=D3RikUWRy4c

http://www.youtube.com/watch?v=BRJv0fuxDbM

http://www.youtube.com/watch?v=BRJv0fuxDbM

Classwork 2.5

1. Write the following using exponents: \(6 \times 12 \times 3 \times 3 \times 6 \times 2 \times 6 \times 6\)

\[6^4 \times 12^1 \times 3^2 \times 2^1\]

2. Write the following using exponents: \(7 \times 7 \times 7 \times 7 \times 2 \times 2 \times 2 \times 3\)

\[2^3 \times 3^1 \times 7^5\]

Simplify each of the following expressions completely by following the order of operations:

3. \(2^4 + (12 - 9)^2 \times 5 + 100 \div 4\)

\[86\]

4. \(325 - (\sqrt{100} - 9)^2 + 36 \div 2\)

\[342\]

5. \(200 - \frac{60}{4} + 18 \times 3\)

\[239\]

6. \(\sqrt{25 + 144} + 4^3 - \frac{20}{10}\)

\[75\]

7. \(\frac{15 \times 8 - 60 \div 4}{\sqrt{25 + 10}}\)

\[7\]

8. \(\frac{120 + 3 \times 80}{\sqrt{16 + 20}}\)

\[60\]

9. \((9 - 7)^5 + (20 \div 4)^2\)

\[57\]

10. \(\sqrt{225} + 4 \times \sqrt{64} - 30 \div 6\)

\[42\]
Homework 2.5

1. Write the following using exponents: \( 2 \times 2 \times 2 \times 5 \times 5 \times 2 \times 3 \times 5 \times 2 \)
   \[ 2^5 \times 5^3 \times 3^1 \]

2. Write the following using exponents: \( 9 \times 9 \times 8 \times 8 \times 2 \times 2 \times 2 \times 7 \)
   \[ 9^3 \times 8^2 \times 2^3 \times 7^1 \]

Simplify each of the following expressions completely by following the order of operations:

3. \( 9^2 + (30 - 25)^2 \times 2 + 70 \div 5 \)  
   \[ 145 \]

4. \( 400 - (\sqrt{144} - 11)^2 + 50 \div 10 \)  
   \[ 404 \]

5. \( 216 \div 9 + \frac{120}{6} - 5 \times 8 \)  
   \[ 4 \]

6. \( 12 \times \sqrt{49} + 51 + 3^3 - \frac{720}{9} \)  
   \[ 67 \]

7. \( \frac{6 \times 7 - 32 \div 8 - 2}{\sqrt{121} - 5} \)  
   \[ 6 \]

8. \( \frac{534 - 5 \times 18}{\sqrt{50} - 46} \)  
   \[ 222 \]

9. \( (17 + 2)^2 - (84 \div 7)^2 \)  
   \[ 217 \]

10. \( \sqrt{400} + 15 \times \sqrt{81} - 321 \div 3 \)  
    \[ 48 \]

For additional practice exercises with solutions on the order of operations, see http://cnx.org/content/m34872/latest/?collection=col10615/latest
Section 2.6 – Evaluating Expressions

In math we often use letters to represent numbers. Letters that are used to represent numbers are called **variables**. You can use any letter you want to represent a number, but once you assign a value to a variable you must be consistent in giving the same value to the variable throughout all the steps you take to simplify an expression.

All operations on numbers \(5^4 = 5 \times 5 \times 5 \times 5\)

also apply to variables \(c^4 = c \times c \times c \times c\)

Knowing that variables represent numbers, we can write down expressions involving addition, subtraction, multiplication, division or any other operation. Moreover, if we know the value of each variable we can **evaluate**, or find the value of, the given expression.

Whenever you evaluate or simplify an expression, you should follow the order of operations that was presented in section 2.5. Please review this section if you need to look at examples on how to apply the order of operations. There are instances when you can use certain properties or laws that do not follow the order of operations. These cases will be discussed in Section 7.3.

**Note:** To avoid confusion between the variable \(x\) and the multiplication symbol \(\times\), we will often use the raised dot \(\cdot\) to represent multiplication. We also mean multiplication when we write two variables together, such as \(rk\), or when we write parentheses together, such as \((r)(k)\). In fact, all of the following expressions represent multiplication of two numbers:

\[
\begin{align*}
  r \cdot k & \quad rk & \quad (r)(k) & \quad 7 \cdot k & \quad 7k & \quad 7 \ast k & \quad 7(k)
\end{align*}
\]
Example 2.6.1

Knowing that $m = 10, k = 5, p = 2$ and $a = 1$ find the value of the following expression:

$$\frac{150}{p} + k^2 - m \cdot k \cdot a$$

Answer: $$\frac{150}{p} + k^2 - m \cdot k \cdot a$$

First, replace each variable with its corresponding value.

$$\frac{150}{2} + 5^2 - 10 \cdot 5 \cdot 1$$

Then follow the order of operations to simplify the above expression completely.

$$\frac{150}{2} + 25 - 10 \cdot 5 \cdot 1$$

$$75 + 25 - 50$$

$$100 - 50 = 50$$

Example 2.6.2

Knowing that $w = 30, r = 4, p = 3$ and $b = 12$ find the value of the following expression:

$$wr + w \div p - 4b$$

Answer: $$wr + w \div p - 4b$$

First, replace each variable with its corresponding value.

$$30 \cdot 4 + 30 \div 3 - 4 \cdot 12$$

Then follow the order of operations to simplify the above expression completely.

$$120 + 30 \div 3 - 4 \cdot 12$$

$$120 + 10 - 48$$

$$130 - 48$$

$$82$$
Example 2.6.3

Knowing that \( m = 1, a = 5, h = 20 \) and \( x = 4 \) find the value of the following expression:

\[(m + x)^2 \cdot h - 2ah\]

Answer:

First, replace each variable with its corresponding value.

\[(1 + 4)^2 \cdot 20 - 2 \cdot 5 \cdot 20\]

Then follow the order of operations to simplify the above expression completely.

\[
5^2 \cdot 20 - 2 \cdot 5 \cdot 20 \\
25 \cdot 20 - 100 \\
500 - 200 \\
300
\]

Example 2.6.4

Knowing that \( t = 16, k = 49, p = 4 \) and \( m = 2 \) evaluate the following expression:

\[
\frac{9p - 2\sqrt{k} + 8\sqrt{t}}{m}
\]

Answer:

First, replace each variable with its corresponding value.

\[
\frac{9 \cdot 4 - 2 \cdot \sqrt{49} + 8 \cdot \sqrt{16}}{2}
\]

Then follow the order of operations to simplify the above expression completely.

\[
\frac{9 \cdot 4 - 2 \cdot 7 + 8 \cdot 4}{2}
\]

\[
\frac{36 - 14 + 32}{2}
\]

\[
\frac{22 + 32}{2} = \frac{54}{2}
\]

\[
27
\]
Example 2.6.5

Knowing that \( x = 50, y = 100, z = 2 \) and \( w = 3 \) evaluate the following expression:

\[
\frac{x + y}{z} + 4w^2 - 60 \div w
\]

Answer: \( \frac{x + y}{z} + 4w^2 - 60 \div w \)

First, replace each variable with its corresponding value.

\[
\frac{50 + 100}{2} + 4 \cdot 3^2 - 60 \div 3
\]

Then follow the order of operations to simplify the above expression completely.

\[
\frac{50 + 100}{2} + 4 \cdot 9 - 60 \div 3
\]

\[
\frac{150}{2} + 4 \cdot 9 - 60 \div 3
\]

\[
75 + 36 - 20
\]

\[
111 - 20
\]

\[
91
\]

Example 2.6.6

Knowing that \( a = 1, b = 3, c = 121 \) and \( d = 144 \) evaluate the following expression:

\[
\frac{b^3 + \sqrt{a} + \sqrt{d}}{\sqrt{c} - 6}
\]

Answer: \( \frac{b^3 + \sqrt{a} + \sqrt{d}}{\sqrt{c} - 6} \)

First, replace each variable with its corresponding value.

\[
\frac{3^3 + \sqrt{1} + \sqrt{144}}{\sqrt{121} - 6}
\]

Then follow the order of operations to simplify the above expression completely.

\[
\frac{3^3 + 1 + 12}{\sqrt{121} - 6}
\]

\[
\frac{11 - 6}{27 + 1 + 12}
\]

\[
\frac{11 - 6}{40}
\]

\[
\frac{5}{5} = 8
\]
Classwork 2.6
Given that \( a = 1, b = 2, c = 3, x = 10, y = 12 \) and \( z = 20 \) simplify each expression using the order of operations.

1. \( \frac{z}{5} + 18b^3 - cx \)
2. \( 34 + 5z - (4x + 3b) + \sqrt{3y} \)
3. \( a^4 + (z - y)^2 \times 2 + z \div 4 \)
4. \( \sqrt{bc + 19} + cy - \frac{5x}{2} + a^8 \)
5. \( \frac{xy - 32 + z + 2}{4b^3 - cx} \)
6. \( (c + a)^4 - (y - x)^3 \)
7. \( 15b + 3y - c^2 \)
8. \( \frac{y}{b} + 5z - \frac{9x}{bc} \)
9. \( 56b - \sqrt{cy \div b} \)
10. \( \frac{\sqrt{5z + x + 8z}}{\sqrt{4x + 9 - (b + c)}} \)

Homework 2.6
Given that \( m = 6, k = 4, c = 5, x = 2, y = 8 \) and \( z = 50 \) simplify each expression using the order of operations.

1. \( \sqrt{k + c} + 2m^2 + z \div x \)
2. \( 8c - (xy - m) + 4 \cdot \sqrt{7y + 25} \)
3. \( 3x^5 + (c - k)^{10} + \frac{216}{y + 1} \)
4. \(40 - 5 \cdot \sqrt{z - 49} + mk\)

5. \(\frac{\sqrt{144+2y+mk}}{cx^2-k^2}\)

6. \(y^2 - x^5 + m^2\)

7. \(9z - 5k - c^2\)

8. \(\frac{4z}{x+y} + \frac{7k+2}{c}\)

9. \(12 \cdot \sqrt{k+c} + c - z \div x\)

10. \(\frac{\sqrt{5y+c+k+c-x}}{\sqrt{k+x+1}}\)
Section 2.7 – Area

If we want to carpet a floor, we need to know how much carpet we are going to need to cover the floor. We would need to compute the area of the floor. Knowing the area of a plot of land is also important in farming. Farmers need this information to know how much land they have available to plant crops.

The area of a closed figure, or region, is the amount of land inside that figure. Area is measured in square units, such as square feet ($ft^2$), square meters ($m^2$), square yards ($yd^2$) and square miles ($mi^2$). To compute the area of a figure, we first select a square unit to measure area with, and then estimate how many square units fit inside the figure. For example, if we use $ft^2$ as the unit to measure area, then the figures shaded green below have the indicated area:

- Area = 72 square units
- Area = 21 square units
- Area ≈ 28 square units

The symbol $\approx$ means “approximately equal to.” Below are the formulas to find the area of a square, a rectangle, a parallelogram and a triangle.

**Square**

Area = Side x Side = $(5m)(5m)=25 \text{ m}^2$
**Rectangle**

Area = Base x Height = (18yd)(12yd) = 216 yd²

**Parallelogram**

Area = Base x Height = (15in)(8in) = 120 in²

**Triangle**

Area = \( \frac{\text{Base} \times \text{Height}}{2} \)

- \( \frac{8\text{ft} \times 10\text{ft}}{2} = \frac{80\text{ft}^2}{2} = 40\text{ ft}^2 \)
- \( \frac{5\text{cm} \times 12\text{cm}}{2} = \frac{60\text{ cm}^2}{2} = 30\text{ cm}^2 \)
- \( \frac{9\text{m} \times 4\text{m}}{2} = \frac{36\text{ m}^2}{2} = 18\text{ m}^2 \)
Example 2.7.1

Determine the area of the green region. Write your answer in square units. You may use an area formula or visually estimate the number of square units inside the figure.

Area = 24 square units

Area = 36 square units

Example 2.7.2

A) Explain the difference between the area and the perimeter of a closed figure. Area is the amount of land inside a closed geometric figure, whereas perimeter is the total distance around a closed geometric figure.

B) Find the area and perimeter of the following rectangle. Assume that each little square represents 1 square meter (1 m²).

Area = (16 m)(9 m) = 144 m²

Perimeter = 16m+9m+16m+9m = 50m
Example 2.7.3
Find the area and perimeter of a square whose sides measure 17 feet each.

Answer:
\[
\text{Area} = 17\text{ft} \times 17\text{ft} = 289\text{ft}^2
\]
\[
\text{Perimeter} = 17\text{ft} + 17\text{ft} + 17\text{ft} + 17\text{ft} = 4 \times 17\text{ft} = 68\text{ft}
\]

Example 2.7.4
Find the area and perimeter of a rectangle which has a base measuring 35 yards and a height measuring 16 yards.

Answer:
\[
\text{Area} = 35\text{yd} \times 16\text{yd} = 560\text{yd}^2
\]
\[
\text{Perimeter} = 35\text{yd} + 16\text{yd} + 35\text{yd} + 16\text{yd} = 102\text{yd}
\]

Example 2.7.5
Find the area and perimeter of a triangle which has a base measuring 6 inches, a height measuring 8 inches and the third side measures 10 inches.

Answer:
\[
\text{Area} = \frac{6\text{ in} \times 8\text{ in}}{2} = \frac{48\text{ in}^2}{2} = 24\text{ in}^2
\]
\[
\text{Perimeter} = 6\text{ in} + 8\text{ in} + 10\text{ in} = 24\text{ in}
\]
The following videos show how to compute the area of various geometric figures.

Square, Rectangle & Parallelogram:

http://www.youtube.com/watch?v=IFRGGD07Cgw
http://www.youtube.com/watch?v=zMHtM5xH8t4

Triangle:  http://www.youtube.com/watch?v=7wskaOGelEA

Classwork 2.7
1. Determine the area of the green region. Write your answer in squared units. You may use an area formula or visually estimate the number of square units inside the figure.

Area = 12 squared units  Area = 30 squared units

2. Find the area and perimeter of a square whose sides measure 25 feet each.

\[ \text{Area} = 625 \text{ ft}^2 \quad \text{Perimeter} = 100 \text{ ft} \]
3. Find the area and perimeter of a rectangle which has a base measuring 5 yards and a height measuring 2 yards.

\[ \text{Area} = 10 \text{ yd}^2 \quad \text{Perimeter} = 14 \text{ yd} \]

4. Find the area and perimeter of a triangle which has a base measuring 4 inches, a height measuring 3 inches and the third side measures 5 inches.

\[ \text{Area} = 6 \text{ in}^2 \quad \text{Perimeter} = 12 \text{ ft} \]

5. Find the area of a parallelogram having a base = 126 cm and a height = 47 cm.

\[ \text{Area} = 5,922 \text{ cm}^2 \]

6. If a square has an area of 196 square meters, how long is each side of the square? Each side measures 14 meters.

7. If a square has a perimeter of 1,132 yards, how long is each side of the square?

Each side measures 283 yards.

8. Find the area of the shaded region.

The shaded region has an area = 580 m².
Homework 2.7

1. Find the area and perimeter of the following region. Assume that each little square represents 1 square meter (1 m²).

   Area = 152 m²
   Perimeter = 54 m

2. Find the area and perimeter of a square whose sides measure 7 feet each.

   \[ \text{Area} = 49 \text{ ft}^2 \quad \text{Perimeter} = 28 \text{ ft} \]

3. Find the area and perimeter of a rectangle which has a base measuring 23 yards and a height measuring 15 yards.

   \[ \text{Area} = 345 \text{ yd}^2 \quad \text{Perimeter} = 76 \text{ yd} \]

4. Find the area and perimeter of a triangle which has a base measuring 10 inches, a height measuring 24 inches and the third side measures 26 inches.

   \[ \text{Area} = 120 \text{ in}^2 \quad \text{Perimeter} = 60 \text{ in} \]

5. Find the area of a parallelogram having a base that measures 85 feet and a height that measures 98 feet.

   \[ \text{Area} = 8,330 \text{ ft}^2 \]

6. A rectangle has a perimeter of 10,128 feet. If one of the longer sides measures 3,128 feet, how long is one of the shorter sides?

   Each of the shorter sides measures 1,936 feet.
7. A rectangle has an area of 300 square meters. If its base measures 12 meters, how long is its height? *The height of the rectangle measures 25 meters.*

8. Find the area of the shaded region. Assume that the outer shape is a rectangle and the inner shape is a parallelogram.

*The shaded region has an area $= 3,456 \text{ ft}^2$.***
Chapter 2 Test

1. Which numbers are factors of 368,270?
   a. 1
   b. 5
   c. 2
   d. 9
   e. 3
   f. 6
   g. 4
   h. 10

2. Which numbers are multiples of 9?
   a. 6,786
   b. 3
   c. 432,012
   d. 57
   e. 9
   f. 36

3. Which numbers are divisible by 6?
   a. 623,014
   b. 4,206
   c. 582
   d. 16,931
   e. 4,678,583
   f. 9,300

4. Which numbers are composite?
   a. 37
   b. 1
   c. 45,762
   d. 712,323
Find the prime factorization of the following numbers.

5. 900
   \[900 = 2^2 \times 3^2 \times 5^2\]

6. 22,275
   \[22,275 = 3^4 \times 5^2 \times 11^1\]

7. 2,760
   \[2,760 = 2^3 \times 3^1 \times 5^1 \times 23^1\]

8. List or make a table of all the factors of 40.

Answer:

\[
\begin{array}{c|c}
  \text{Factor} & 40 \\
  \hline
  1 & 40 \\
  2 & 20 \\
  4 & 10 \\
  5 & 8 \\
\end{array}
\]

Find the Greatest Common Factor (GCF) of each set of numbers.

9. 28, 80 and 98
   \[\text{GCF} = 2\]

10. 360, 1,200 and 720
    \[\text{GCF} = 120\]

Find the Least Common Multiple (LCM) of each set of numbers.

11. 9, 30 and 40
    \[\text{LCM} = 360\]

12. 80 and 100
    \[\text{LCM} = 400\]

Compute the following square roots.

13. \(\sqrt{324}\)
    \[18\]

14. \(\sqrt{81}\)
    \[9\]

Simplify each expression by using the order of operations.

15. \(690 \div 3 \cdot 2 + 145 + 50 \div 2 - 4^2\)
    \[614\]

16. \(\frac{32 + 4 + \sqrt{100} - 6}{\sqrt{49} - 65 \div 13}\)
    \[6\]
17. Evaluate the expression \( \frac{k}{m} + g^3 - r^7 + (m - g)^4 \) given that \( k = 120, m = 6, g = 4, r = 1 \).

18. Find the area and perimeter of a rectangle that has a base = 67 meters and a height = 25 meters.

\[
\text{Area} = 1675 \text{ m}^2 \quad \text{Perimeter} = 184 \text{ m}
\]

19. Find the area and perimeter of a parallelogram that has a base = 23 yards, a height = 12 yards and a side that is not a base measures 13 yards.

\[
\text{Area} = 276 \text{ yd}^2 \quad \text{Perimeter} = 72 \text{ yd}
\]

20. Find the area and perimeter of the following triangle:

\[
\text{Area} = 588 \text{ ft}^2 \quad \text{Perimeter} = 112 \text{ ft}
\]
Cumulative Review – Chapters 1-2

Perform each operation.

1) Add: $3,897,204 + $634,911 + $76,521 + $3,669

$4,612,305

2) Subtract: $47,620 - $38,751

$8,869

3) 

\[
\begin{array}{c}
6 & 2 & 8 & 5 \\
7 & 1 & 5 & 4 \\
8 & 0 & 0 & 2 \\
+ & 1 & 4 & 3 \\
\hline
21,584
\end{array}
\]

4) 

\[
\begin{array}{c}
9 & 3 & 2 & 1 & 1 \\
2 & 3 & 1 & 8 & 7 \\
7 & 7 & 0 & 6 \\
+ & 5 & 6 & 7 & 9 \\
\hline
129,783
\end{array}
\]

5) 

\[
\begin{array}{c}
8 & 2 & 6 & 3 & 5 \\
9 & 9 & 4 & 0 & 0 \\
9 & 7 & 7 & 5 \\
+ & 6 & 2 & 7 \\
\hline
192,437
\end{array}
\]

6) 

\[
\begin{array}{c}
9 & 3 & 0 & 0 \\
- & 5 & 6 & 7 & 1 \\
\hline
3,629
\end{array}
\]

7) 

\[
\begin{array}{c}
1 & 6 & 4 & 0 & 5 \\
- & 9 & 3 & 8 & 5 \\
\hline
7,020
\end{array}
\]

8) 

\[
\begin{array}{c}
6 & 0 & 0 & 0 & 0 \\
- & 5 & 1 & 0 & 8 & 6 \\
\hline
8,914
\end{array}
\]

9) Subtract 67,821 from 100,000.

32,179

10) 

\[
\begin{array}{c}
2 & 9 & 6 \\
x & 8 & 0 \\
\hline
23,680
\end{array}
\]

11) 

\[
\begin{array}{c}
3 & 5 & 9 & 7 & 1 \\
x & 8 & 4 \\
\hline
3,021,564
\end{array}
\]

12) 

\[
\begin{array}{c}
8 & 4 & 0 \\
x & 7 & 5 & 1 \\
\hline
630,840
\end{array}
\]

13) Multiply: 4,000 \times 100 = 400,000

14) 

\[7 \div 9 \ 7 \ 6 \ 5 \]
15. \( \frac{218}{R50} \)  \\
16. \( 875,918 \div 6 \)  \\
17. Divide: \( \frac{3,816}{4} = 954 \)

18) List or make a table of all the factors of 200.

Answer:

<table>
<thead>
<tr>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 200</td>
</tr>
<tr>
<td>2 100</td>
</tr>
<tr>
<td>4 50</td>
</tr>
<tr>
<td>5 40</td>
</tr>
<tr>
<td>8 25</td>
</tr>
<tr>
<td>10 20</td>
</tr>
</tbody>
</table>

19) Circle the numbers that are divisible by 7:

\[ 34, \ 80, \ 616, \ 21, \ 80, \ 56, \ 2422 \]  

\[ 616, 21, 56, 2422 \]

Simplify using the order of operations. Reduce any fractional answers.

20) \( (520 \div 40)^2 - 360 \div 120 \cdot 4 + 5^3 \)  

\( 282 \)

21) \( \frac{6 \cdot \sqrt{9 - (3^2 + 3^2)}}{\sqrt{144 + 6 \cdot 7^2 + 1}} \)  

\( 0 \)

22) \( 2000 - 2 \cdot \left( \frac{40}{10} \right)^4 + \sqrt{\frac{100}{25}} \)  

\( 1,490 \)

23) \( \frac{3500}{25} + \frac{720}{9} - \frac{52}{4} \)  

\( 207 \)

24) \( 16 \times 10^3 - 28 \times 10^2 \)  

\( 13,200 \)
Find the greatest common factor (GCF) of each set of numbers. If the GCF is 1, write “relatively prime.”

25) 2800, 350 and 140  
   GCF = 70  
   The numbers are relatively prime

Find the least common multiple (LCM) of each set of numbers.

26) 64, 121 and 81  
   LCM = 600

Write a sentence to answer the following question.

27) Jim’s work location is 23 miles from his home. If he works Monday through Friday, how many miles does Jim drive in total each week?
   
   *Jim drives a total of 230 miles each week.*

28) Bethany bought 3 dresses that cost $95 each and two pairs of shoes that cost $79 each. What was the total cost, before the sales tax, of the items that Bethany bought?
   
   *The total cost, before the sales tax, of the items Bethany bought was $443.*

30) It would take Lloyd 125 months to pay the $125,000 difference, which is a little over 10 years.

31) Lloyd is considering buying a house. The house he is interested in buying has a selling price of $225,000. He already secured a loan for $100,000. If he were to pay the difference in monthly payments of $1,000, how many months would it take him to pay the difference?
   
   *It would take Lloyd 125 months to pay the $125,000 difference, which is a little over 10 years.*

32) William is selling his old video game console for $50 and each game for $7. If William sold the console and 15 video games, how much did he earn?
   
   *William earned $155 selling the video game console and 15 video games.*

33) Martha is flying from Los Angeles to Chicago for a science conference. The cost of the roundtrip flight is $478. She’s staying at a hotel that charges $179 per night. If Martha stays at the hotel 3 nights, what will be the total expense for the flight and hotel?
   
   *The total expense for the flight and hotel will be $1,015.*
34) Chris scores an average of 18 points per game. At this rate, how many points will he score in total if he stays healthy and plays 82 games?

*Chris will score 1,476 points if he plays 82 games.*

35) Alice needs to pay $6,000 from a student loan. If she makes monthly payments of $250, how long will it take her to pay off this debt?

*It will take Alice 24 months (2 years) to pay off her student loan.*

36) T / F : 24 is a multiple of 3.  
*True*

37) T / F : 80 is a factor of 560.  
*True*

38) T / F : 60,471 is divisible by 9.  
*True*

39) T / F : 5 is a factor of 741,032.  
*False*

40) T / F: 4,521 is a prime number.  
*False*
Chapter 3

Multiplying and Dividing Fractions
# Chapter 3 Overview

By the end of this chapter, you will achieve mastery of the following concepts:

<table>
<thead>
<tr>
<th>Concept</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Understanding Fractions</strong></td>
<td>When a whole item is divided into equal parts, we refer to each piece as a fraction of the total. Here we have ( \frac{5}{6} ) of a pizza remaining (red) and ( \frac{1}{6} ) has been eaten (white).</td>
<td>( \frac{48}{60} \div \frac{4}{4} = \frac{12}{15} \div \frac{3}{3} = \frac{4}{5} )</td>
</tr>
<tr>
<td><strong>Reducing a Fraction to Lowest Terms</strong></td>
<td>A fraction can be simplified to an equivalent fraction by dividing the numerator and denominator by a common factor.</td>
<td>( \frac{48}{60} \div \frac{4}{4} = \frac{12}{15} \div \frac{3}{3} = \frac{4}{5} )</td>
</tr>
<tr>
<td><strong>Proper Fraction</strong></td>
<td>A proper fraction has a numerator that is smaller than the denominator:</td>
<td>( \frac{1}{2}, \frac{10}{15}, \frac{52}{100}, \frac{31}{40} )</td>
</tr>
<tr>
<td><strong>Improper Fraction</strong></td>
<td>An improper fraction has a numerator equal to or greater than the denominator:</td>
<td>( \frac{3}{2}, \frac{19}{8}, \frac{12}{12}, \frac{125}{71} )</td>
</tr>
<tr>
<td><strong>Mixed Number</strong></td>
<td>A mixed number is a sum of a whole number and a fraction.</td>
<td>( 6 \frac{1}{2} = 6 + \frac{1}{2}, 34 \frac{7}{8} = 34 + \frac{7}{8}, 1 \frac{5}{9} = 1 + \frac{5}{9} )</td>
</tr>
<tr>
<td><strong>Improper Fraction to Mixed Number</strong></td>
<td>To write an improper fraction as a mixed number, we divide the numerator by the denominator, using the remainder and divisor to construct a fraction.</td>
<td></td>
</tr>
<tr>
<td><strong>Multiplying Fractions</strong></td>
<td>We first write any mixed numbers as improper fractions. We then cross-cancel, if possible, before multiplying the numerators and denominators.</td>
<td>( \frac{4}{2} \times \frac{8}{9} )</td>
</tr>
<tr>
<td><strong>Dividing Fractions</strong></td>
<td>We first write any mixed numbers as improper fractions. We then multiply the first fraction by the reciprocal of the second fraction, by following the procedure for multiplication of fractions.</td>
<td>( \frac{1}{3} \div \frac{2}{3} = \frac{25}{3} \div \frac{5}{5} )</td>
</tr>
</tbody>
</table>
A fraction represents a part of a whole item. For example, we can talk about eating a portion of a cake or a fraction of a cake. The difference between a “portion” versus a “fraction” is that when we talk about fractions it is assumed that the whole item (cake) was divided into equal parts.

For a rectangular cake, we will let a white piece represent a fraction of the cake that has been eaten. It is assumed that the cake has been cut into 8 equal pieces.

\[
\begin{align*}
\frac{0}{8} &= 0 \text{ no pieces eaten} \\
\frac{1}{8} &= 1 \text{ piece out of 8 was eaten} \\
\frac{2}{8} &= 2 \text{ pieces out of 8 were eaten} \\
\frac{3}{8} &= 3 \text{ pieces out of 8 were eaten} \\
\frac{4}{8} &= 4 \text{ pieces out of 8 were eaten} \\
\frac{6}{8} &= 6 \text{ pieces out of 8 were eaten}
\end{align*}
\]

Now, let’s assume that more than 1 cake was eaten:

\[
\begin{align*}
\text{1 cake eaten} + \text{1 cake eaten} + \frac{5}{8} \text{ cake eaten}
\end{align*}
\]

5 out of the 8 pieces were eaten
We represent the total number of cakes eaten as:

\[
1 + 1 + \frac{5}{8} = 2\frac{5}{8}
\]

We say that “two whole cakes and five eights of a third cake were eaten.”

**Definitions Involving Fractions**

**Numerator**: tells how many parts we are talking about (7 pieces eaten)

\[
\frac{7}{8}
\]

**Denominator**: tells into how many equal parts one whole cake was divided (8)

\[
\frac{1}{2} \text{ “one half”} \quad \frac{1}{3} \text{ “one third”} \quad \frac{1}{4} \text{ “one fourth”} \quad \frac{1}{5} \text{ “one fifth”}
\]

\[
\frac{1}{20} \text{ “one twentieth”} \quad \frac{1}{50} \text{ “one fiftieth”} \quad \frac{1}{100} \text{ “one hundredth”} \quad \frac{1}{1000} \text{ “one thousandth”}
\]

The following applet allows you to visualize a fraction of a whole item:

http://nlvm.usu.edu/en/nav/frames_asid_103_g_2_t_1.html?from=topic_t_1.html

You can practice naming fractions by following this link:

http://nlvm.usu.edu/en/nav/frames_asid_104_g_2_t_1.html?from=topic_t_1.html

\[
\frac{5}{12}
\]

This is a **proper fraction** because the numerator (5) is less than the denominator (12).

That is, \(5 < 12\). Other examples of proper fractions: \(\frac{19}{20}, \frac{85}{100}, \frac{1}{50}\)
A fraction is an **improper fraction** if the numerator is equal to or greater than the denominator. \( \frac{4}{3} \) is an improper fraction because \( 4 > 3 \). Other examples of improper fractions are: \( \frac{10}{8}, \frac{20}{20}, \frac{13}{5}, \frac{1}{1}, \frac{14}{7} \).

\( 2\frac{5}{8} \) This is called a **mixed number** because it consists of a whole number (2) and a proper fraction \( \frac{5}{8} \). Other examples of mixed numbers include:

\[ 1\frac{7}{13}, \quad 5\frac{2}{6}, \quad 7\frac{4}{50}, \quad 3\frac{1}{16} \]

**Note:** A mixed number is always greater than 1 and represents a sum of one or more whole items plus a fraction of a whole item.

For example, based on the fact that a mixed number represents a sum of a whole number and a fraction, it is clear that all mixed numbers are greater than 1.

\[ 1\frac{7}{13} = 1 + \frac{7}{13} \quad 5\frac{2}{6} = 5 + \frac{2}{6} \]

\[ 7\frac{4}{50} = 7 + \frac{4}{50} \quad 3\frac{1}{16} = 3 + \frac{1}{16} \]

**Note:** Improper fractions, which have the numerator greater than or equal to the denominator, can always be written as a mixed number or as a whole number.
Example 3.1.1

Write \( \frac{13}{5} \) as a mixed number.

Remembering that the fraction bar represents division \((13 \div 5)\), we see that after dividing 13 by 5 we get a quotient of 2 and a reminder of 3. This means that

\[
\frac{13}{5} = 2 \frac{3}{5}
\]

because

\[
\frac{13}{5} = 2 \frac{3}{5}
\]

Example 3.1.2

Write \( \frac{50}{15} \) as a mixed number.

This improper fraction represents the division \( 50 \div 15 \). We see that after dividing 50 by 15 we get a quotient of 3 and a reminder of 5. This means that

\[
\frac{50}{15} = 3 \frac{5}{15}
\]

because

\[
\frac{50}{15} = 3 \frac{5}{15}
\]

Example 3.1.3

Simplify \( \frac{4}{4} \) completely.

This fraction is the same as \( 4 \div 4 \). After dividing 4 by 4 we get a quotient of 1 and a reminder of 0. Therefore,

\[
\frac{4}{4} = 1 \frac{0}{4} = 1 + \frac{0}{4} = 1 + 0 = 1
\]
To write any mixed number, such as $3 \frac{1}{2}$, as an improper fraction we multiply the denominator times the whole number, and then add the numerator. We leave the denominator the same when we write the improper fraction.

$$3 \frac{1}{2} = \frac{7}{2}$$

For $10 \frac{7}{8}$ we have

$$10 \frac{7}{8} = \frac{87}{8}$$

Example 3.1.4

Write $8 \frac{5}{6}$ as an improper fraction.

$$8 \frac{5}{6} = \frac{53}{6}$$

Example 3.1.5

Write $12 \frac{1}{4}$ as an improper fraction.

$$12 \frac{1}{4} = \frac{49}{4}$$

Since the fraction bar represents division, whole numbers can also be written as fractions. For example, $5 = \frac{5}{1}$ because this fraction represents $5 \div 1 = 5$.

$$8 = \frac{8}{1} \quad 36 = \frac{36}{1} \quad 1 = \frac{1}{1} \quad 0 = \frac{0}{1}$$
Notice that there are many ways to write whole numbers as fractions

\[
1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots = \frac{25}{25} = \ldots = \frac{73}{73} = \ldots
\]

\[
2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = \ldots = \frac{24}{12} = \ldots = \frac{60}{30} = \ldots
\]

\[
3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} = \frac{15}{5} = \ldots = \frac{63}{21} = \ldots = \frac{150}{50} = \ldots
\]

Proper fractions, which have the numerator smaller than the denominator, cannot be written as a mixed number because all proper fractions are less than 1 whole. For example,

\[
\frac{3}{4} < 1 \quad \frac{9}{10} < 1 \quad \frac{73}{120} < 1
\]

On the other hand, we can always write an improper fraction as a mixed number because all improper fractions are greater than or equal to 1 whole. For example,

\[
\frac{6}{5} = 1 \frac{1}{5} > 1 \quad \frac{27}{4} = 6 \frac{3}{4} > 1
\]

\[
\frac{32}{8} = 4 \frac{0}{8} = 4 > 1 \quad \frac{12}{12} = 1 \frac{0}{12} = 1
\]
Note: \( \frac{0}{8} = 0 \) because this means that we divide zero candies into 8 people \((0 \div 8)\), so each person gets zero candies because there are no candies at all. However, \( \frac{8}{0} \) has an undefined value because it makes no sense to divide 8 candies into zero people. This is so because we can’t distribute something when there is no one to give it to.

Here is a video on how to convert an improper fraction to a mixed number and a mixed number to an improper fraction:  
http://www.youtube.com/watch?v=5dAl5BSr-eY

### Classwork 3.1

Write each improper fraction as a mixed number or whole number.

1. \( \frac{38}{17} \)  
2. \( \frac{106}{5} \)  
3. \( \frac{42}{6} \)  
4. \( \frac{9}{1} \)

Write each mixed number as an improper fraction.

5. \( 2 \frac{1}{10} \)  
6. \( 8 \frac{5}{9} \)  
7. \( 1 \frac{3}{8} \)  
8. \( 12 \frac{3}{5} \)

Determine whether each statement is true or false.

9. \( \frac{12}{19} > 1 \)  
10. \( \frac{40}{2} = 20 \)  
11. \( \frac{5}{2} > 1 \)  
12. \( \frac{18}{3} > 7 \)

1) 2  
2) 21  
3) 7  
4) 9  
5) \( \frac{21}{10} \)  
6) \( \frac{77}{9} \)  
7) \( \frac{11}{8} \)  
8) \( \frac{63}{5} \)  
9) F  
10) T  
11) T  
12) F

### Homework 3.1

Write each improper fraction as a mixed number or whole number.

1. \( \frac{7}{3} \)  
2. \( \frac{234}{7} \)  
3. \( \frac{64}{4} \)  
4. \( \frac{17}{1} \)

Write each mixed number as an improper fraction.

5. \( 6 \frac{2}{5} \)  
6. \( 7 \frac{2}{3} \)  
7. \( 2 \frac{1}{5} \)  
8. \( 18 \frac{2}{3} \)

Determine whether each statement is true or false.

9. \( \frac{6}{25} > 1 \)  
10. \( \frac{45}{134} < 1 \)  
11. \( \frac{35}{7} > 8 \)  
12. \( \frac{60}{5} = 12 \)

1) 2  
2) 33  
3) 16  
4) 17  
5) \( \frac{32}{5} \)  
6) \( \frac{23}{3} \)  
7) \( \frac{11}{5} \)  
8) \( \frac{56}{3} \)  
9) F  
10) T  
11) F  
12) T
Section 3.2 – Simplifying Fractions

Whenever possible, we simplify a fraction by writing an equivalent fraction that represents the same amount of the whole item, but that uses smaller numbers in the numerator and in the denominator. This is called simplifying or reducing a fraction to its lowest terms.

Question: Is there an easy method to simplify a fraction? Yes. You divide the numerator and denominator by any factor greater than 1 that they have in common. You then continue dividing the new numerator and new denominator by a common factor until the only common factor is 1. That is, until the numerator and denominator become relatively prime, in which case the fraction is said to be in lowest terms.

For example, cutting a pizza into 8 equal pieces and eating 2 of them is the same as cutting the pizza into 4 equal pieces and eating 1 piece, as you can see from the figures above. This means that \( \frac{2}{8} \) is equal to \( \frac{1}{4} \), and we write this fact as \( \frac{2}{8} = \frac{1}{4} \). We have reduced the fraction \( \frac{2}{8} \) to its lowest terms: \( \frac{1}{4} \).

In order to reduce a fraction to its lowest terms, you must know your times table well and must understand the concept of factor, common factor and multiple. Please read Section 2.1 if you need to review these concepts. Additionally, the rules of divisibility presented in Section 2.2 will be extremely useful when trying to simplify a fraction to its lowest terms.
Simplify \( \frac{240}{400} \) completely.

\[
\begin{align*}
\frac{240}{400} &= \frac{24 \div 2}{40 \div 2} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5} \\
\end{align*}
\]

To reduce \( \frac{240}{400} \) to its lowest terms, we made use of the divisibility rule for 10 and the divisibility rule for 2 to verify that 10 and 2 are common factors of the numerator and denominator.

One way to determine whether any two fractions \( \frac{a}{b} \) and \( \frac{c}{d} \) are equivalent is to check whether \( a \cdot d = b \cdot c \). This is called cross multiplication and should not to be confused with cross cancellation, which you will learn in the next section. In the example above, we obtained the result \( \frac{240}{400} = \frac{3}{5} \). Since \( 240 \times 5 = 1200 \) and \( 400 \times 3 = 1200 \), this confirms that \( \frac{240}{400} \) and \( \frac{3}{5} \) are indeed equivalent fractions.

**Question:** Is there a faster method to simplify a fraction? Yes. You divide the numerator and denominator by the greatest common factor (GCF). Doing this leads to a fraction that is in lowest terms in one step.

For example, when we prime factorize 240 and 400 we see that

\[
\begin{align*}
240 &= 5^1 \cdot 2^4 \cdot 3^1 \\
400 &= 5^2 \cdot 2^4 \\
\end{align*}
\]

Hence, the GCF of 240 and 400 is \( 5^1 \cdot 2^4 = 80 \).

Therefore, we divide the numerator and denominator by 80 to reduce the original fraction to lowest terms in one step:

\[
\begin{align*}
\frac{240 \div 80}{400 \div 80} &= \frac{3}{5} \\
\end{align*}
\]
Simplify \( \frac{72}{90} \) completely.

A process to reduce this fraction that many students consider easy, is to use the divisibility rules and the knowledge of the times tables to find common factors. For example, 36 and 45 are both divisible by 3 based on the divisibility rule for 3.

Dividing by 3 we get

\[
\frac{72}{90} = \frac{24}{30}
\]

Applying the divisibility rule for 3 again we get

\[
\frac{24}{30} = \frac{8}{10}
\]

Applying the divisibility rule for 2 gives

\[
\frac{8}{10} = \frac{4}{5}
\]

To verify that \( \frac{72}{90} = \frac{4}{5} \), we multiply \( 72 \times 5 = 360 \) and \( 90 \times 4 = 360 \). Since both multiplications give the same product (360), this result confirms that \( \frac{72}{90} \) and \( \frac{4}{5} \) are equivalent fractions.

The faster method is to find the greatest common factor (GCF) of 72 and 90 and then divide 72 and 90 by their GCF. Since the GCF of 72 and 90 is 18, we divide the numerator and denominator by 18.

\[
\frac{72}{90} = \frac{4}{5}
\]

Since 3 and 5 are relatively prime, we have reduced the original fraction to its lowest terms.

Since 4 and 5 are relatively prime, we have reduced the original fraction to its lowest terms.

Example 3.2.2

Simplify \( \frac{72}{90} \) completely.

Since 3 and 5 are relatively prime, we have reduced the original fraction to its lowest terms.

You may use any method you prefer to reduce a fraction to lowest terms. Some students prefer to divide by the GCF right away, while others prefer using the divisibility rules. In any case, you will get the correct answer as long as you divide the numerator and denominator by the same number until the numerator and the denominator are as small as possible.
denominator become relatively prime. That is, until the greatest common factor becomes 1.

\[ \frac{375}{900} \]

\[ \begin{align*}
375 & \div 5 = 75 \\
900 & \div 5 = 180
\end{align*} \]

\[ \begin{align*}
180 & \div 5 = 36 \\
36 & \div 3 = 12 \\
15 & \div 3 = 5
\end{align*} \]

Since 5 and 12 are relatively prime, we have reduced the original fraction to its lowest terms.

To verify that \( \frac{375}{900} = \frac{5}{12} \), we cross multiply \( 375 \times 12 = 4500 \) and \( 900 \times 5 = 4500 \). Since both multiplications give the same product (4500), this result confirms that \( \frac{375}{900} \) and \( \frac{5}{12} \) are equivalent fractions.

\[ \frac{198}{264} \]

\[ \begin{align*}
198 & \div 2 = 99 \\
264 & \div 2 = 132
\end{align*} \]

\[ \begin{align*}
99 & \div 3 = 33 \\
132 & \div 3 = 44
\end{align*} \]

\[ \begin{align*}
33 & \div 11 = 3 \\
44 & \div 11 = 4
\end{align*} \]

Since 3 and 4 are relatively prime, we have reduced the original fraction to its lowest terms.

To verify that \( \frac{198}{264} = \frac{3}{4} \), we cross multiply \( 198 \times 4 = 792 \) and \( 264 \times 3 = 792 \). Since both multiplications give the same product (792), this result confirms that \( \frac{198}{264} \) and \( \frac{3}{4} \) are equivalent fractions.
Example 3.2.5

Reduce \(\frac{32}{64}\) to lowest terms.

\[
\begin{align*}
\frac{32}{64} &= \frac{32 \div 32}{64 \div 32} = \frac{1}{2} \\
&= \frac{64}{32} \div 2
\end{align*}
\]

Since 1 and 2 are relatively prime, we have reduced the original fraction to its lowest terms.

To verify that \(\frac{32}{64} = \frac{1}{2}\), we cross multiply \(32 \times 2 = 64\) and \(64 \times 1 = 64\). Since both multiplications give the same product (64), this result confirms that \(\frac{32}{64}\) and \(\frac{1}{2}\) are equivalent fractions.

Example 3.2.6

Reduce \(\frac{162}{234}\) to lowest terms.

\[
\begin{align*}
\frac{162}{234} &= \frac{162 \div 2}{234 \div 2} = \frac{81 \div 9}{117 \div 9} = \frac{9}{13} \\
&= \frac{234}{162} \div 2
\end{align*}
\]

Since 9 and 13 are relatively prime, we have reduced the original fraction to its lowest terms.

To verify that \(\frac{162}{234} = \frac{9}{13}\), we cross multiply \(162 \times 13 = 2106\) and \(234 \times 9 = 2106\). Since both multiplications give the same product (2106), this result confirms that \(\frac{162}{234}\) and \(\frac{9}{13}\) are equivalent fractions.

Example 3.2.6

Reduce \(\frac{60}{420}\) to lowest terms.

\[
\begin{align*}
\frac{60}{420} &= \frac{60 \div 10}{420 \div 10} = \frac{6 \div 6}{42 \div 6} = \frac{1}{7} \\
&= \frac{420}{60} \div 10
\end{align*}
\]

Since 1 and 7 are relatively prime, we have reduced the original fraction to its lowest terms.
To verify that \( \frac{60}{420} = \frac{1}{7} \), we cross multiply \( 60 \times 7 = 420 \) and \( 420 \times 1 = 420 \). Since both multiplications give the same product (420), this result confirms that \( \frac{60}{420} \) and \( \frac{1}{7} \) are equivalent fractions.

**Warning:** \( \frac{1}{7} \neq 7 \). Any fractional answers like \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \), etc. should be left as they are. Remember that these are proper fractions because the numerator is smaller than the denominator. **All proper fractions are less than 1.** That is why to say that \( \frac{1}{2} = 2 \) or \( \frac{1}{3} = 3 \) would be incorrect.

Here’s an instructional video that shows two ways to reduce a fraction to lowest terms.  
http://www.youtube.com/watch?v=3QHf3k2x3gk

**Classwork 3.2**

Simplify completely. If the simplified expression is undefined, say so. Write improper fractions as mixed numbers.

1) \[ \frac{30}{70} \]
2) \[ \frac{48}{\sqrt{64}} \]
3) \[ \frac{81}{108} \]

4) \[ \frac{60}{150} \]
5) \[ \frac{28}{98} \]
6) \[ \frac{50}{1000} \]

7) \[ \frac{12+2+10}{15+5-1} \]
8) \[ \frac{25-21+3}{\sqrt{16}} \]
9) \[ \frac{5^2-3^2}{3(7+1)} \]
Simplify completely. Write improper fractions as mixed numbers. If the simplified expression is undefined, say so.

1) \( \frac{20}{24} \)  
2) \( \frac{480}{540} \)  
3) \( \frac{22}{100} \)  

4) \( \frac{55}{99} \)  
5) \( \frac{77}{63} \)  
6) \( \frac{64}{256} \)

7) \( \frac{2^3 \cdot 5}{2 \cdot 5^2} \)  
8) \( \frac{32 \div 2 - 16}{10 \div 5} \)  
9) \( \frac{25 - 10}{32 - 2^5} \)

10) \( \frac{\sqrt{100}}{2} + \frac{21 - 4^2}{5} \)  
11) \( \frac{80 \div 4 + \sqrt{9} - 5}{32 - 14 \div 7} \)  
12) \( \frac{12 \cdot 6 + 4}{200 \div 20} \)

1) \( \frac{5}{6} \)  
2) \( \frac{8}{9} \)  
3) \( \frac{11}{50} \)  
4) \( \frac{5}{9} \)  
5) \( \frac{1}{2} \)  
6) \( \frac{1}{4} \)  
7) \( \frac{4}{5} \)  
8) \( 0 \)  
9) undefined  
10) 6  
11) \( \frac{3}{5} \)  
12) \( \frac{3}{5} \)

The following website has many additional exercises with solutions on how to reduce a fraction to lowest terms, as well as checking whether two fractions are equivalent:

[http://cnx.org/content/m34927/latest/?collection=col10615/latest](http://cnx.org/content/m34927/latest/?collection=col10615/latest)
Section 3.3 – Multiplying Fractions

In this section you will learn how to multiply fractions. Having learned to reduce fractions to lowest terms in Section 3.2, you will now be able to put that skill into practice. Not only will you be able to simplify the fractional answers you get, but the concept of common factors and the rules of divisibility will lead to an easy method to reduce fractions before multiplying them known as cross cancelling.

If you need to review how to reduce a fraction to lowest terms, please read Section 3.2. Also, the rules of divisibility presented in Section 2.2 will be very useful when we perform cross cancelling prior to multiplying the fractions. Please make sure you understand the material that was presented in Section 2.2 and Section 3.2 before continuing reading.

Just like we can multiply whole numbers such as $6 \times 8$, in real life applications we often need to multiply fractions such as $6\frac{1}{4} \times 8\frac{2}{5}$. The method to multiply fractions is outlined below.

<table>
<thead>
<tr>
<th>Method to Multiply Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Write the mixed numbers, if any, as improper fractions.</td>
</tr>
<tr>
<td>$\frac{1}{4} \times \frac{2}{5}$</td>
</tr>
<tr>
<td>$\frac{6}{4} \times \frac{5}{2}$</td>
</tr>
<tr>
<td><strong>Step 2:</strong> (Optional) Simplify the fractions by cross cancelling, or by separately reducing each fraction to lowest terms, before multiplying the fractions.</td>
</tr>
<tr>
<td>$25 \div 5 = 5$</td>
</tr>
<tr>
<td>$42 \div 2 = 21$</td>
</tr>
<tr>
<td>$4 \div 2 = 2$</td>
</tr>
<tr>
<td>$5 \div 5 = 1$</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Multiply all the numerators and then multiply all the denominators.</td>
</tr>
<tr>
<td>$\frac{5 \times 21}{2} = \frac{105}{2}$</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Ensure that your fractional answer is in lowest terms and that an improper fraction is written as a mixed number.</td>
</tr>
<tr>
<td>$\frac{105}{2} = 52\frac{1}{2}$</td>
</tr>
</tbody>
</table>
The cross cancelling step, although optional, is a time saver and we recommend that you always perform this step whenever possible. An important skill that will be needed to implement this step is to be able to identify common factors of numbers. For example, recall that 4 and 42 have 1 and 2 as common factors. Hence, a fraction such as \( \frac{42}{4} \) can be simplified in one step by dividing 42 and 63 by their GCF which is 2:

\[
\frac{42 \div 2}{4 \div 2} = \frac{21}{2}
\]

Let’s now look at some examples involving multiplication of fractions.

**Example 3.3.1**

Multiply \( \frac{5}{20} \cdot \frac{3}{7} \) and simplify your answer completely.

**Student 1**

\[
\frac{5 \cdot 3}{20 \cdot 7} = \frac{15}{140}
\]

\[
15 \div 5 = 3
\]

\[
140 \div 5 = 28
\]

3 and 28 are relatively prime.

**Student 2**

\[
\frac{5 \cdot 3}{20 \cdot 7}
\]

\[
5 \div 5 = 3
\]

\[
20 \div 5 = 7
\]

\[
\frac{1 \cdot 3}{4 \cdot 7} = \frac{3}{28}
\]

3 and 28 are relatively prime.

Student 1 skipped step 2 of the method for multiplying fractions. After multiplying the numerators and denominators, he still had to reduce the fraction to get it into lowest terms. Student 2 did step 2 before multiplying the numerators and denominators. Consequently, his fractional answer was already completely reduced.
**Example 3.3.2**

Multiply $\frac{9}{12} \cdot \frac{5}{15}$ and simplify your answer completely.

**Student 1**

\[
\frac{9}{12} \cdot \frac{5}{15} = \frac{45}{180}
\]

\[
\frac{45}{180} \div 5 = \frac{9}{36}
\]

\[
\frac{9}{36} \div 9 = \frac{1}{4}
\]

1 and 4 are relatively prime.

**Student 2**

\[
\frac{9}{12} \div \frac{3}{15} \cdot \frac{5}{15} \div 5
\]

\[
\frac{3}{4} \cdot \frac{1}{3} \div 3
\]

\[
\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4}
\]

1 and 4 are relatively prime.

Again, student 1 skipped step 2. After multiplying the numerators and denominators, he still had to reduce the fraction to get it into its lowest terms. Student 1 did step 2 before multiplying the numerators and denominators. Consequently, his fractional answer was already in lowest terms.

**Example 3.3.3**

Multiply $\frac{80}{100} \cdot \frac{20}{50}$ and simplify your answer completely.

**Student 1**

\[
\frac{80}{100} \cdot \frac{20}{50} = \frac{1600}{5000}
\]

\[
\frac{1600}{5000} \div 100 = \frac{16}{50}
\]

\[
\frac{16}{50} \div 2 = \frac{8}{25}
\]

8 and 25 are relatively prime.

**Student 2**

\[
\frac{80}{100} \div \frac{20}{50} \cdot \frac{20}{10}
\]

\[
\frac{80}{100} \div 20 \cdot \frac{20}{10}
\]

\[
\frac{4}{5} \cdot \frac{8}{25}
\]

8 and 25 are relatively prime.
Student 1 multiplied the numerators and the denominators and then reduced the fractional answer completely. Student 2 first reduced each fraction separately and then multiplied the numerators and denominators. The fractional answer was already reduced.

Example 3.3.4

Multiply $\frac{32}{125} \cdot \frac{5}{14}$ and simplify your answer completely.

<table>
<thead>
<tr>
<th>Student 1</th>
<th>Student 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{32}{125} \cdot \frac{5}{14} = \frac{160}{1750}$</td>
<td>$\frac{32}{125} \div 2 \cdot \frac{5 \div 5}{14 \div 2}$</td>
</tr>
<tr>
<td>$160 \div 10 = 16$</td>
<td>$16 \div 1 = 16$</td>
</tr>
<tr>
<td>$\frac{1750}{10} = \frac{175}{1}$</td>
<td>$\frac{25}{7} = \frac{16}{175}$</td>
</tr>
</tbody>
</table>

16 and 175 are relatively prime.

Student 1 multiplied the numerators and the denominators and then reduced the fractional answer completely. Student 2 first performed cross cancelling, and then multiplied the numerators and denominators. Consequently, the fractional answer was already reduced.
Example 3.3.5

Multiply \(5 \frac{1}{4} \cdot \frac{8}{35}\) and simplify your answer completely.

**Student 1**

\[
\begin{align*}
&5 \frac{1}{4} \cdot \frac{8}{35} \\
&= \frac{21}{4} \cdot \frac{8}{35} \\
&= \frac{168}{140} \\
&= \frac{84}{70} \\
&= \frac{42}{35} \\
&= \frac{6}{5} \\
&= 1 \frac{1}{5}
\end{align*}
\]

**Student 2**

\[
\begin{align*}
&5 \frac{1}{4} \cdot \frac{8}{35} \\
&= \frac{21}{4} \cdot \frac{8}{35} \\
&= \frac{21 \div 7}{4} \cdot \frac{8 \div 4}{35 \div 7} \\
&= \frac{3}{1} \cdot \frac{2}{5} \\
&= \frac{6}{5} = 1 \frac{1}{5}
\end{align*}
\]
Example 3.3.6

Multiply \( \frac{7}{9} \cdot 6\frac{3}{4} \) and simplify your answer completely.

<table>
<thead>
<tr>
<th>Student 1</th>
<th>Student 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{7}{9} \cdot \frac{3}{4} )</td>
<td>( \frac{7}{9} \cdot \frac{3}{4} )</td>
</tr>
<tr>
<td>( \frac{25}{9} \cdot \frac{27}{4} = 675 )</td>
<td>( \frac{25}{9} \cdot \frac{27}{9} = 27 \div 9 )</td>
</tr>
<tr>
<td>( \frac{9}{4} = \frac{36}{4} )</td>
<td>( \frac{9}{4} = \frac{36}{4} )</td>
</tr>
<tr>
<td>( 675 \div 9 = 75 )</td>
<td>( 25 \div 9 \cdot \frac{75}{4} )</td>
</tr>
<tr>
<td>( 36 \div 9 = \frac{4}{4} )</td>
<td>( \frac{25}{1} \cdot \frac{3}{4} = \frac{75}{4} )</td>
</tr>
<tr>
<td>( 75 \div 4 = 18 \frac{3}{4} )</td>
<td>( \frac{75}{4} = 18 \frac{3}{4} )</td>
</tr>
</tbody>
</table>

If you follow all of the steps of the method for multiplying fractions, including cross cancelling, you usually will not have to work with large numbers in the fractions. That is why it is recommended to perform cross cancelling, whenever possible, before multiplying the fractions. However, as you can see, there are other ways to get the same answer. Hence, you should adopt an approach that you are comfortable with.

**Note:** Never leave answers as improper fractions, unless you are told to do so. Always write answers that are improper fractions as mixed numbers. Also, don’t forget that proper fractions cannot be expressed as mixed numbers.
The next example shows what happens when we multiply two fractions that are flipped versions of each other. They are called **reciprocals** of each other.

**Example 3.3.7**

Multiply \( \frac{8}{17} \cdot \frac{17}{8} \) and simplify your answer completely.

<table>
<thead>
<tr>
<th><strong>Student 1</strong></th>
<th><strong>Student 2</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{8}{17} \cdot \frac{17}{8} = \frac{136}{136} = 1 )</td>
<td>( \frac{8}{17} \cdot \frac{17}{8} = \frac{8 \div 8}{17 \div 17} \cdot \frac{17 \div 17}{8 \div 8} = \frac{1 \cdot 1}{1 \cdot 1} = \frac{1}{1} = 1 )</td>
</tr>
</tbody>
</table>

**Note:** Whenever we multiply reciprocals of each other, the answer is always 1. For example,

\[
\frac{1 \cdot 5}{5 \cdot 1} = 1, \quad \frac{131 \cdot 54}{54 \cdot 131} = 1, \quad \frac{20 \cdot 64}{64 \cdot 20} = 1
\]

\[
\frac{1 \cdot 9}{9 \cdot 1} = 1, \quad \frac{1540 \cdot 2000}{2000 \cdot 1540} = 1, \quad \frac{3 \cdot 8}{8 \cdot 3} = 1
\]

Instructional videos on multiplication of fractions:

http://www.youtube.com/watch?v=BgH5xbvBJwo

http://www.youtube.com/watch?v=HQ3EXyW36es
Perform each multiplication. Simplify your answer completely. Write improper fractions as mixed numbers.

1) \(7 \cdot \frac{9}{14}\) 
2) \(\frac{8}{15} \cdot \frac{20}{32}\) 
3) \(5 \frac{1}{2} \cdot \frac{8}{33}\) 
4) \(2 \frac{1}{4} \cdot 5 \frac{1}{3}\) 
5) \(9 \frac{1}{3} \cdot 1 \frac{1}{2}\) 
6) \(\frac{430}{78} \cdot \frac{78}{430}\)

1) \(4 \frac{1}{2}\) 
2) \(1 \frac{1}{3}\) 
3) \(1 \frac{1}{3}\) 
4) 12 
5) 14 
6) 1

Perform each multiplication. Simplify your answer completely. Write improper fractions as mixed numbers.

1) \(3 \frac{1}{5} \cdot \frac{2}{4}\) 
2) \(\frac{8}{15} \cdot 9\) 
3) \(8 \frac{2}{5} \cdot \frac{5}{14}\) 
4) \(\frac{79}{4} \cdot \frac{4}{79}\) 
5) \(4 \frac{1}{3} \cdot 2 \frac{1}{4}\) 
6) \(\frac{3}{8} \cdot \frac{2}{6}\)

1) \(1 \frac{3}{5}\) 
2) \(4 \frac{4}{5}\) 
3) 3 
4) 1 
5) 9 \frac{3}{4} 
6) \(1 \frac{1}{8}\)

The following website has additional exercises with solutions involving multiplication of fractions:
http://cnx.org/content/m34928/latest/?collection=col10615/latest
Section 3.4 – Dividing Fractions

In the previous section you learned how to multiply fractions. The good news is that the method to divide fractions is similar to the method to multiply fractions, with one slight modification. Before you do the cross cancelling, you must flip the second fraction and then change the division sign to multiplication. You would then continue with the steps for multiplying fractions that you learned in the last section. That’s it. Below is the method to divide fractions.

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>Write the mixed numbers, if any, as improper fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{1}{4} \div \frac{2}{8} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{33}{4} \div \frac{17}{8} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2:</th>
<th>Flip the second fraction and change the division sign to a multiplication sign. The division problem now becomes a multiplication problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{33}{4} \cdot \frac{8}{17} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3: (Optional)</th>
<th>Simplify the fractions by cross cancelling, or by separately reducing each fraction to lowest terms, before multiplying the fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{33}{4} \div \frac{4}{17} ) \rightarrow ( \frac{33}{4} \cdot \frac{2}{17} ) \rightarrow ( \frac{66}{17} ) = ( 3\frac{15}{17} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4:</th>
<th>Multiply all the numerators and then multiply all the denominators.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{33}{4} \cdot \frac{2}{17} = \frac{66}{17} ) = ( 3\frac{15}{17} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 5:</th>
<th>Ensure that your fractional answer is in lowest terms and that an improper fraction is written as a mixed number.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{66}{17} = 3\frac{15}{17} )</td>
</tr>
</tbody>
</table>

Step 3, although optional, is a time saver. You should always do this step whenever possible.

Let’s now look at some examples involving division of fractions.
Example 3.4.1

Perform the division \( \frac{12}{25} \div \frac{4}{5} \) and simplify your answer completely.

\[
\begin{align*}
\text{Answer:} & \quad \frac{12}{25} \div \frac{4}{5} \\
& \rightarrow \quad 12 \div 4 \cdot \frac{5}{25} \div \frac{4}{4} \\
& \quad \frac{3}{5} \cdot \frac{1}{1} = \frac{3}{5}
\end{align*}
\]

3 and 5 are relatively prime.

Example 3.4.2

Perform the division \( \frac{3}{24} \div \frac{21}{40} \) and simplify your answer completely.

\[
\begin{align*}
\text{Answer:} & \quad \frac{3}{24} \div \frac{21}{40} \\
& \rightarrow \quad 3 \div 3 \cdot \frac{40}{24} \div \frac{21}{3} \\
& \quad \frac{5}{3} \cdot \frac{1}{7} = \frac{5}{21}
\end{align*}
\]

5 and 21 are relatively prime.
Example 3.4.3

Divide \( \frac{4\frac{1}{2}}{1\frac{2}{7}} \) and simplify your answer.

Answer:

\[
\frac{4\frac{1}{2}}{1\frac{2}{7}} = \frac{9}{2} \div \frac{9}{7}
\]

\[
\frac{9 \cdot 7}{2 \cdot 9} \rightarrow \frac{9 \div 9}{2} \cdot \frac{7}{9 \div 9}
\]

\[
\frac{1}{2} \cdot \frac{7}{1} = \frac{7}{2}
\]

\[
\frac{7}{2} = 3\frac{1}{2}
\]

Example 3.4.4

Divide \( \frac{84}{90} \div \frac{20}{36} \) and simplify your answer.

Answer:

\[
\frac{84}{90} \div \frac{20}{36}
\]

\[
\frac{84 \cdot 36}{90 \cdot 20} \rightarrow \frac{84 \div 2}{90 \div 2} \cdot \frac{36 \div 2}{20 \div 2}
\]

\[
\frac{42 \cdot 18}{45 \cdot 10} \rightarrow \frac{42 \div 2}{45 \div 9} \cdot \frac{18 \div 9}{10 \div 2}
\]

\[
\frac{21}{5} \cdot \frac{2}{5} = \frac{42}{25}
\]

\[
\frac{42}{25} = 1\frac{17}{25}
\]
Example 3.4.5

Divide \( 9 \frac{1}{3} \div \frac{14}{15} \) and simplify your answer.

\[
\begin{align*}
\text{Answer:} & \quad 9 \frac{1}{3} \div \frac{14}{15} \\
& \quad \frac{28 \cdot 15}{3 \cdot 14} \rightarrow \frac{28 \div 14}{3 \div 3} \cdot \frac{15 \div 3}{14 \div 14} \\
& \quad \frac{2 \cdot 5}{1 \cdot 1} = \frac{10}{1} = 10
\end{align*}
\]

Example 3.4.6

A complex fraction is a fraction that has a fraction in the numerator and/or denominator instead of a whole number.

Simplify the complex fraction completely:

\[
\begin{align*}
\text{Answer:} & \quad \frac{8}{\frac{9}{2}} \div \frac{6}{2} \\
& \quad \frac{8 \div 2}{\frac{9}{2}} \div \frac{6 \div 2}{2} \\
& \quad \frac{4 \cdot 2}{3 \cdot 1} = \frac{8}{3} \\
& \quad \frac{8}{3} = \frac{2}{3}
\end{align*}
\]
Note: Never leave answers as improper fractions, unless you are told to do so. Always write answers that are improper fractions as mixed numbers. Finally, don’t forget that proper fractions cannot be expressed as mixed numbers.

Instructional videos on dividing fractions:
http://www.youtube.com/watch?v=uMTqaEbXQ6k
http://www.youtube.com/watch?v=hSK04S6J3rI
http://www.youtube.com/watch?v=aV6gYM4cEJs

Classwork 3.4
Perform each division. Simplify your answer completely. Write improper fractions as mixed numbers.

1) \( \frac{18}{25} \div \frac{9}{10} \)  
2) \( \frac{4}{2} \div \frac{27}{10} \)  
3) \( \frac{7}{35} \div 14 \)

4) \( 7 \frac{1}{5} \div 4 \frac{1}{5} \)  
5) \( 20 \div 3 \frac{1}{5} \)  
6) \( \frac{3}{8} \div \frac{3}{8} \)

1) \( 1 \frac{4}{5} \)  
2) \( 1 \frac{2}{3} \)  
3) \( \frac{1}{70} \)  
4) \( 1 \frac{5}{7} \)  
5) \( 6 \frac{1}{4} \)  
6) \( 1 \)

Homework 3.4
Perform each division. Simplify your answer completely. Write improper fractions as mixed numbers.

1) \( \frac{14}{15} \div \frac{21}{30} \)  
2) \( \frac{3}{8} \div \frac{20}{30} \)  
3) \( \frac{4}{13} \div 6 \)

4) \( 12 \frac{1}{4} \div 2 \frac{1}{3} \)  
5) \( 12 \div 4 \frac{1}{2} \)  
6) \( \frac{5}{6} \div \frac{6}{5} \)

1) \( 1 \frac{1}{3} \)  
2) \( 4 \frac{11}{16} \)  
3) \( \frac{2}{39} \)  
4) \( 5 \frac{1}{4} \)  
5) \( 2 \frac{2}{3} \)  
6) \( \frac{25}{36} \)

The following website has additional exercises with solutions involving division of fractions:
http://cnx.org/content/m34929/latest/?collection=col10615/latest
Section 3.5 – Word Problems Involving Multiplication and Division of Fractions

Now that you have learned how to multiply and divide fractions, the challenge with word problems is to become good at determining whether the problem requires you to add, subtract, multiply or divide. In this section we will focus on problems involving multiplication and division of fractions. You should always try to look for keywords that serve as a hint as to what operation should be performed. Below are some examples of keywords that indicate that you should multiply or divide.

<table>
<thead>
<tr>
<th>Keyword / Phrase</th>
<th>Operation to Perform</th>
</tr>
</thead>
<tbody>
<tr>
<td>of</td>
<td>multiplication</td>
</tr>
<tr>
<td>times</td>
<td>multiplication</td>
</tr>
<tr>
<td>product</td>
<td>multiplication</td>
</tr>
<tr>
<td>find the area</td>
<td>multiplication</td>
</tr>
<tr>
<td>at this rate</td>
<td>multiplication</td>
</tr>
<tr>
<td>per (a rate is given)</td>
<td>multiplication</td>
</tr>
<tr>
<td>each (a rate is given)</td>
<td>multiplication</td>
</tr>
<tr>
<td>distribute</td>
<td>division</td>
</tr>
<tr>
<td>divide</td>
<td>division</td>
</tr>
<tr>
<td>cut into</td>
<td>division</td>
</tr>
<tr>
<td>separate/group into</td>
<td>division</td>
</tr>
<tr>
<td>per (asked to find a rate)</td>
<td>division</td>
</tr>
<tr>
<td>each (asked to find a rate)</td>
<td>division</td>
</tr>
</tbody>
</table>

There are instances when there is no explicit keyword in the word problem, and you will have to use your experience or the context of the problem to determine the appropriate operation to use.
Example 3.5.1

Tilly’s cookie recipe calls for \( \frac{5}{8} \) cup of sugar to make a batch of cookies. If we want to make \( \frac{1}{4} \) of a batch of cookies, how much sugar do we need?

The keyword “of” tells us that we must multiply the fractions because we need to find \( \frac{1}{4} \) of \( \frac{5}{8} \).

Answer:

\[
\frac{1}{4} \cdot \frac{5}{8} = \frac{5}{32}
\]

We need \( \frac{5}{32} \) cup of sugar to make \( \frac{1}{4} \) of a batch of cookies.

Example 3.5.2

For a party, 4 large pizzas weighing \( 3 \frac{3}{4} \) lb each were bought. What was the total weight of the 4 pizzas combined?

Answer:

\[
4 \cdot 3 \frac{3}{4} = \frac{4 \cdot 15}{4} = \frac{4 \div 4 \cdot 15}{1} = 15\frac{1}{1} = 15 \text{ lb}
\]

The total weight of the 4 pizzas is 15 pounds.
Example 3.5.3

A rectangle has a base that measures $5 \frac{3}{8}$ in and a height that measures $2 \frac{2}{5}$ in. What is the area of the rectangle?

Answer:

$$5 \frac{3}{8} \cdot 2 \frac{2}{5}$$

$$\frac{43}{8} \cdot \frac{12}{5} \rightarrow \frac{43}{8 \div 4} \cdot \frac{12 \div 4}{5}$$

$$\frac{43 \cdot 3}{2 \cdot 5} = \frac{129}{10}$$

$$\frac{129}{10} = 12 \frac{9}{10} \text{ in}^2$$

The area of the rectangle is $12 \frac{9}{10}$ squared inches.
Example 3.5.4

A tape measuring $15\frac{3}{4}$ feet must be cut into 6 pieces of equal length. How long must each piece be?

Answer:

$$15\frac{3}{4} \div 6$$

$$\frac{63}{4} \div \frac{6}{1}$$

$$\frac{63}{4} \cdot \frac{1}{6} \rightarrow \frac{63}{4} \div \frac{3}{6} \div \frac{1}{3}$$

$$\frac{21}{4} \cdot \frac{1}{2} = \frac{21}{8}$$

$$\frac{21}{8} = 2\frac{5}{8} \text{ in}$$

Each piece of tape must measure $2\frac{5}{8}$ inches.

Classwork 3.5

1. If a faucet leaks $1\frac{1}{4}$ quarts of water every hour, how much does it leak in 24 hours?

   The faucet leaks 30 quarts in 24 hours (1 day).

2. A piece of candy weighs $\frac{1}{8}$ pound. If Mark bought 32 pieces, how many pounds of candy did he buy?

   Mark bought 4 pounds of candy.
3. Maria needs \(\frac{3}{4}\) of a cup of sugar for one serving of her recipe. How many cups of sugar will she need for 5 servings?

\[\text{Maria will need } \frac{3}{4} \text{ cups of sugar for 5 servings of her recipe.}\]

4. A recipe for banana oat muffins calls for \(\frac{3}{4}\) cup of oats. You are making \(\frac{1}{2}\) of the recipe. How much oats should you use?

\[\text{I will need } \frac{3}{8} \text{ cup of oats to make half of the recipe.}\]

5. Giselle is covering the kitchen shelves in her new apartment. She has 12 \(\frac{1}{2}\) feet of shelving paper. How many shelves can she cover if each shelf is 1 \(\frac{1}{4}\) feet long?

\[\text{Giselle can cover exactly 10 shelves with the amount of shelving paper she has.}\]

6. Carolyn wants to divide 2 \(\frac{1}{2}\) yards of ribbon into three equal pieces. How long will each piece be?

\[\text{Each piece of ribbon will be } \frac{5}{6} \text{ yards long.}\]

7. A 20-inch ribbon needs to be cut into 3 equal pieces. What will be the length of each piece?

\[\text{Each piece of ribbon will be } 6 \frac{2}{3} \text{ inches long.}\]

8. A car was originally priced at $24,000. It is now selling for \(\frac{3}{4}\) of the original price. What is the new price of the car?

\[\text{The new price of the car is } $18,000.\]

9. The area of a rectangle is 14 cm\(^2\). If its base measures 6 \(\frac{2}{9}\) cm, how long is its height?

\[\text{The height of the rectangle measures } 2 \frac{1}{4} \text{ cm.}\]

10. A rectangle has a base measuring 2 \(\frac{1}{5}\) yards. If its height is twice as long as its base, what is the area of the rectangle?

\[\text{The area of the rectangle is } 9 \frac{17}{25} \text{ yd}^2.\]
Homework 3.5

1. A bicycle is on sale for $\frac{2}{3}$ of its original price. If the original price was $354, what is the new price of the bicycle?
   
   The new price of the bicycle is $236.

2. A chemical experiment requires $\frac{1}{3}$ ounce of sulfide. How many experiments can be run with $1\frac{2}{3}$ ounces of sulfide?
   
   Five experiments can be run with the amount of sulfide that is available.

3. A square has sides of length $3\frac{1}{2}$ cm. What is the area of the square?

   The area of the square is $12\frac{1}{4}$ cm².

4. A rectangle has a base measuring $8\frac{1}{4}$ ft and a height measuring $5\frac{1}{3}$ ft. What is the area of the rectangle?

   The area of the rectangle is $44$ ft².

5. A long stick of wood measures $7\frac{3}{5}$ ft. It needs to be cut into 4 equal pieces. How long will each piece be?

   Each piece of wood will be $1\frac{9}{10}$ feet long.

6. Suppose you need to divide $1\frac{3}{8}$ cups of cough syrup into 4 doses. How many cups of cough syrup will each dose consist of?

   Each dose will consist of $\frac{11}{32}$ cup of cough syrup.

7. To pass a math test, Joseph needs to answer correctly at least $\frac{4}{5}$ of the total number of questions. The exam consists of 40 questions. What is the minimum number of questions that Joseph must answer correctly to pass the math test?

   To pass the math test, Joseph needs to answer at least 32 questions correctly.

8. Pat is a biology tutor and she gives several tutoring sessions each day. If she works $4\frac{1}{2}$ hours a day, and if each tutoring session lasts $1\frac{1}{2}$ hr, how many tutoring sessions does she give each day?

   Pat gives 3 tutoring sessions each day.
9. Suzy used \(2 \frac{3}{8}\) gallons of paint to paint her bedroom. She needed \(\frac{1}{2}\) of that amount to paint the hallway. How much paint did Suzy use to paint the hallway?

\[\text{Suzy used } 1 \frac{3}{16} \text{ gallons of paint to paint the hallway.}\]

10. Michael has \(14 \frac{2}{3}\) yards of matting material. If he needs to cut mats that are \(\frac{5}{8}\) yd long, how many full mats can he make?

\[\text{Michael can make 23 full mats.}\]
Chapter 3 Test

1. Write each of the following improper fractions as a mixed number or whole number.
   \[ \frac{17}{5} \quad \frac{3}{5} \]
   \[ \frac{186}{19} \quad \frac{9}{19} \]
   \[ \frac{52}{4} \quad 13 \]
   \[ \frac{32}{32} \quad 1 \]

2. Write each of the following mixed numbers as an improper fraction.
   \[ 6 \frac{1}{2} \quad \frac{13}{2} \]
   \[ 14 \frac{2}{3} \quad \frac{44}{3} \]
   \[ 1 \frac{4}{7} \quad \frac{11}{7} \]
   \[ 9 \frac{7}{8} \quad \frac{79}{8} \]

3. Determine whether each of the following statements is true or false.
   \[ \frac{13}{6} > 2 \quad True \]
   \[ \frac{4}{7} > 5 \quad False \]
   \[ \frac{135}{5} < 20 \quad False \]
   \[ \frac{32}{77} > 1 \quad False \]
   \[ \frac{21}{35} < 1 \quad True \]

4. Circle the fractions that represent less than 1 whole.
   \[ \frac{5}{16} \]
   \[ \frac{12}{11} \]
   \[ \frac{78}{216} \]
   \[ 1 \frac{3}{100} \]
Simplify each of the following fractions by reducing it to lowest terms.

5. $\frac{720}{880}$
6. $\frac{350}{2800}$
7. $\frac{525}{615}$
8. $\frac{24}{36}$

Perform each multiplication. Simplify your answer completely. If your answer is an improper fraction, write it as a mixed number.

9. $\frac{8}{45} \cdot \frac{20}{24}$
10. $2\frac{2}{5} \cdot 1\frac{1}{4}$
11. $9\frac{5}{8} \cdot \frac{80}{121}$
12. $\frac{32}{45} \cdot 2\frac{1}{4}$

Perform each division. Simplify your answer completely. If your answer is an improper fraction, write it as a mixed number.

13. $\frac{6}{125} \div \frac{36}{50}$
14. $4\frac{1}{5} \div 1\frac{5}{7}$
15. $12\frac{1}{2} \div 3\frac{1}{8}$
16. $\frac{60}{300} \div 5$

17. Connie works as a teaching assistant at UC Irvine Monday through Friday. If she works $3\frac{3}{4}$ hours each day, how many hours does she work per week? Express your answer as a mixed number.

Connnie works $18\frac{3}{4}$ hours each week.

18. Blake needs to cut a long piece of wood that measures $10\frac{1}{2}$ feet into 4 equal pieces. How long will each piece be?

Each piece will be $2\frac{5}{8}$ feet long.

19. Find the area of a square whose sides measure $9\frac{3}{4}$ inches long. Express your answer as a mixed number.

$Area = 95\frac{1}{16} \text{in}^2$.

20. Find the area of a triangle whose base measures 14 meters and has a height that measures $20\frac{1}{7}$ meters. Simplify your answer completely.

$Area = 141 \text{m}^2$. 
Cumulative Review – Chapters 1-3

1) Add: $679 + $28,504 + $192,000
   221,183

2) Subtract: $34,661 minus $28,783
   $5,878

3) Multiply: 53 x 10^4
   530,000

4) Divide: 87,519 ÷ 13
   6732 R3

5) List or make a table of all the factors of 300.
   Answer:
   \[
   \begin{array}{c|c}
   \text{Factor} & \text{Product} \\
   \hline
   1 & 300 \\
   2 & 150 \\
   3 & 100 \\
   4 & 75 \\
   5 & 60 \\
   6 & 50 \\
   10 & 30 \\
   12 & 25 \\
   15 & 20 \\
   \end{array}
   \]

6) Circle the numbers that are prime:
   13, 16, 1, 327, 29, 65, 741

7) \( \frac{832 + 30 \cdot 4}{\sqrt{900 - 5^2 + 1}} \)  
   \( 158 \frac{2}{3} \)

8) 5 \cdot 6 + 2 - 2^3 + 4 \cdot (42 - 20)  
   112
9) \(100 - 2 \cdot \left(\frac{60}{30}\right)^5 + 8 \cdot \sqrt{256}\)  
10) \(\frac{54}{9} + \frac{320}{8} - \frac{27}{9}\)  
11) \(\frac{12000}{5} \div (40 \div 4) + (5 \times 10^2)\)  

Find the greatest common factor (GCF) of each set of numbers. If the GCF is 1, write “relatively prime.”

12) 27, 54  
13) 60, 12, 48  
14) 620 and 121  
15) 400, 80 and 1000

Relatively prime

Find the least common multiple (LCM) of each set of numbers.

16) 30, 50 and 100  
17) 85 and 20

Write a sentence to answer the following question.

18) A mother has a sick child and needs to divide liquid cough medicine into 6 equal doses. If she has \(1 \frac{1}{2}\) cups of cough syrup, how much cough syrup will each dose contain? Write your answer in terms of cups.

Each dose will consist of \(\frac{1}{4}\) cup of cough syrup.

Multiply or divide. Simplify your answer completely. If your answer is an improper fraction, write it as a mixed number.

19) \(\frac{9}{25} \cdot \frac{100}{27}\)  
20) \(\frac{14}{35} \cdot \frac{35}{14}\)  
21) \(4 \frac{2}{5} \cdot \frac{10}{77}\)  
22) \(\left(2 \frac{3}{4}\right)^2\)
23) \( \frac{60}{125} \div \frac{12}{45} \) 
24) \( 6 \frac{1}{2} \div \frac{26}{50} \) 
25) \( 8 \div 2 \frac{2}{3} \cdot 5 \) 

\[ \begin{align*} 
&= 1 \frac{4}{5} \\
&= 12 \frac{1}{2} \\
&= 15 \\
\end{align*} \]

26) Tiffany puts $135 in her savings account each month. She currently has $2,800 in her savings account. If she continues to save the same amount each month, how much will she have in her savings account in 6 months? $3,610

27) Ray needs to divide a stick of plastic measuring \( 2 \frac{3}{4} \) meters into 6 equal pieces. How long will each piece be? \( \frac{11}{24} \) meters long

28) Carmen has paid $18,500 of a $30,000 loan. She would like to pay the rest of the balance in monthly payments of $125. How many payments will she need to make? 92 payments

29) Lynn spent a total of \( 8 \frac{3}{4} \) hours working on her homework this week. Express the total time that Lynn spent on her homework in minutes. 525 minutes

30) In a chemistry course, \( \frac{1}{5} \) of the students received an “A” in Exam 1 and \( \frac{3}{8} \) of the students received a “B.” If a total of 40 students took Exam 1,

\begin{itemize}
  \item how many students received a grade of “A”? 8 students
  \item how many students received a grade of “B”? 15 students
  \item what fraction of the students did not receive an “A” or a “B”? \( \frac{17}{40} \)
\end{itemize}
Chapter 4

Adding and Subtracting Fractions
# Chapter 4 Overview

By the end of this chapter, you will achieve mastery of the following concepts:

- **Like Fractions**
  - Fractions that have the same denominator, such as \( \frac{3}{10} \) and \( \frac{2}{10} \), are called like fractions. They can be added (or subtracted) by adding (subtracting) their numerators, and keeping the common denominator. We then reduce our answer and write it as a mixed number, if possible.
  - \[ \frac{3}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2} \]
  - \[ \frac{3}{10} - \frac{2}{10} = \frac{1}{10} \]

- **Adding and Subtracting Like Fractions**
  - To add or subtract unlike fractions, we must first write the original fractions using equivalent fractions that have the same denominator. We can then add (or subtract) these like fractions by adding (subtracting) their numerators, keeping the common denominator. We then reduce our answer and write it as a mixed number, if possible.
  - \[ \frac{6}{7} + \frac{4}{5} \rightarrow \frac{6 \cdot 5}{7 \cdot 5} + \frac{4 \cdot 7}{5 \cdot 7} \rightarrow \frac{30}{35} + \frac{28}{35} = \frac{58}{35} = \frac{1}{23} \]
  - \[ \frac{8}{25} - \frac{1}{10} \rightarrow \frac{8 \cdot 2}{25 \cdot 2} + \frac{1 \cdot 5}{10 \cdot 5} \rightarrow \frac{16}{50} + \frac{5}{50} = \frac{21}{50} \]

- **Adding and Subtracting Fractions Having Different Denominators**
  - When adding mixed numbers, we may add the whole numbers and the fractions separately, following the procedure for adding fractions described above.
  - \[ 8 \frac{3}{4} + 5 \frac{1}{6} \rightarrow (8 + 5) + \left( \frac{3}{4} + \frac{1}{6} \right) \rightarrow 13 + \frac{3 \cdot 3}{4 \cdot 3} + \frac{1 \cdot 2}{6 \cdot 2} \]
  - \[ = 13 + \frac{9}{12} + \frac{2}{12} = 13 + \frac{11}{12} = 13 \frac{11}{12} \]

- **Subtracting Mixed Numbers**
  - When subtracting mixed numbers, the second fraction must be equal to or smaller than the first fraction. Otherwise, we will have to “borrow.” Once the borrowing step is complete, we then subtract the whole numbers and the fractions separately, following the procedure for subtracting fractions described above.
  - \[ 20 \frac{4}{27} - 15 \frac{1}{3} \rightarrow 20 \frac{4}{27} - 15 \frac{1}{3} \cdot 9 \rightarrow 20 \frac{4}{27} - 15 \frac{9}{27} \]
  - We must borrow because \( \frac{4}{27} < \frac{9}{27} \). Since 20 = 19 + 1, we have
  - \[ 19 + 1 + \frac{4}{27} - 15 \frac{1}{3} \rightarrow 19 \frac{31}{27} - 15 \frac{9}{27} = \frac{4}{27} \]
Section 4.1 – Adding and Subtracting Fractions Having the Same Denominator

Whenever we need to add or subtract fractions that have the same denominator, all we need to do is add all the numerators, but we keep the denominator the same. For example,

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

On the other hand, it would be incorrect to say that

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{8} = \frac{1}{2}.$$  

The figures above make it clear that $$\frac{3}{4} + \frac{1}{4} = 1$$, not $$\frac{1}{2}$$.

Below is an outline of the procedure you should follow when adding or subtracting fractions that already have the same denominator.

<table>
<thead>
<tr>
<th><strong>Adding &amp; Subtracting Fractions that Have the Same Denominator</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Add or subtract the numerators. Leave the denominator the same.</td>
</tr>
<tr>
<td>Write the answer as a fraction having the same denominator as the original fractions.</td>
</tr>
</tbody>
</table>

| **Step 2:** Reduce the fractional answer to lowest terms. If the fraction is improper, write it as a mixed number. | \( \frac{10}{20} = \frac{1}{2} \) |
| | \( \frac{4}{20} = \frac{1}{5} \) |
Example 4.1.1

Find the perimeter of this rectangle.

\[
P = \frac{3}{16} \text{ in} + \frac{7}{16} \text{ in} + \frac{3}{16} \text{ in} + \frac{7}{16} \text{ in} = \frac{20}{16} \text{ in}
\]

\[
\frac{20 \div 4}{16 \div 4} = \frac{5}{4} \text{ inches}
\]

\[
\frac{5}{4} = 1 \frac{1}{4} \text{ inches}
\]

The perimeter of the rectangle is \( 1 \frac{1}{4} \) inches.

Example 4.1.2

Find the difference between \( \frac{37}{50} \) and \( \frac{22}{50} \).

Note: The phrase “find the difference between \( a \) and \( b \)” means \( a - b \), not \( b - a \).

For example, to find the difference between 45 and 29 we take away 29 from 45:

\[
45 - 29 = 16
\]
Example 4.1.3
The following list is the number of hours that Kevin spent each day this week doing science homework. How many total hours did Kevin spend on his science homework this week?

\[ \text{M: } \frac{3}{4} \text{ hr} \quad \text{T: } \frac{3}{4} \text{ hr} \quad \text{W: } \frac{1}{4} \text{ hr} \quad \text{Th: } \frac{1}{4} \text{ hr} \quad \text{F: } \frac{3}{4} \text{ hr} \]

Answer:
\[
\frac{3}{4} \text{ hr} + \frac{3}{4} \text{ hr} + \frac{1}{4} \text{ hr} + \frac{1}{4} \text{ hr} + \frac{3}{4} \text{ hr} = \frac{11}{4} \text{ hr}
\]
\[
\frac{11}{4} = 2 \frac{3}{4} \text{ hours}
\]

Kevin spent a total of \(2 \frac{3}{4}\) hours on his science homework this week. This means that Kevin spent 2 hours and 45 minutes on his science homework.

Example 4.1.4
Subtract \(\frac{17}{60}\) from \(\frac{33}{60}\)

Answer:
\[
\frac{33}{60} - \frac{17}{60} = \frac{16}{60}
\]
\[
16 \div 2 = 8
\]
\[
\frac{60}{2} = 30
\]
\[
8 \div 2 = 4
\]
\[
\frac{30}{2} = 15
\]

Note: The phrase “subtract \(a\) from \(b\)” means \(b - a\), not \(a - b\).

For example, to subtract 13 from 20 means to take away 13 from 20: \(20 - 13 = 7\).
Here’s an instructional video on how to add and subtract fractions that have the same denominator:
http://www.youtube.com/watch?v=HbjMM2DeTq4

**Classwork 4.1**

Perform each addition or subtraction and reduce your fractional answer to lowest terms. If the answer is an improper fraction, write it as a mixed number.

1) \(\frac{38}{90} - \frac{23}{90}\)  
2) \(\frac{1}{6} + \frac{5}{6} + \frac{7}{6}\)  
3) \(\frac{87}{120} - \frac{47}{120}\)  
4) \(\frac{29}{48} - \frac{9}{48}\)

5) \(\frac{321}{520} + \frac{79}{520}\)  
6) \(\frac{8}{11} + \frac{3}{11}\)  
7) \(\frac{13}{20} - \frac{7}{20}\)  
8) \(\frac{37}{15} - \frac{7}{15}\)

1) \(\frac{1}{6}\)  
2) \(3\frac{1}{3}\)  
3) \(\frac{1}{3}\)  
4) \(\frac{5}{12}\)  
5) \(\frac{10}{13}\)  
6) \(1\)  
7) \(\frac{3}{10}\)  
8) \(2\)

**Homework 4.1**

Perform each addition or subtraction and reduce your fractional answer to lowest terms. If the answer is an improper fraction, write it as a mixed number.

1) \(\frac{3}{44} + \frac{8}{44}\)  
2) \(\frac{1}{9} + \frac{8}{9} + \frac{2}{9}\)  
3) \(\frac{15}{32} - \frac{9}{32}\)  
4) \(\frac{137}{180} - \frac{49}{180}\)

5) \(\frac{13}{14} + \frac{29}{14}\)  
6) \(\frac{17}{28} - \frac{5}{28}\)  
7) \(\frac{16}{23} - \frac{16}{23}\)  
8) \(\frac{17}{20} + \frac{3}{20}\)

1) \(\frac{1}{4}\)  
2) \(1\frac{2}{9}\)  
3) \(\frac{3}{16}\)  
4) \(\frac{22}{45}\)  
5) \(3\)  
6) \(\frac{3}{7}\)  
7) \(0\)  
8) \(1\)

The following website has additional exercises with solutions involving addition and subtraction of fractions that have the same denominator:
http://cnx.org/content/m34934/latest/?collection=col10615/latest
Section 4.2 – Adding and Subtracting Fractions Having Different Denominators

When we need to add or subtract fractions, we must make sure that all the fractions have the same denominator. If this is not the case, we must find a way to rewrite the fractions to have the same (common) denominator, without changing the original quantity that each fraction represents. For example, we already know that we can reduce a fraction to its lowest terms without changing the actual amount that the fraction represents. Take for example $\frac{30}{60}$

\[
\frac{30 \div 2}{60 \div 2} = \frac{15 \div 3}{30 \div 3} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}
\]

Therefore,

\[
\frac{30}{60} = \frac{1}{2}
\]

The fractions $\frac{30}{60}$, $\frac{15}{30}$, $\frac{5}{10}$ and $\frac{1}{2}$ are equivalent to each other because they were obtained by dividing the numerator and denominator by the same number. Thus, each of these fractions represents the same amount. For example, cutting a pizza into 60 equal pieces and eating 30 is the same as cutting the pizza into 2 equal parts and eating 1 piece. In both cases, we would be eating half of the pizza.

**Note:** We can obtain an equivalent fraction by multiplying the numerator and denominator by the same number.

For example,

\[
\frac{1\times30}{2\times30} = \frac{30}{60} \quad \frac{5\times6}{10\times6} = \frac{30}{60} \quad \frac{15\times2}{30\times2} = \frac{30}{60}
\]

Therefore, when we need to add or subtract fractions that have different denominators, we can multiply the numerator and denominator of one or more fractions by the same number to make all the fractions have the same denominator. An outline of this procedure is presented below.
### Adding & Subtracting Fractions that Have Different Denominators

| Step 1: | Change the original fractions to an equivalent form so that all the fractions have the same denominator. Ideally, the new fractions should have the lowest common denominator (LCD), but this is not required. | \[
\begin{array}{ccc}
\frac{1}{12} + \frac{1}{15} & \quad & \frac{13}{20} - \frac{1}{4} \\
1 \cdot \frac{5}{12} + 1 \cdot \frac{4}{15} & \quad & 13 \cdot \frac{1 \cdot 5}{20} - \frac{4 \cdot 5}{4} \\
\frac{5}{60} + \frac{4}{60} & \quad & \frac{13}{20} - \frac{5}{20}
\end{array}
\] |
|---|---|---|
| Step 2: | Add or subtract the numerators. Leave the common denominator the same. | \[
\begin{array}{c}
\frac{5}{60} + \frac{4}{60} = \frac{9}{60} \\
\frac{13}{20} - \frac{5}{20} = \frac{8}{20}
\end{array}
\] |
| Step 3: | If possible, reduce the fractional answer to lowest terms. If the fraction is improper, write it as a mixed number. | \[
\begin{array}{c}
\frac{9}{60} \div \frac{3}{20} = \frac{3}{20} \\
\frac{8}{20} \div \frac{4}{20} = \frac{2}{5}
\end{array}
\] |

---

### Example 4.2.1

Find the perimeter of the rectangle.

\[
P = \frac{7}{16} \text{ in} + \frac{7}{8} \text{ in} + \frac{7}{16} \text{ in} + \frac{7}{8} \text{ in}
\]

\[
= \frac{7}{16} + \frac{7 \cdot 2}{8 \cdot 2} + \frac{7}{16} + \frac{7 \cdot 2}{8 \cdot 2}
\]

\[
= \frac{7}{16} + \frac{14}{16} + \frac{7}{16} + \frac{14}{16} = \frac{42}{16} \text{ inches}
\]

\[
\frac{42}{16} \div 2 = \frac{21}{8} \text{ inches}
\]

\[
\frac{21}{8} = 2 \frac{5}{8} \text{ inches}
\]
Example 4.2.2
Find the difference between $\frac{29}{30}$ and $\frac{13}{15}$.

Answer:

\[
\begin{align*}
\frac{29}{30} - \frac{13}{15} &= \frac{29}{30} - \frac{13 \cdot 2}{15 \cdot 2} \\
&= \frac{29}{30} - \frac{26}{30} = \frac{3}{30} \\
&= \frac{3}{30} ÷ 3 = \frac{1}{10}
\end{align*}
\]

Example 4.2.3
The following list is the daily number of hours that Cynthia volunteered at the local hospital this week. How many total hours of volunteer work did Cynthia complete this week?

\begin{align*}
M: \frac{7}{8} \text{ hr} & \quad T: \frac{1}{2} \text{ hr} & \quad W: \frac{1}{4} \text{ hr} & \quad Th: \frac{3}{8} \text{ hr} & \quad F: \frac{7}{12} \text{ hr}
\end{align*}

Answer:

\[
\begin{align*}
\frac{7 \cdot 3}{8 \cdot 3} + \frac{1 \cdot 12}{2 \cdot 12} + \frac{1 \cdot 6}{4 \cdot 6} + \frac{3 \cdot 3}{8 \cdot 3} + \frac{7 \cdot 2}{12 \cdot 2} \\
&= \frac{21}{24} + \frac{12}{24} + \frac{6}{24} + \frac{9}{24} + \frac{14}{24} = \frac{62}{24} \\
&= \frac{62 ÷ 2}{24 ÷ 2} = \frac{31}{12} \\
&= 2 \frac{7}{12}
\end{align*}
\]

Cynthia volunteered a total of $2 \frac{7}{12}$ hours this week. This means that Cynthia volunteered a total of 2 hours and 35 minutes.
Perform the following subtraction: \[ \frac{5}{6} - \frac{35}{72} \]

Answer:

\[
\begin{align*}
\frac{5 \cdot 12}{6 \cdot 12} & - \frac{35}{72} \\
\frac{60}{72} - \frac{35}{72} & = \frac{25}{72}
\end{align*}
\]

25 and 72 are relatively prime.

We end this section by noting that an important skill when adding or subtracting fractions that have different denominators is being able to find equivalent fractions that have the same denominator. Therefore, you should become adept at writing fractions that are equivalent to the given fractions.

The following video provides an overview of how to find equivalent fractions:
http://www.youtube.com/watch?v=uh_tyCaf9T8

Here’s an instructional video on how to add or subtract fractions that have different denominators:
http://www.youtube.com/watch?v=N0chS7qYj0Y
http://www.youtube.com/watch?v=yUiWvCM_zI8

The following videos show how to apply the order of operations to simplify an expression that involves fractions:
http://www.youtube.com/watch?v=ApEksFuB49I
http://www.youtube.com/watch?v=SfjwohV5Olc
Perform each addition or subtraction. Simplify your answer completely.

1) \( \frac{67}{90} - \frac{17}{45} \)  
2) \( \frac{1}{15} + \frac{2}{3} + \frac{9}{10} + \frac{1}{2} \)  
3) \( \frac{5}{8} - \frac{25}{48} \)  
4) \( \frac{15}{55} - \frac{2}{11} \)

5) \( \frac{32}{49} + \frac{6}{7} \)  
6) \( \frac{10}{16} + \frac{1}{4} \)  
7) \( \frac{34}{40} - \frac{12}{25} \)  
8) \( \frac{1}{2} - \frac{7}{30} \)

1) \( \frac{11}{30} \)  
2) \( \frac{2}{15} \)  
3) \( \frac{5}{48} \)  
4) \( \frac{1}{11} \)  
5) \( \frac{125}{49} \)  
6) \( \frac{7}{8} \)  
7) \( \frac{37}{100} \)  
8) \( \frac{4}{15} \)

Homework 4.2

Perform each addition or subtraction. Simplify your answer completely.

1) \( \frac{9}{14} - \frac{1}{7} \)  
2) \( \frac{2}{9} + \frac{3}{4} + \frac{5}{6} + \frac{1}{3} \)  
3) \( \frac{13}{21} - \frac{20}{42} \)  
4) \( \frac{79}{80} - \frac{1}{5} \)

5) \( \frac{6}{25} + \frac{1}{10} \)  
6) \( \frac{30}{64} + \frac{5}{32} \)  
7) \( \frac{17}{20} - \frac{2}{5} \)  
8) \( \frac{1}{3} - \frac{1}{42} \)

1) \( \frac{1}{2} \)  
2) \( \frac{5}{36} \)  
3) \( \frac{1}{7} \)  
4) \( \frac{63}{80} \)  
5) \( \frac{17}{50} \)  
6) \( \frac{5}{8} \)  
7) \( \frac{9}{20} \)  
8) \( \frac{13}{42} \)

The following website has additional exercises with solutions involving addition and subtraction of fractions that have different denominators:

http://cnx.org/content/m34935/latest/?collection=col10615/latest
Section 4.3 – Adding and Subtracting Mixed Numbers

When we add mixed numbers, we add the whole numbers and the fractions separately, and then put the results together as a mixed number. Finally, we must check that the fractional part of the mixed number is in lowest terms.

Here is the procedure to add mixed numbers:

<table>
<thead>
<tr>
<th>Adding Mixed Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Add the whole numbers and the fractions separately.</td>
</tr>
<tr>
<td>$7 \frac{1}{5} + 9 \frac{3}{10}$</td>
</tr>
<tr>
<td>$7 + 9 + \frac{1}{5} + \frac{3}{10}$</td>
</tr>
<tr>
<td>$16 + \frac{1}{5} + \frac{3}{10}$</td>
</tr>
<tr>
<td><strong>Step 2:</strong> If the fractions have different denominators, write down equivalent fractions that have the least common denominator.</td>
</tr>
<tr>
<td>$16 + \frac{1 \cdot 2}{5 \cdot 2} + \frac{3}{10}$</td>
</tr>
<tr>
<td>$16 + \frac{2}{10} + \frac{3}{10}$</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Add the fractions by adding their numerators and leave their common denominator the same. Write the sum of the whole number and fraction as a mixed number.</td>
</tr>
<tr>
<td>$16 + \frac{5}{10} = 16 \frac{5}{10}$</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Make sure that the fractional part is a proper fraction in reduced form.</td>
</tr>
<tr>
<td>$16 \frac{5}{10} \div 5 = 16 \frac{1}{2}$</td>
</tr>
</tbody>
</table>

It should be clear to the student that a mixed number represents the sum of a whole number and a fraction. For example,

$7 \frac{6}{13} = 7 + \frac{6}{13}$

$10 \frac{8}{25} = 10 + \frac{8}{25}$

$3 \frac{12}{12} = 3 + \frac{12}{12} = 3 + 1 = 4$

You can break up a whole number any way you want: $5 = 4 + 1 = 4 + \frac{8}{8} = 4 \frac{8}{8}$
Let’s now look at a couple of examples that involve addition of mixed numbers.

**Example 4.3.1**
Find the perimeter of the triangle.

![Diagram of a triangle with sides labeled as follows: 2 3/16 m, 4 1/8 m, 4 1/8 m.]

Answer:

\[
P = 2 \frac{3}{16} m + 4 \frac{1}{8} m + 4 \frac{1}{8} m
\]

\[
2 + 4 + 4 + \frac{3}{16} + \frac{1}{8} + \frac{1}{8}
\]

\[
10 + \frac{3}{16} + \frac{2}{8} \cdot 2 + \frac{1}{8} \cdot 2
\]

\[
10 + \frac{2}{16} + \frac{2}{16}
\]

\[
10 \frac{7}{16} \text{ meters}
\]

The perimeter of the triangle is \( 10 \frac{7}{16} \) meters.

**Example 4.3.2**
Add the following mixed numbers:

\[
6 \frac{3}{32} + 1 \frac{1}{4} + \frac{15}{16}
\]

Answer:

\[
6 + \frac{3}{32} + \frac{1}{4} + \frac{15}{16}
\]

\[
6 + 1 + \frac{3}{32} + \frac{1}{4} + \frac{15}{16}
\]

\[
7 + \frac{3}{32} + \frac{1}{4} \cdot 8 + \frac{15 \cdot 2}{16 \cdot 2}
\]

\[
7 + \frac{3}{32} + \frac{8}{32} + \frac{30}{32} = 7 \frac{41}{32}
\]

\[
7 \frac{41}{32} = 7 + \frac{9}{32} = 8 \frac{9}{32}
\]
When we subtract mixed numbers, there will be times when we may have to borrow, as we sometimes did with subtraction of whole numbers.

### Subtracting Mixed Numbers

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong></td>
<td>Rewrite the fractions so that they all have a common denominator.</td>
<td>$\frac{10}{5} - \frac{3}{8}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{10 \cdot 8}{5 \cdot 8} - \frac{7 \cdot 5}{8 \cdot 5}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{10 \cdot 8}{40} - \frac{3 \cdot 35}{40}$</td>
</tr>
<tr>
<td><strong>Step 2:</strong></td>
<td>If the first fraction is smaller than the second fraction, we cannot subtract yet. We must first borrow.</td>
<td>$9 + \frac{1}{40} - \frac{3 \cdot 35}{40}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$9 + \frac{8}{40} - \frac{3 \cdot 35}{40}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$9 \frac{48}{40} - \frac{3 \cdot 35}{40}$</td>
</tr>
<tr>
<td><strong>Step 3:</strong></td>
<td>Subtract the whole numbers and the fractions separately.</td>
<td>$6 \frac{13}{40}$</td>
</tr>
<tr>
<td><strong>Step 4:</strong></td>
<td>Make sure that the fractional part is a proper fraction and that it is in reduced form.</td>
<td>13 and 40 are relatively prime so the final answer is $6 \frac{13}{40}$.</td>
</tr>
</tbody>
</table>

### Example 4.3.3
Perform the following subtraction:

$$25\frac{3}{4} - 14\frac{5}{8}$$

**Answer:**

$$25\frac{3}{4} - 14\frac{5}{8}$$

$$25\frac{3 \cdot 2}{4 \cdot 2} - 14\frac{5}{8}$$

$$25\frac{6}{8} - 14\frac{5}{8}$$

$$11\frac{1}{8}$$
Example 4.3.4

Perform the following subtraction: \( 5 \frac{7}{20} - 1 \frac{4}{5} \)

Answer:

\[
5 \frac{7}{20} - 1 \frac{4}{5} = 4 \frac{27}{20} - 1 \frac{16}{20} = 3 \frac{11}{20}
\]
Example 4.3.5

Perform the following subtraction: \[ 13 \frac{5}{18} - 5 \frac{11}{12} \]

Answer:

\[
\begin{align*}
13 \frac{5}{18} & \quad - \quad 5 \frac{11}{12} \\
13 \quad \frac{5 \cdot 2}{18 \cdot 2} & \quad - \quad 5 \quad \frac{11 \cdot 3}{12 \cdot 3} \\
13 \quad \frac{10}{36} & \quad - \quad 5 \quad \frac{33}{36} \\
12 + 1 + \frac{10}{36} & \quad - \quad 5 \quad \frac{33}{36} \\
12 + \frac{46}{36} & \quad - \quad 5 \quad \frac{33}{36} = 7 \frac{13}{36}
\end{align*}
\]
**Example 4.3.6**

Subtract $8\frac{2}{15}$ from 30.

\[
\begin{align*}
30 - 8\frac{2}{15} & = 29 + 1 - 8\frac{2}{15} \\
& = 29 + \frac{15}{15} - 8\frac{2}{15} \\
& = 29\frac{15}{15} - 8\frac{2}{15} = 21\frac{13}{15}
\end{align*}
\]

Here are some instructional videos on how to add and subtract mixed numbers:

http://www.youtube.com/watch?v=yf2CmffGB0g

http://www.youtube.com/watch?v=TqlD5fl1v3I

http://www.youtube.com/watch?v=0Z3ZD3YillA

http://www.youtube.com/watch?v=el8tqAKElJg

If you need to “borrow” when subtracting mixed numbers and prefer to avoid this step, take a look at the following video. It shows you how you can avoid borrowing when you subtract any two mixed numbers.

http://www.youtube.com/watch?v=QpRLs-PkP6k
**Classwork 4.3**

Perform each addition or subtraction. Simplify your answer completely. If the answer is an improper fraction, write it as a mixed number.

1) \(3 \frac{1}{2} - 1 \frac{1}{8}\)  
2) \(10 \frac{12}{27} + 7 \frac{1}{9}\)  
3) \(8 \frac{23}{30} + 2 \frac{11}{12}\)  
4) \(15 \frac{1}{3} - 9 \frac{3}{5}\)

5) \(16 - 5 \frac{3}{7}\)  
6) \(9 + 15 \frac{1}{7}\)  
7) \(3 \frac{7}{12} + 1 \frac{5}{12}\)  
8) \(1 - \frac{8}{29}\)

1) \(2 \frac{3}{8}\)  
2) \(17 \frac{5}{9}\)  
3) \(11 \frac{41}{60}\)  
4) \(5 \frac{11}{15}\)  
5) \(10 \frac{4}{7}\)  
6) \(24 \frac{1}{7}\)  
7) \(5\)  
8) \(\frac{21}{29}\)

**Homework 4.3**

Perform each addition or subtraction. Simplify your answer completely. If the answer is an improper fraction, write it as a mixed number.

1) \(20 \frac{35}{49} - 16 \frac{2}{7}\)  
2) \(8 \frac{16}{33} + 4 \frac{9}{11}\)  
3) \(1 \frac{13}{15} - \frac{11}{12}\)  
4) \(18 - 6 \frac{1}{2}\)

5) \(13 \frac{3}{16} - 2 \frac{5}{8}\)  
6) \(3 \frac{5}{13} + 4 \frac{8}{13}\)  
7) \(6 \frac{1}{10} - 3 \frac{1}{4}\)  
8) \(12 \frac{3}{5} - \frac{1}{6}\)

1) \(4 \frac{3}{7}\)  
2) \(13 \frac{10}{33}\)  
3) \(\frac{19}{20}\)  
4) \(11 \frac{1}{2}\)  
5) \(10 \frac{9}{16}\)  
6) \(8\)  
7) \(2 \frac{17}{20}\)  
8) \(12 \frac{13}{30}\)

The following website has additional exercises with solutions involving addition and subtraction of mixed numbers. The method used here involves writing each mixed number as an improper fraction and then adding/subtracting the improper fractions by expressing them with a common denominator. **This method avoids the borrowing step!**

http://cnx.org/content/m34936/latest/?collection=col10615/latest
Section 4.4 – Word Problems Involving Addition and Subtraction of Fractions

In this section we will present applications of fractions and mixed numbers involving addition and subtraction. You should try to look for keywords that serve as a hint as to what operation you should perform. Below are some examples of keywords that indicate that you should add or subtract to solve the word problem.

<table>
<thead>
<tr>
<th>Keyword / Phrase</th>
<th>Operation to Perform</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>addition</td>
</tr>
<tr>
<td>how much</td>
<td>addition</td>
</tr>
<tr>
<td>sum</td>
<td>addition</td>
</tr>
<tr>
<td>cumulative</td>
<td>addition</td>
</tr>
<tr>
<td>together/altogether</td>
<td>addition</td>
</tr>
<tr>
<td>combined</td>
<td>addition</td>
</tr>
<tr>
<td>at this rate</td>
<td>addition</td>
</tr>
<tr>
<td>perimeter</td>
<td>addition</td>
</tr>
<tr>
<td>in all</td>
<td>addition</td>
</tr>
<tr>
<td>difference</td>
<td>subtraction</td>
</tr>
<tr>
<td>how much more</td>
<td>subtraction</td>
</tr>
<tr>
<td>how many more</td>
<td>subtraction</td>
</tr>
<tr>
<td>deduct</td>
<td>subtraction</td>
</tr>
<tr>
<td>decrease by</td>
<td>subtraction</td>
</tr>
<tr>
<td>more than / less than</td>
<td>subtraction</td>
</tr>
</tbody>
</table>

There are instances when there is no explicit keyword in the word problem, and you will have to use your experience or the context of the problem to determine the appropriate operation to use.
Find the perimeter of the following trapezoid.

The perimeter is the total distance around a closed figure. Therefore, we must add the lengths of the 4 sides:

\[ \frac{7}{8}m + \frac{3}{8}m + \frac{1}{8}m + \frac{3}{8}m \]

We add the whole numbers and the fractions separately:

\[ 16 + \frac{14}{8} = 16 \frac{14}{8} = 16 \frac{7}{4} \text{ meters} \]

The mixed number \(16 \frac{7}{4}\) has an improper fraction \(\frac{7}{4}\). We can simplify the mixed number as follows:

\[ 16 \frac{7}{4} = 16 + \frac{7}{4} = 16 + 1 \frac{3}{4} = 17 \frac{3}{4} \]

The total distance around the trapezoid is \(17 \frac{3}{4}\) meters.
Example 4.4.2

Joanna went to the store and bought $3\frac{1}{2}$ pounds of apples, $5\frac{7}{12}$ pounds of oranges and $1\frac{3}{4}$ pounds of bananas. How many pounds of fruit did Joanna buy?

Answer:

$$3\frac{1}{2} + 5\frac{7}{12} + 1\frac{3}{4}$$

$$\frac{3\cdot 6}{2\cdot 6} + \frac{5\cdot 7}{12} + \frac{3\cdot 3}{4\cdot 3}$$

$$\frac{3}{6} + \frac{5}{12} + \frac{9}{12}$$

$$3 + 5 + 1 + \frac{6}{12} + \frac{7}{12} + \frac{9}{12}$$

$$9 + \frac{22}{12} = 9 + \frac{11}{6}$$

$$9 + \frac{11}{6} = 9 + 1\frac{5}{6}$$

$$10\frac{5}{6} \text{ pounds}$$

Joanna bought $10\frac{5}{6}$ pounds of fruit.
Last week Adrian worked $36\frac{3}{4}$ hours. This week he worked $39\frac{1}{2}$ hours. How many more hours did Adrian work this week than last week?

**Example 4.4.3**

\[
\begin{align*}
39 \frac{1}{2} & - 36 \frac{3}{4} \\
&= 39 \frac{1 \cdot 2}{2 \cdot 2} - 36 \frac{3}{4} \\
&= 39 \frac{2}{4} - 36 \frac{3}{4} \\
&= 38 + 1 + \frac{2}{4} - 36 \frac{3}{4} \\
&= 38 + 1\frac{2}{4} - 36 \frac{3}{4} \\
&= 38\frac{6}{4} - 36 \frac{3}{4} \\
&= 2\frac{3}{4} \text{ hours}
\end{align*}
\]

Adrian worked $2\frac{3}{4}$ more hours this week than last week. This means that Adrian worked 2 hours and 45 minutes more this week than last week.
Example 4.4.4

A roll of duct tape has $15\frac{13}{20}$ feet of tape. Anthony then uses $2\frac{1}{2}$ feet of tape. How much duct tape is left?

\[
\begin{align*}
15\frac{13}{20} &- 2\frac{1}{2} \\
15\frac{13}{20} &- 2\cdot\frac{10}{10} \\
15\frac{13}{20} &- 2\cdot\frac{10}{20} \\
13\frac{3}{20} &\text{ feet}
\end{align*}
\]

There are $13\frac{3}{20}$ feet of duct tape left.

Classwork 4.4

1. Mr. Carter planted $3\frac{3}{4}$ acres of wheat and $1\frac{7}{8}$ acres of barley. How much more wheat did he plant than barley.

\[Mr.\ Carter\ planted\ 1\frac{7}{8}\ more\ of\ wheat\ than\ of\ barley.\]

2. Today John spent $1\frac{1}{12}$ hours doing science homework and $1\frac{1}{8}$ working on his English homework. How many hours in total did John spent working on his homework today?

\[Today\ John\ spent\ a\ total\ of\ 2\frac{5}{24}\ hours\ working\ on\ his\ homework.\]
3. It usually takes Marie $\frac{4}{5}$ hr to drive to work. Today it took her $1\frac{1}{2}$ hr to get to work. How much longer did it take Marie to get to work today?

Today it took Marie $\frac{7}{10}$ hr longer to get to work.

4. According to a recipe, Naomi needs to mix $3\frac{1}{4}$ cups of flour with $1\frac{5}{16}$ cups of ground nuts. How many cups of flour and ground nuts combined will Naomi use?

Naomi will use $4\frac{9}{16}$ cups of flour and ground nuts combined.

5. In a chemistry class consisting of 30 students, 5 received an A for the course, 10 received a B and 7 received a C. What fraction of all the students received a C or better?

$\frac{11}{15}$ of the 30 students received a C or better.

6. A cooking recipe calls for $\frac{1}{3}$ hr of cooking time for each pound of turkey. If a turkey weighs 15 pounds, what is the total cooking time that is required for this turkey?

According to the recipe, a 15 lb turkey requires 5 hours of cooking time.

**Homework 4.4**

1. Lilly bought $4\frac{4}{5}$ lb of pears and $2\frac{7}{8}$ lb of apples. How many more pounds of pears than apples did Lilly buy?

Lilly bought $1\frac{37}{40}$ lb more of pears than of apples.

2. Find the perimeter of a square with sides measuring $6\frac{4}{5}$ inches.

The perimeter of the square is $27\frac{1}{5}$ inches.

3. Brenda is $5\frac{5}{16}$ feet tall. Tiffany is $5\frac{3}{8}$ feet. Who is taller and by how much?

Tiffany is taller than Brenda by $\frac{1}{16}$ feet.
4. Yesterday Emily spent 2 hours doing her homework. Today she spent $1 \frac{7}{12}$ hours working on her homework. How much more time did Emily spend working on her homework yesterday?

*Emily spent $\frac{5}{12}$ hours more working on her homework yesterday than today.*

5. Today, Melissa drove $8 \frac{1}{5}$ miles from her home to work. After work, she drove $6 \frac{2}{3}$ miles to the supermarket. Finally, she drove $4 \frac{5}{6}$ miles from the supermarket to her home. How many miles in total did Melissa drive today?

*Today, Melissa drove a total of $19 \frac{7}{10}$ miles.*

6. A month ago, Alan’s weight was $162 \frac{9}{16}$ lbs. He now weighs $158 \frac{1}{4}$ lbs. How many pounds did Alan lose this past month?

*Alan lost $4 \frac{5}{16}$ pounds this past month.*
Chapter 4 Test

1. What is the least common denominator (LCD) of $\frac{12}{25}$ and $\frac{3}{10}$? Circle one.
   a. 25
   b. 250
   c. 50
   d. 10

2. What is the least common denominator (LCD) of $\frac{5}{56}$ and $\frac{3}{8}$? Circle one.
   a. 8
   b. 56
   c. 7
   d. 448

3. What is the least common denominator (LCD) of $\frac{2}{9}$ and $\frac{5}{12}$? Circle one.
   a. 9
   b. 12
   c. 108
   d. 36

Add or subtract as indicated. Simplify your answer completely and write any improper fractions as a mixed number.

4. $\frac{15}{42} + \frac{19}{42} - \frac{7}{42} = \frac{9}{14}$

5. $\frac{3}{5} + \frac{4}{5} + \frac{1}{5} = \frac{3}{5}$

6. $\frac{56}{125} - \frac{21}{125} = \frac{7}{25}$

7. $\frac{12}{30} - \frac{7}{30} + \frac{25}{30} = 1$

8. $\frac{1}{12} + \frac{11}{12} + \frac{9}{12} = 1 \frac{3}{4}$
Simplify each expression completely using the order of operations whenever necessary. If your answer is an improper fraction, write it as a mixed number.

9. \( \frac{6}{7} + \frac{4}{21} = \frac{11}{21} \)

10. \( \frac{8}{15} - \frac{4}{9} = \frac{4}{45} \)

11. \( \frac{5}{8} + \frac{1}{12} \cdot \frac{3}{4} = \frac{53}{96} = \frac{7}{12} \)

12. \( 9 - 6 \frac{5}{12} = 2 \frac{7}{12} \)

13. \( 12 \frac{2}{5} - 4 \frac{7}{10} = 7 \frac{7}{10} \)

14. \( 6 ÷ \frac{1}{3} - 12 \frac{2}{9} = 5 \frac{7}{9} \)

15. \( 8 \frac{5}{32} - 2 \frac{1}{16} = 6 \frac{3}{32} \)

16. \( \frac{7}{15} \cdot \frac{15}{7} - \frac{1}{5} = \frac{4}{5} \)

17. Pauline works as a volunteer at the local hospital. Pauline volunteered 4 \( \frac{1}{4} \) hours on Monday, 5 hours on Tuesday, 4 \( \frac{3}{4} \) hours on Wednesday, 4 hours on Thursday, and 5 \( \frac{3}{4} \) hours on Friday. How many hours of volunteer service did she complete this week? Simplify your answer and express it as a mixed number if necessary.

Pauline completed 23 \( \frac{3}{4} \) hours of volunteer service at the local hospital.

18. Alma had 4 \( \frac{3}{16} \) lb of flour, but then used 1 \( \frac{5}{8} \) lb of flour to bake a cake. How much flour was left after Alma baked the cake?

There were 2 \( \frac{9}{16} \) lb of flour left.

19. Gregory went to the market to buy fruit. He bought 4 \( \frac{5}{16} \) pounds of plums, 3 \( \frac{1}{4} \) pounds of apples and 2 \( \frac{5}{8} \) pounds of kiwis. How many pounds of fruit did Gregory buy?

Gregory bought a total of 10 \( \frac{3}{16} \) pounds of fruit.

20. Find the area and perimeter of a rectangle whose base measures 3 \( \frac{1}{2} \) inches and whose height measures 2 \( \frac{3}{4} \) inches. Simplify your answer completely. Express your answer as a mixed number.

\[ \text{Area} = 9 \frac{5}{8} \text{ in}^2 \]
\[ \text{Perimeter} = 12 \frac{1}{2} \text{ in} \]
Cumulative Review – Chapters 1-4

1) Add: $356,986 + $67,055 + $1,529

$425,570

2) Subtract: $65,000 - $26,762

$38,238

3) Multiply: 695 x 340

236,300

4) Divide: 7,164,018 ÷ 38

188,526 R30

5) Write the first five multiples of 15.

15, 30, 45, 60, 75

6) Circle the numbers that are composite:

82, 1, 63, 411, 915, 37, 1210

7) Simplify using the order of operations. Reduce any fractional answers.

7) $\sqrt{81} + 30 ÷ 6 \div \sqrt{64 + 2^3}$

$\frac{7}{8}$

8) 420 ÷ 6 · 2 – (36 ÷ 9)³

76

9) 680 – 42 · 5 + 7 · $\left(\frac{30}{10}\right)^4$

1,037

10) 56 · 1¹² + (\sqrt{225} + 5)²

456

Write each improper fraction as a mixed number and simplify your answer completely:

11) $\frac{76}{28}$

2 $\frac{5}{7}$

12) $\frac{1080}{240}$

4 $\frac{1}{2}$
Round 64,799,984 to the indicated place.

13) ten millions 60,000,000
14) hundreds 64,800,000

Simplify completely. Write improper fractions as mixed numbers.

15) \( \frac{245}{5} + \frac{32}{8} - \frac{40}{10} \)
16) \( \frac{32}{21} + \frac{9}{21} - \frac{12}{21} = \frac{17}{21} \)

17) \( \frac{3}{5} + \frac{1}{8} = \frac{29}{40} \)
18) \( \frac{29}{40} - \frac{1}{10} = \frac{5}{8} \)
19) \( 2 \frac{7}{10} \cdot \frac{5}{9} = \frac{1}{2} \)

20) \( 6 \frac{2}{5} \div 1 \frac{1}{7} = 5 \frac{3}{5} \)
21) Round $46.7153 to the nearest cent (hundredth) $46.72$

Find the perimeter and area of each geometric figure. Show the appropriate units in your answers.

*Hint: Remember that perimeter is the total distance around a figure. Area is the total amount of “land” or “region” in square units inside a geometric figure.*

22) The parallelogram shown on the right.

Perimeter = 92 yd
Area = 416 yd$^2$
23) The triangle shown on the right.

Perimeter = \(261 \, m\)

Area = \(2,730 \, m^2\)

24) Ralph needs to divide a sac of beans whose weight is 50 lb into 12 equal portions. How much will each portion weigh? Express your answer as a mixed number in reduced form.

\(4 \frac{1}{6} \, lb\)

25) Yvonne can run a mile in \(8 \frac{3}{4}\) minutes. At that rate, how long will it take Yvonne to run 5 miles? Express your answer as a mixed number.

\(43 \frac{3}{4} \, minutes\)

26) Edwin participated in a 28-mile marathon and was able to reach the finish line. If Karl was able to complete only \(\frac{5}{7}\) of the entire course, what was the total distance that Karl ran?

\(20 \, miles\)

27) Sandra invested $6,500 in a company that sells automobiles. So far, she has lost \(\frac{3}{20}\) of the amount she originally invested. How much did Sandra lose?

$975
28) Tatiana needs to complete 20 hours of community service as part of her graduation requirements. So far, she has completed $14\frac{1}{4}$ hours of community service. How many hours does Tatiana still need to complete to meet this graduation requirement?

$\frac{3}{4}$ hours

29) Roger is a sales manager. He used to drive $20\frac{3}{10}$ miles from his home to work each day. He was recently promoted and now works at a different store. He now drives only $13\frac{4}{5}$ miles to work. How much farther was the former work location from his home compared to the new location?

$6\frac{1}{2}$ miles

30) Cynthia needs to divide $4\frac{1}{2}$ ounces of cough syrup into 5 doses. How many ounces of cough syrup will each dose consist of?

$\frac{9}{10}$ of an ounce
Chapter 5

Working with Decimals
## Chapter 5 Overview

By the end of this chapter, you will achieve mastery of the following concepts:

<table>
<thead>
<tr>
<th>Concept</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Writing Decimal Numbers</strong></td>
<td>The number 879.352 is written as: “Eight hundred seventy-nine and three hundred fifty-two thousandths.”</td>
</tr>
</tbody>
</table>
| **Place and Place Value**        | 8 has place value 800  
7 has place value 70  
9 has place value 9  
3 has place value 0.3  
5 has place value 0.05  
2 has place value 0.002 |
| **Rounding Decimals**            | Round 32,704,935.529 to the nearest  
- Thousand: 32,705,000  
- Tenth: 32,704,935.5  
- Ten: 32,704,940  
- Hundred thousand: 32,700,000  
- Hundredth: 32,704,935.53  
- Ten million: 30,000,000 |
| **Adding Decimals**              | 3 1  
8 . 9 4  
1 . 7 7  
9 . 6 5  
+ 0 . 8 1  
2 1 . 1 7  
We must line up the digits vertically according to their place. |
| **Subtracting Decimals**         | 3 16 1 13  
4 6 2 . 3 5  
- 2 8 0 . 8 1  
1 8 1 . 5 4  |
| **Multiplying Decimals**         | The process is the same as for multiplying whole numbers, but we must now account for the decimal digits.  
Note: 107.010 = 107.01 |
| **Dividing Decimals**            | You can also convert any fraction to a decimal by dividing the numerator by the denominator and adding a decimal point to continue dividing.  
\[
\begin{align*}
13 & \div 20 \\
\ell & = \overbrace{16}^{\text{dividend}} \\
4 \downarrow & \quad \text{divisor} \\
0 & \\
12 & \\
12 & \\
0 & \quad \text{remainder}
\end{align*}
\]
\[
\text{Note: } 13 \div 20 = 0.65
\]
Section 5.1 – An Introduction to Decimals

We encounter decimal numbers frequently, such as when we go grocery shopping or conduct financial transactions. We also encounter decimals in applications involving measurement, construction, medical applications, and sports statistics. Just like whole numbers, decimals consist of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 and each digit is given a place value depending on its place or position.

**Example 5.1.1**

The number 106,537.8249 is read

“One hundred six thousand, five hundred thirty-seven and eight thousand, two hundred forty-nine ten thousandths.”

\[106,537.8249 = 100,000 + 6,000 + 500 + 30 + 7 + 0.8 + 0.02 + 0.004 + 0.0009\]
In Section 3.1 you learned how to read fractions:

Here is how to read decimals:

This means that $\frac{1}{10}$ and 0.1 both represent one tenth, $\frac{1}{100}$ and 0.01 both represent one hundredth, and so on. This gives the following list of equivalent fractions and decimals:

When working with decimals, it is very important to understand the decimal place where each digit is located. Understanding the decimal place will allow you to easily express a fraction as a decimal, as well as be able to write a decimal as a fraction. This is illustrated in the next two examples.
Example 5.1.2

Write the following fractions as decimals: \( \frac{7}{10}, \frac{59}{100}, \frac{31}{1000}, \frac{127}{10000}, 4 \frac{3}{100}, 20 \frac{5}{10} \)

Answer:

Begin by saying each fraction and then write the equivalent decimal expression by making sure that the rightmost digit of each numerator is written in the corresponding decimal place:

\[
\begin{align*}
\frac{7}{10} & \quad \text{seven tenths} & \quad 0.7 \\
\frac{59}{100} & \quad \text{fifty nine hundredths} & \quad 0.59 \\
\frac{31}{1000} & \quad \text{thirty one thousandths} & \quad 0.031 \\
\frac{127}{10000} & \quad \text{one hundred twenty seven ten thousandths} & \quad 0.0127 \\
\frac{4}{100} & \quad \text{four and three hundredths} & \quad 4.03 \\
\frac{20}{10} & \quad \text{twenty and five tenths} & \quad 20.5
\end{align*}
\]

The decimal point is read as “and.” It separates the whole number from the decimal part. An expression like \( 4 \frac{3}{100} \) is called a mixed number because it consists of a whole number and a fraction added together: \( 4 + \frac{3}{100} \). Similarly, 4.03 can be considered to be a mixed decimal because it is the sum of a whole number and a decimal: \( 4 + 0.03 \).
Note: We do not use the word “and” when we say or write the whole number part of a number. For example, for the amount $3,054.78 we do not say “three thousand and fifty four dollars and seventy eight cents.” The correct way to say $3,054.78 is “three thousand fifty four dollars and seventy eight cents.” Notice that the word “and” represents the decimal point and separates the dollars from the cents.

Example 5.1.3

Write the following fractions as decimals: \( \frac{98}{10}, \frac{306}{100}, \frac{247}{1000}, \frac{5791}{10000}, \frac{380}{1000}, \frac{9}{1000} \)

Answer:

Begin by saying each fraction and then write the equivalent decimal expression by making sure that the rightmost digit of each numerator is written in the corresponding decimal place:

\[
\begin{align*}
\frac{98}{10} & \quad \text{ninety eight tenths} & 9.8 \\
\frac{306}{100} & \quad \text{three hundred six hundredths} & 3.06 \\
\frac{247}{1000} & \quad \text{two hundred forty seven thousandths} & 0.247 \\
\frac{5791}{10000} & \quad \text{five thousand seven hundred ninety one ten thousandths} & 0.5791 \\
\frac{9}{1000} & \quad \text{nine thousandths} & 0.009 \\
\frac{380}{1000} & \quad \text{three hundred eighty thousandths} & 0.380
\end{align*}
\]
**Note:** Pay attention to the fact that \( \frac{380}{1000} = \frac{38}{100} \) and \( 0.380 = 0.38 \).

This means that “three hundred eighty thousandths” is equivalent to “thirty eight hundredths.”

---

**Example 5.1.4**

Write the following decimals as fractions or mixed numbers:

- 0.23, 0.0015, 7.3, 84.013, 2.8, 3.209

**Answer:**

Begin by saying each decimal and then write the equivalent fraction or mixed number. That is, *write what you say*.

- 0.23 \( \text{twenty three hundredths} \) \( \frac{23}{100} \)
- 0.0015 \( \text{fifteen ten thousandths} \) \( \frac{15}{10000} \)
- 7.3 \( \text{seven and three tenths} \) \( 7 \frac{3}{10} \)
- 84.013 \( \text{eighty four and thirteen thousandths} \) \( 84 \frac{13}{1000} \)
- 2.8 \( \text{two and eight tenths} \) \( 2 \frac{8}{10} \)
- 3.209 \( \text{three and two hundred nine thousandths} \) \( 3 \frac{209}{1000} \)

Of course, you would then have to reduce your fractional answers to lowest terms.
Write the following decimals as fractions or mixed numbers:

17.05, 1.008, 16.771, 5.9, 0.09, 0.0105

Answer:

Begin by saying each decimal and then write the equivalent fraction or mixed number. That is, write what you say.

17.05 twenty three hundredths

\[ \frac{17}{100} \]

1.008 fifteen ten thousandths

\[ \frac{1}{1000} \]

16.771 seven and three tenths

\[ \frac{16771}{1000} \]

5.9 eighty four and thirteen thousandths

\[ \frac{59}{10} \]

0.09 two and eight tenths

\[ \frac{9}{100} \]

0.0105 three and two hundred nine thousandths

\[ \frac{105}{10000} \]

Of course, you would then have to reduce your fractional answers to lowest terms.

Sometimes it happens that a decimal repeats without end. We can write repeating decimals using a bar above the block of digits that repeats:

\[ 0.7111111 \ldots = 0.7\overline{1} \]

\[ 0.3333333 \ldots = 0.\overline{3} \]
Here are some videos that show how to write a fraction as a decimal:

http://www.youtube.com/watch?v=JcA29HoxozU
http://www.youtube.com/watch?v=6pXa7PYNPwU
http://www.youtube.com/watch?v=do_IbHld2Os

Here are some videos that show how to write a decimal as a fraction:

http://www.youtube.com/watch?v=pwsskZpcZOU
http://www.youtube.com/watch?v=Ogz3RQW_kOI

**Classwork 5.1**

1. Fill in the blanks regarding the number given below:

   \[ \begin{array}{cccccccc}
   6 & 2 & 1 & , & 5 & 9 & 0 & \text{and} & 3 & 4 & 8 & 7 \\
   \end{array} \]

   The 2 is in the *ten thousands* place, and its place value is 20,000.
   The 9 is in the *tens* place, and its place value is 90.
   The 4 is in the *hundredths* place, and its place value is 0.04.
   The 6 is in the *hundred thousands* place, and its place value is 600,000.
   The 3 is in the *tenths* place, and its place value is 0.3.
   The 8 is in the *thousandths* place, and its place value is 0.008.
   The 0 is in the *ones* place, and its place value is 0.
2. For the number 48,107.3592 determine which number is in the
   a. Hundreds place 1
   b. Ones place 7
   c. Ten thousandths place 2
   d. Tenths place 3
   e. Hundredths place 5

3. For the number 5,817.203 determine which number is in the
   a. Thousandths place 3
   b. Tens place 1
   c. Thousands place 5
   d. Hundreds place 8
   e. Hundredths place 0

4. For the number 972.085 determine the place of each number
   a. 0 tenths
   b. 9 hundreds
   c. 5 thousandths
   d. 7 tens
   e. 8 hundredths
   f. 2 ones

5. For the number 56,790.4832 determine the place value of each number
   a. 7 700
   b. 3 0.003
   c. 6 6,000
   d. 9 90
   e. 0 0
   f. 4 0.4
   g. 2 0.0002
6. Write each fraction as a decimal.

   A) \( \frac{7}{10} \)  
   B) \( \frac{33}{1000} \)  
   C) \( \frac{467}{100} \)  
   D) \( \frac{8}{10} \)  
   E) \( \frac{29}{10000} \)  
   F) \( \frac{956}{10} \)  

   A) 0.7  
   B) 0.033  
   C) 4.67  
   D) 6.08  
   E) 24.0029  
   F) 95.6  

7. Write each decimal as a fraction. Reduce your answer if possible.

   A) 0.03  
   B) 0.268  
   C) 0.0089  
   D) 34.23  
   E) 6.01  
   F) 18.013  

   A) \( \frac{3}{100} \)  
   B) \( \frac{67}{250} \)  
   C) \( \frac{89}{10000} \)  
   D) \( \frac{34}{100} \)  
   E) \( \frac{61}{100} \)  
   F) \( \frac{1813}{1000} \)  

---

**Homework 5.1**

1. Fill in the blanks regarding the number given below:

   309, 576, and 1248

The 2 is in the *hundredths* place, and its place value is 0.02.
The 9 is in the *thousands* place, and its place value is 9,000.
The 4 is in the *thousandths* place, and its place value is 0.004.
The 6 is in the *ones* place, and its place value is 6.
The 3 is in the *hundred thousands* place, and its place value is 300,000.
The 8 is in the *ten thousandths* place, and its place value is 0.0008.
The 0 is in the *ten thousands* place, and its place value is 0.
2. For the number 29,376.0154 determine which number is in the
   a. Ten thousands place  2
   b. Ones place  6
   c. Ten thousandths place  4
   d. Tenths place  0
   e. Hundredths place  1

3. For the number 63,450.971 determine which number is in the
   a. Thousandths place  1
   b. Tens place  5
   c. Thousands place  3
   d. Hundreds place  4
   e. Hundredths place  7

4. For the number 284.731 determine the place of each number
   a. 1 thousandths
   b. 8 tens
   c. 4 ones
   d. 3 hundredths
   e. 2 hundreds
   f. 7 tenths

5. For the number 4,850,362.971 determine the place value of each number
   a. 7 0.07
   b. 3 300
   c. 6 60
   d. 9 0.9
   e. 8 800,000
   f. 4 4,000,000
   g. 1 0.001
6. Write each fraction as a decimal.

A) \( \frac{19}{100} \)  B) \( \frac{85}{1000} \)  C) \( \frac{137}{10} \)  D) \( \frac{3}{1000} \)  E) \( \frac{13}{10000} \)  F) \( \frac{431}{100} \)

A) 0.19  B) 0.085  C) 13.7  D) 2.003  E) 8.0013  F) 4.31

7. Write each decimal as a fraction. Reduce your answer if possible.

A) 0.776  B) 0.04  C) 4.0005  D) 12.57  E) 0.0091  F) 2.9

A) \( \frac{97}{125} \)  B) \( \frac{1}{25} \)  C) \( 4 \frac{1}{2000} \)  D) \( 12 \frac{57}{100} \)  E) \( \frac{91}{10000} \)  F) \( 2 \frac{9}{10} \)
Section 5.2 – Rounding Decimals

When you put gas on your car, you may have noticed that the screen on the fuel pump display shows the price as $3.869 per gallon. However, the 9 is in the thousandths place and has a place value of 0.009, which is less than one cent. For practical purposes, you round the amount and assume you will be paying $3.87 per gallon. What you’ve done is to “round” to the nearest cent, or hundredth.

To round a quantity in a particular way, you first need to decide to which place you want to round the amount. Do you want to round the amount to the nearest tenth, to the nearest hundredth, or to a different place? In this section we review the method of rounding whole numbers that was introduced in Section 1.2 and we adapt it to round decimals.

**Method to Round Decimals:**

**Step 1:** Determine to which decimal place (tenths, hundredths, thousandths, etc.) you want to round the decimal number.

**Step 2:** Find the digit that is located in that decimal place. We call this digit the place digit. Then draw an “imaginary wall” to the right of the place digit.

**Step 3:** Look at the digit immediately to the right of the wall you drew in step 2. If this digit is 5 or more (5, 6, 7, 8 or 9), increase the place digit in step 2 by 1. Then, look at all the digits to the right of the wall and make all whole number digits into zeros and eliminate all decimal digits. Otherwise, if the digit immediately to the right of the wall is less than 5 (0, 1, 2, 3 or 4), leave the place digit as it is. Then, look at all the digits to the right of the wall and make all whole number digits into zeros and eliminate all decimal digits.

We will now illustrate the method of rounding decimals.
Example 5.2.1

Suppose you want to round $67.342 to the nearest hundredth. We notice that the digit 4 is in the hundredths place. Therefore, 4 is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is 2 (it’s less than 5), we leave the 4 as is and eliminate the 2 because it is a decimal digit and is on the right side of the wall. Therefore, $67.342 rounded to the nearest hundredth is $67.34.

Original number: $67.342
Rounded to the nearest hundredth: $67.34

Example 5.2.2

Suppose you want to round $6,874.251 to the nearest tenth. We notice that the digit 2 is in the tenths place. Therefore, 2 is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is 5 (it’s 5 or greater), we increase the place digit 2 by one, which now becomes a 3. We then eliminate the 5 and the 1 because they are decimal digits and they are on the right side of the wall. Therefore, $6,874.251 rounded to the nearest tenth is $6,874.3.

Original number: $6,874.251
Rounded to the nearest tenth: $6,874.3
Example 5.2.3

Say you want to round $971.82 to the nearest dollar \((\text{ones place})\). We notice that the digit 1 is in the \(\text{ones place}\). Therefore, 1 is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is 8 (it’s 5 or greater), we increase the place digit by one, which now becomes 2, and eliminate all the decimal digits to the right of the wall. Therefore, $971.82 rounded to the nearest dollar is $972.

Original number: $971.82
Rounded to the nearest dollar: $972

Example 5.2.4

Let’s round $867,541.68 to the nearest hundred thousand. We notice that the digit 8 is in the \(\text{hundred thousands place}\). Therefore, 8 is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is 6 (it’s 5 or greater), we increase the place digit by one, which now becomes 9. We then make all the whole number digits to the right of the wall zero and eliminate the decimal digits. Therefore, $867,541.68 rounded to the nearest hundred thousand is $900,000.

Original number: $867,541.68
Rounded to the nearest hundred thousand: $900,000
Example 5.2.5

Let’s round $527.96 to the nearest tenth. We notice that in the tenths place there is a 9. Therefore, 9 is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is a 6 (it’s 5 or greater), we increase the place digit by one, which now becomes 10, but there is no way to fit a 10 in a single place. Ignoring the decimal point for a moment, we add 1 to the number 5279, which now becomes 5280 and put back the decimal point to make it 528.0. We then eliminate the decimal digit 6 because it is on the right side of the wall. Therefore, the final answer is $528.0 (not $528.00).

Original number: $527.96
Rounded to the nearest tenth: $528.0

Example 5.2.6

Let’s round $4,703.995 to the nearest cent (hundredths place). We notice that in the hundredths place there is a 9. Therefore, the 9 in the hundredths place is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of the wall is a 5 (it’s 5 or greater), we add 1 to the place digit 9, which now becomes 10, but there is no way to fit a 10 in a single place. Ignoring the decimal point for a moment, we add 1 to 470399, which now becomes 470400 and put back the decimal point to make it 4704.00. We then eliminate the decimal digit to the right of the wall. Therefore, the final answer is $4704.00 (not $4704.000).

Original number: $4,703.995
Rounded to the nearest hundredth: $4,704.00
**Example 5.2.7**

Let’s round $247.38$ to the nearest *hundred*. We notice that in the *hundreds place* there is a 2. Therefore, 2 is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of 2 is 4 (it’s less than 5), we leave the 2 as is. We then turn the whole number digits that are on the right side of the wall into zeros, and eliminate all the decimal digits that are on the right side of the wall. Therefore, $217.38$ rounded to the nearest hundred is $200$ (not $200.0$ or $200.00$).

Original number: $247.38$

Rounded to the nearest *hundred*: $200$

**Example 5.2.8**

Let’s round $612,193.78$ to the nearest *hundred thousand*. We notice that in the *hundred thousands place* there is a 6. Therefore, 6 is the place digit. We draw a wall to the right of the place digit. Since the digit to the right of 6 is 1 (it’s less than 5), we leave the 6 as is. We then turn the whole number digits to the right of the wall into zeros, and eliminate all decimal digits on the right side of the wall. Therefore, $612,193.78$ rounded to the nearest hundred thousand is $600,000$ (not $600,000.0$ or $600,000.00$).

Original number: $612,193.78$

Rounded to the nearest *hundred thousand*: $600,000$
1. Round the number 628,670.435 to the nearest
   a. thousand 629,000
   b. hundred thousand 600,000
   c. ten 628,670
   d. tenth 628,670.4
   e. hundredth 628,670.44
   f. ten thousand 630,000

2. Round the number 381.257 to the nearest
   a. hundredth 381.26
   b. whole number 381
   c. tenth 381.3
   d. thousandth 381.257

3. Round the number 147,936.054 to the nearest
   a. hundred thousand 100,000
   b. hundred 147,900
   c. million 0
   d. tenth 147,936.1
   e. hundredth 147,936.05
1. Round the number 4,261,599.955 to the nearest
   a. thousand                  4,262,000
   b. hundred thousand                4,300,000
   c. whole number                 4,261,600
   d. hundredth                     4,261,599.96
   e. tenth                   4,261,600.0
   f. million                  4,000,000

2. Round the number 3,995.4294 to the nearest
   a. hundredth        3,995.43
   b. tenth         3,995.4
   c. thousand            4,000
   d. thousandth        3,995.429
   e. whole number       3,995

3. Round the number 790,358.241 to the nearest
   a. tenth                 790,358.2
   b. hundred thousand 800,000
   c. hundredth        790,358.24
   d. million        1,000,000
   e. ten thousand       790,000
   f. ten                 790,360
Section 5.3 – Adding and Subtracting Decimals

Vanessa went to the mall and bought some clothing. Based on the price of each item, what is the total cost without including the sales tax?

<table>
<thead>
<tr>
<th>Item</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skirt</td>
<td>$54.99</td>
</tr>
<tr>
<td>Blouse</td>
<td>$32.50</td>
</tr>
<tr>
<td>Socks</td>
<td>$12.95</td>
</tr>
<tr>
<td>Slippers</td>
<td>$23.75</td>
</tr>
</tbody>
</table>

To find the total amount that Vanessa spent, we need to add the cost of each item. This problem involves addition of decimals. To add or subtract decimals, we have to follow the same procedure we used to add whole numbers. In particular, we must line up each digit vertically according to its place. We will be adding cents plus cents, dimes plus dimes, dollars plus dollars, etc. This sum is represented as follows.

```
 1 3 1
5 4 . 9 9
3 2 . 5 0
1 2 . 9 5
+ 2 3 . 7 5
```

The total cost of the clothing before the sales tax was $124.19.

Now suppose that after sales tax the total amount Vanessa needs to pay is $134.13. If she has $200, how much will she receive in change?

To determine how much Vanessa will receive in change, we need to subtract $134.13 from $200. To subtract decimals, we follow the same procedure that we
used to subtract whole numbers. We line up the digits vertically according to their place value, and subtract the digits keeping in mind that we will have to borrow if necessary.

\[
\begin{array}{c}
199910 \\
200.00 \\
- 134.13 \\
\hline
065.87
\end{array}
\]

Vanessa will receive $65.87 in change.

**Note:** Addition and subtraction of decimals is performed the same way as when we add and subtract whole numbers. The procedure is based on lining each digit vertically according to its decimal place. If we need to borrow when subtracting decimals, the same borrowing approach that was presented in Section 1.4 can be used. Please refer to Section 1.4 if you need to review this borrowing technique.

**Example 5.3.1**

Mark earned $2,519.86 last month. This month he earned $3,078.23. How much did Mark earn these past two months? Also, how much more did he earn this month compared to last month?

To find how much Mark earned these past two months, we must add the amount that he earned last month and this month.

\[
\begin{array}{c}
11 \\
2519.86 \\
+ 3078.23 \\
\hline
5598.09
\end{array}
\]
Mark earned $5,598.09 in the last two months. To determine how much more Mark earned this month than the previous month, we must subtract $2,519.86 from $3,078.23.

\[
\begin{array}{ccccc}
2 & 10 & 6 & 7 & 11 \\
3 & 0 & 7 & 8 & . & 2 & 3 \\
- & 2 & 5 & 1 & 9 & . & 8 & 6 \\
0 & 5 & 5 & 8 & . & 3 & 7 \\
\end{array}
\]

Mark earned $558.37 more this month than the previous month.

**Example 5.3.2**

Find the perimeter of the following park.

The total distance around the park is 4.26 miles.
Perform the indicated subtraction:

$32,508 − $21,999.73

Answer:

Here is a video that shows how to add and subtract decimals:

http://www.youtube.com/watch?v=OE3r5QO8Dos

Classwork 5.3

Perform each operation.

1. $8.29 + 6.89 = 15.18$

2. $9.78 + 9.25 + 7.60 + 0.91 = 27.54$

3. $37.0 + 29.8 + 80.6 = 147.4$
4. \[ \begin{array}{c}
7.921 \\
5.913 \\
3.297 \\
\hline
8.566
\end{array} \]

5. \[ \begin{array}{c}
8.47 \\
-7.21 \\
\hline
\end{array} \] 12.6

6. \[ \begin{array}{c}
78.7 \\
-48.9 \\
\hline
\end{array} \] 29.8

7. \[ \begin{array}{c}
9.904 \\
-6.959 \\
\hline
\end{array} \] 2.945

8. \[ \begin{array}{c}
900.19 \\
-325.36 \\
\hline
\end{array} \] 574.83

9. Subtract $3,581.78 from $10,000 $6,418.22

10. Perform the operation: $1002.09 minus $723.16 $278.93
Perform each operation.

1. 
   \[
   \begin{array}{c}
   83.9 \\
   + 47.4 \\
   \end{array}
   \]
   \[131.3\]

2. 
   \[
   \begin{array}{c}
   2.71 \\
   4.75 \\
   6.80 \\
   + 0.63 \\
   \end{array}
   \]
   \[14.89\]

3. 
   \[
   \begin{array}{c}
   73.2 \\
   48.7 \\
   + 9.7 \\
   \end{array}
   \]
   \[131.6\]

4. 
   \[
   \begin{array}{c}
   2.625 \\
   7.829 \\
   0.034 \\
   + 6.876 \\
   \end{array}
   \]
   \[17.364\]
5. Subtract $18,809.76 from $25,000  $6,190.24

10. Perform the operation:  $3500 minus $1859.99  $1640.01

The following website has additional exercises with solutions involving addition and subtraction of decimals:
http://cnx.org/content/m34960/latest/?collection=col10615/latest
Section 5.4 – Multiplying Decimals

In this section we will go over the method for multiplying decimals. Before describing this process, let’s practice rounding decimals in multiplication problems. This will allow us to get a quick estimate of the actual answer, and will allow us to check whether our actual answer is reasonable.

Suppose you work part-time as a salesperson earning $9.48 per hour. This week you worked 35.7 hours and would like to estimate your earnings. To do that, you decide to round $9.48 to the nearest dollar. Since 9 is in the ones place, you look at the digit to its right which is a 4 (it’s less than 5). You must leave the 9 as it is and eliminate all the decimal digits. The estimate is $9. Similarly, 35.7 hr rounded to the nearest hour is 36 hr. Therefore, you multiply $9 times 36 to obtain an estimate of your earnings

\[
\begin{array}{c}
5 \\
3.6 \\
x 9.48 \\
324
\end{array}
\]

Based on this estimate, you earned approximately $324 this week. To compute the exact amount, you need to multiply 9.48 times 35.7. The table below describes how to multiply decimals.

<table>
<thead>
<tr>
<th>Method to Multiply Decimals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Write each decimal number on a separate line. You do not need to line up the decimal points vertically.</td>
</tr>
</tbody>
</table>
| 3.57 | 3 5 . 7  \\
x 9.48 | x 9 . 4 8 |
| **Step 2:** Ignore the decimal points for a moment, and multiply the whole numbers as you learned to do in Section 1.5. |
| 3 5 7 | 3 5 7  \\
x 9 4 8 | x 9 4 8 |
| 2 8 5 6 | 2 8 5 6  \\
1 4 2 8 | 1 4 2 8  \\
+ 3 2 1 3 | + 3 2 1 3  \\
3 3 8 4 3 6 | 3 3 8 4 3 6 |
**Step 3:** Count the total number of decimal digits in the original numbers. In this example, the original numbers are 35.7 and 9.48, which have a total of 3 decimal places combined.

Correct the answer you obtained in step 2 by moving the decimal point of the whole number to the left the total number of decimal places.

**Step 4:** If the question involves money, round your answer to the nearest cent, unless instructed otherwise.

Note: rounding to the nearest cent means rounding to the nearest hundredth.

The exact earnings for this week are $338.44.

**Example 5.4.1**

Brenda works as a student assistant. She is paid $15.32 per hour. How much did she earn if she worked 8.5 hours this week?

\[
15.32 \times 8.5
\]

We first multiply the decimals as if they were whole numbers.

\[
1532 \times 85
\]

Since the original decimal numbers (15.32 and 8.5) have 3 decimal places combined, we move the decimal point of the whole number 130220 three places to the left. Therefore, Brenda earned $130.22 this week.
Multiply: $6.5 \times 25 \times 1.4$

The order in which we multiply the numbers does not affect the answer. Therefore, we can start by multiplying any two of the three numbers, and then multiply the answer times the third number.

The final and exact answer is $6.5 \times 25 \times 1.4 = 22750 = 227.50$.

If we were only interested in obtaining a quick estimate of the product, we could have rounded the factors to the nearest whole number and then multiplied:

$7 \times 25 \times 1 = 175$

Notice that estimates do not always give an answer that is close to the exact answer. They are useful if we want a rough estimate of the exact answer.
Multiplying Numbers by Powers of 10

If you *multiply* any number such as 18.761 by 10, 100, 1000, 10000, etc. you will see that the final answer has the same digits but the decimal point ends up further to the right. For example, having learned how to multiply decimals you will see that

\[
\begin{align*}
18.761 \times 1 &= 18.761 & \text{the decimal remained in the same place} \\
18.761 \times 10 &= 187.61 & \text{the decimal moved 1 place to the right} \\
18.761 \times 100 &= 1876.1 & \text{the decimal moved 2 places to the right} \\
18.761 \times 1,000 &= 18761. = 18,761 & \text{the decimal moved 3 places to the right} \\
18.761 \times 10,000 &= 187,610 & \text{the decimal moved 4 places to the right} \\
18.761 \times 100,000 &= 1,876,100 & \text{the decimal moved 5 places to the right}
\end{align*}
\]

**Note:** The quick way to multiply a whole number or decimal times 10, 100, 1000 etc. is to simply move the decimal to the right the same number of spaces as there are zeros. If you run out of numbers when you move the decimal point, you should write additional zeros.

Of course, you may multiply say, 18.761 \times 1,000 by writing down the numbers and following the steps outlined for multiplying decimals. However, you will see that this multiplication process takes longer than simply moving the decimal point to the right.

The following video illustrates the process to multiply decimals:

[http://www.youtube.com/watch?v=TtJWsJ3OEGg](http://www.youtube.com/watch?v=TtJWsJ3OEGg)
Perform each multiplication.

1. \[\begin{array}{c}
7.25 \\
x 0.9
\end{array}\]
   \[6.525\]

2. \[\begin{array}{c}
4.6 \\
x 0.2
\end{array}\]
   \[0.92\]

3. \[\begin{array}{c}
0.5 \\
x 3.0
\end{array}\]
   \[15\]

4. \[\begin{array}{c}
1.08 \\
x 0.16
\end{array}\]
   \[0.1728\]

5. \[\begin{array}{c}
25.2 \\
x 8
\end{array}\]
   \[201.6\]
6. $437.056 \times 10,000 = 4,370,560$

7. $0.34 \times 100 = 34$

8. $0.52 \times 1,000 = 520$

**Homework 5.4**

Perform each multiplication.

1. \[ \begin{array}{c}
1 & 6 & . & 2 \\
\times & 0 & . & 3 \\
\hline
4 & 8 & 6 \\
\end{array} \]

2. \[ \begin{array}{c}
0 & . & 6 & 3 \\
\times & 0 & . & 8 \\
\hline
0 & . & 4 & 0 & 4 & 1 \\
\end{array} \]

3. \[ \begin{array}{c}
4 & 7 & . & 3 \\
\times & 0 & . & 1 & 7 \\
\hline
8 & 0 & 4 & 1 \\
\end{array} \]
4. \[0.038 \times 0.16 = 0.00608\]

5. \[2.5 \times 48 = 120\]

6. \[0.0579 \times 100 = 5.79\]

7. \[6.001 \times 10,000 = 60,010\]

8. \[75.3 \times 10 = 753\]

The following website has additional exercises with solutions involving multiplication of decimals:

http://cnx.org/content/m34963/latest/?collection=col10615/latest
Section 5.5 – Dividing Decimals

In this section we describe how to divide decimals. As you will soon see, dividing decimals involves almost the same steps as dividing whole numbers. In you need to review long division, please refer to Section 1.6 where we describe the process step by step. You should also strive to memorize the time tables up to 12. Knowing the times table by memory will facilitate your learning and will make the process of dividing much easier.

An application of division occurs in carpeting or matting of a floor. Suppose that Anthony has a roll consisting of 30.75 feet of material used for floor mats. He needs to cut floor mats that are 2.5 feet long. How many full mats can he make with the amount of floor mat material that he has available?

\[
\text{30.75 ft} \quad \text{2.5 ft}
\]

The long way to answer this question is to subtract 2.5 from 30.75 repeatedly until we don’t have enough material to make another mat. A faster way to obtain the answer is to divide:  

\[
30.75 \div 2.5
\]

Note: Remember that 30.75 ÷ 2.5 is also written as  

\[
2.5 \overline{)30.75} \quad \text{or as } \quad \frac{30.75}{2.5}
\]

Since the division problem above involves decimals, we will now describe the method of dividing decimals step by step. The key requirement is that whenever we perform long division, the divisor must be a whole number or must be turned into a whole number in order to make the division process easier.
Method to Divide Decimals

**Step 1:**
Write the division problem \( a \div b \) as \( b \overline{a} \).

Remember that \( a \) is called the dividend and \( b \) is called the divisor.

**Step 2:**
If the divisor \( b \) is a decimal, move the decimal point all the way to the right to turn the divisor into a whole number. Then, move the decimal point of the dividend to the right the same number of spaces.

Add zeros if you run out of digits when you move the decimal point of the divided.

**Step 3:**
Now that the divisor is a whole number, you may perform long division.

If the new dividend from step 2 is not a whole number or if you want to continue dividing to get a decimal quotient, you must put a decimal point in the quotient right above the decimal point of the new dividend.

Based on the above result, we conclude that \( 30.75 \div 2.5 = 12.3 \). This means that Anthony can make 12 complete mats with the material he has. The 0.3 represents material that is left over and that is not sufficient to make an additional mat.

**Note:** If you encounter a division where the divisor is already a whole number, you do not need to move any decimal points to the right. After performing long division, all you need to do is write a decimal point in the quotient right above the decimal point of the dividend.

We will now illustrate the method of dividing decimals with some examples.
Sarahi needs to cut a rope that is 20.7 meters long into 6 pieces of equal length. How long will each piece of rope be?

We must perform the division $20.7 \div 6$

Since the divisor is already a whole number (6), we did not have to move any decimals to the right. We proceeded to perform long division, and wrote a decimal point in the quotient right above the decimal point of the dividend.

Answer:

Each employee will receive $467.75 for his work.
Perform the following division: $25 \div 0.016$

\[
\begin{array}{c}
16 \overline{)25.0000}\hspace{1cm}1\hspace{1cm}5\hspace{1cm}6\hspace{1cm}2.\hspace{0.5cm}5 \\
-16\hspace{3.5cm}
\hline
90\hspace{3.5cm}40\hspace{3.5cm}0
-80\hspace{3.5cm}320\hspace{3.5cm}80
\hline
10000\hspace{3.5cm}8000\hspace{3.5cm}0000
-8000\hspace{3.5cm}
\hline
0
\end{array}
\]

In this division problem, the divisor was a decimal (0.016) with 3 decimal places. Thus, we moved the decimal point in the divisor and the dividend 3 places to the right. We performed long division and then wrote a decimal point in the quotient, right above the decimal point of the dividend. We added an extra zero to the dividend to continue dividing.

**Answer:** $25 \div 0.016 = 1,562.5$

---

**Dividing Numbers by Powers of 10**

If you divide any number such as 348.75 by 10, 100, 1000, 10000, etc. you will see that the final answer has the same digits but the decimal point ends up further to the left. For example, by performing the long division process for decimals that you learned in this section, you will see that

\[
\begin{align*}
348.75 \div 1 &= 348.75 & \text{the decimal remained in the same place} \\
348.75 \div 10 &= 34.875 & \text{the decimal moved 1 place to the left} \\
348.75 \div 100 &= 3.4875 & \text{the decimal moved 2 places to the left} \\
348.75 \div 1,000 &= 0.34875 & \text{the decimal moved 3 places to the left} \\
348.75 \div 10,000 &= 0.034875 & \text{the decimal moved 4 places to the left} \\
348.75 \div 100,000 &= 0.0034875 & \text{the decimal moved 5 places to the left}
\end{align*}
\]
**Note:** The quick way to divide a whole number or decimal by 10, 100, 1000 etc. is to simply move the decimal to the left the same number of spaces as there are zeros. If you run out of numbers when you move the decimal point, you should write additional zeros.

Of course, you may divide, say 13.921 ÷ 1,000 by writing down the numbers and following the steps outlined above for dividing decimals. However, you will see that this division process takes longer than simply moving the decimal point to the left.

In Section 5.1 we described how to write fractions having a power of 10 in the denominator as a decimal. We employed the “*read and write*” approach, which allowed us to see that \( \frac{1}{10} = 0.1 \), \( \frac{99}{100} = 0.89 \), \( \frac{39}{1000} = 0.039 \) and so on. However, in Section 5.1 we did not discuss how to write a fraction that does not have a power of 10 in the denominator as a decimal, such as \( \frac{5}{12} \). To obtain the decimal equivalent of \( \frac{5}{12} \), we perform long division and continue dividing by writing 5 with its decimal point. The process is shown below:

```
12 | 0.4 1 6 6 6
   | 5.0 0 0 0 0
-4 8
  2 0
-1 2
  8 0
-7 2
  8 0
-7 2
  8
```

**Answer:**

Since we get a repeating decimal for \( 5 \div 12 \), we write a bar above the repeating digit and express the answer as 0.416... or as \( 0.4\overline{16} \).
Any fraction can be written as a decimal by dividing the numerator by the denominator. Always remember that you must place the numerator inside the division box, while the denominator must be placed outside the box: \( \frac{a}{b} \) is equivalent to \( b \overline{a} \).

### Equivalent Fractions, Decimals and Percents

<table>
<thead>
<tr>
<th>(Chapters 3-4) Fraction</th>
<th>(Chapter 5) Decimal</th>
<th>(Chapter 6) Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>0.5</td>
<td>50%</td>
</tr>
<tr>
<td>( \frac{1}{3} )</td>
<td>0.33333... = 0.\overline{3}</td>
<td>33.\overline{3}%</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>( \frac{1}{5} )</td>
<td>0.2</td>
<td>20%</td>
</tr>
<tr>
<td>( \frac{1}{6} )</td>
<td>0.166666... = 0.\overline{16}</td>
<td>16.\overline{6}%</td>
</tr>
<tr>
<td>( \frac{1}{7} )</td>
<td>0.143</td>
<td>14.3%</td>
</tr>
<tr>
<td>( \frac{1}{8} )</td>
<td>0.125</td>
<td>12.5%</td>
</tr>
<tr>
<td>( \frac{1}{9} )</td>
<td>0.11111... = 0.\overline{1}</td>
<td>11.\overline{1}%</td>
</tr>
<tr>
<td>( \frac{1}{10} )</td>
<td>0.1</td>
<td>10%</td>
</tr>
<tr>
<td>( \frac{1}{20} )</td>
<td>0.05</td>
<td>5%</td>
</tr>
</tbody>
</table>
The following videos show how to divide decimals:

http://www.youtube.com/watch?v=S7QGTFvhHXA

http://www.youtube.com/watch?v=WFATCQCyVH0

This video shows how to simplify expressions involving fractions and decimals together: http://www.youtube.com/watch?v=qxWLJYpRglk

Classwork 5.5

Perform each division.

1. \[ \frac{1}{10} \div 253.5 \] \[ \boxed{16.9} \]

2. \[ \frac{1}{8} \div 9586.26 \] \[ \boxed{10,651.4} \]
3. \(4.25 \div 85\)  
4. \(25 \div 6\)  
5. \(0.12 \div 0.0564\)  
6. \(368.5 \div 0.4\)  
7. \(5.67 \div 1000\)  
8. \(0.037 \div 100\)  
9. \(9785 \div 10\)

Write each fraction as a decimal.

10. \(\frac{19}{100}\)  
11. \(\frac{13}{10}\)  
12. \(\frac{38}{10000}\)  
13. \(\frac{35}{100}\)  
14. \(\frac{15}{36}\)  
15. \(\frac{43}{16}\)

10) 0.19  
11) 13.4  
12) 0.0038  
13) 5.35  
14) 0.4166...  
15) 2.6875

Write each decimal as a fraction or mixed number. If possible, simplify your answer to lowest terms.

16) 0.084  
17) 6.32  
18) 2.0017  
19) 25.875  
20) 0.65  
21) 38.6

16) \(\frac{21}{250}\)  
17) \(\frac{8}{25}\)  
18) \(\frac{17}{10000}\)  
19) \(\frac{7}{8}\)  
20) \(\frac{13}{20}\)  
21) \(\frac{38}{5}\)

**Homework 5.5**

Perform each division.

1. \(8 \div 895.36\)  

1) 111.92
2. \(0.015 \div 3.265\)  \(= 217.6\)

3. \(8 \div 3\)  \(= 0.375\)

4. \(40 \div 684.2\)  \(= 17.105\)

5. \(0.3 \div 0.00144\)  \(= 0.0048\)

6. \(29.08 \div 0.08\)  \(= 363.5\)

7. \(12.996 \div 100\)  \(= 0.12996\)

8. \(1.034 \div 1000\)  \(= 0.001034\)

9. \(24,765 \div 10\)  \(= 2,476.5\)

Write each fraction as a decimal.

10. \(\frac{78}{10}\)  \(= 7.8\)
11. \(1 \frac{32}{1000}\)  \(= 1.032\)
12. \(\frac{576}{1000}\)  \(= 0.576\)
13. \(8 \frac{7}{10}\)  \(= 8.7\)
14. \(\frac{23}{84}\)  \(= 0.2738\)
15. \(\frac{15}{28}\)  \(= 0.5357\)

Write each decimal as a fraction or mixed number. If possible, simplify your answer to lowest terms.

16. \(4.346\)  \(= \frac{4173}{500}\)
17. \(9.008\)  \(= 9 \frac{1}{125}\)
18. \(0.0095\)  \(= \frac{19}{2000}\)
19. \(14.54\)  \(= 14 \frac{27}{50}\)
20. \(42.5\)  \(= 42 \frac{1}{2}\)
21. \(0.64\)  \(= \frac{16}{25}\)

The following website has additional exercises with solutions involving division of decimals:  [http://cnx.org/content/m34968/latest/?collection=col10615/latest](http://cnx.org/content/m34968/latest/?collection=col10615/latest)
Section 5.6 – Word Problems Involving Decimals

In this section we present applications of decimals. The aim of this section is to help you gain experience translating sentences into mathematical expressions. As you solve the classroom and homework exercises, pay close attention to what you are being asked to find and try to identify any keywords that indicate the operations that may be used to solve the problem. As you gain experience solving word problems, you will see that you can use similar strategies to determine what operation to use, regardless of whether the problem involves whole numbers, fractions or decimals.

Example 5.6.1

Caroline bought 6 boxes of salted crackers and paid a total of $14.58. How much did each box of salted crackers cost?

The keyword here is “each” and refers to the cost of an individual box. Since we know the total cost and the number of items bought, we can divide to determine the cost of one item.

\[
\begin{array}{c}
\text{2.43} \\
\hline
\text{6} & \text{14.58} \\
\text{12} \\
\text{25} \\
\text{24} \\
\text{18} \\
\text{18} \\
\text{0}
\end{array}
\]

Since the divisor (6) is a whole number, there is no need to move any decimal points to the right. All we need to do is perform the long division and write a decimal point in the quotient, right above the decimal point of the dividend (14.58).

Answer:

Each box of salted crackers costs $2.43.
Marcos is going to rent an SUV for 7 weeks. If it costs $159.35 per week to rent the SUV he selected, how much will Marcos pay for the seven weeks?

The keywords here are “how much” and refer to a total amount. We can answer this question by repeated addition (adding $159.35 seven times), or multiplying $159.35 times seven. The latter approach is faster, so we will perform a multiplication.

\[
\begin{array}{c}
159.35 \\
\times 7 \\
\hline
1115.45
\end{array}
\]

Answer:
We initially ignore the decimal point and multiply the whole numbers. We then correct our answer by moving the decimal point twice to the left. This gives 159.35 \times 7 = $1,115.45

Example 5.6.3
A rectangular plot of land has a base of 2.81 km and a height of 1.5 km. What is the perimeter and area of the park in km²? Express the perimeter in kilometers and the area in kilometers squared (km²).

The perimeter is the total distance around the plot of land. We have to add the lengths of the 4 sides. To add decimals, we must line up the digits according to their decimal place.

\[
\begin{align*}
2.81 & \\
2.81 & \\
1.50 & \\
\hline
8.62 &
\end{align*}
\]

The perimeter of the park is 8.62 km.
To find the area of the rectangular plot, we must multiply one base times one height. This gives the amount of land inside the rectangular plot.

\[ 4 \times 2.81 \]
\[ \underline{+ 1.5} \]
\[ 1405 \]
\[ 281 \]
\[ \underline{4215} \]

Answer:

\[ 2.81 \text{km} \times 1.5 \text{km} = 4.215 \text{ km}^2 \]

The area of the park is 4.215 \( \text{km}^2 \).

**Example 5.6.4**

Judith bought two cantaloupes at the local market. The first cantaloupe weighs 12.86 lb and the second one weighs 15.3 lb.

A) By how many more pounds is the larger cantaloupe heavier than the smaller one?
Here, the keywords are “how many more” and they indicate that we must subtract the larger weight by the smaller one to find the difference. To subtract decimals, we must line up the digits according to their decimal place.

\[
\begin{array}{c}
12.86 \\
- 15.30 \\
\hline
-2.44
\end{array}
\]

Answer:

The larger cantaloupe weighs 2.44 lb more than the smaller one.

B) If cantaloupes cost $0.35 per pound, how much does the large cantaloupe cost? Round your answer to the nearest cent (hundredths place).
Here the keywords are “per” and “how much” and they indicate that in this context we should multiply the price per pound times the number of pounds.

\[
\begin{array}{ccc}
1 & 5 & 3 \\
\times & 3 & 5 \\
\hline
7 & 6 & 5 \\
4 & 5 & 9 \\
\hline
5 & 3 & 5 & 5
\end{array}
\quad \quad \quad
\begin{array}{ccc}
1 & 5.3 \\
\times & .3 & 5 \\
\hline
7 & 6 & 5 \\
4 & 5 & 9 \\
\hline
5 & 3 & 5 & 5
\end{array}
\]

Answer:

Rounded to the nearest cent, the cost of the larger cantaloupe is $5.36.

**Example 5.6.5**

Valerie and Josh drove downtown to see a musical. Before they left the house the car’s odometer read 78,516.7 mi. When they arrived at their destination, the odometer read 78,539.4 mi. How far is the downtown theater from their home?

This question asks us to find the difference between the mileage on the car before leaving the house and the new mileage displayed on the odometer once they arrived at the theater. Thus, we perform a subtraction by lining up the digits according to their decimal place.

\[
\begin{array}{c}
7 & 8 & 5 & 1 & 6 & . & 7 \\
- & 7 & 8 & 5 & 3 & 9 & . & 4 \\
\hline
0 & 0 & 0 & 2 & 2 & . & 7
\end{array}
\]

Answer:

The downtown theater that Valerie and Josh attended is 22.7 miles from their home.
1. Alfred bought $32.78 worth of vegetables at the local supermarket. If he paid with a $50 bill, how much change did he receive?
   *Alfred received $17.22 in change.*

2. Kathy bought a dress for $115.99 and a pair of shoes for $87.99. What was the total cost of the two items?
   *The total cost of the two items was $203.98.*

3. Robert is selling his old CD’s for $2.75 each. If he sold 24 CD’s today, how much money did he make today?
   *Robert made $66 today selling CD’s.*

4. If a pound of salmon costs $8.25, how much will 4 pounds cost?
   *Four pounds of salmon will cost $33.*

5. Lizzy has $2,104.32 left to pay on her car loan. If she is going to pay this amount in 12 monthly payments, how much will she pay per month?
   *Lizzy will pay $175.36 per month.*

6. Jerry wants to divide 8.75 meters of tape into 5 equal pieces. How long will each piece be?
   *Each piece will measure 1.75 meters.*

7. A store sells pants for $38.99 each, and shirts for $25.75 each. If Frank bought 3 pairs of pants and 6 shirts, what was the total cost of the 3 pants and the 6 shirts before the sales tax?
   *The total cost of the items before the sales tax was $271.47.*

8. Jonathan wants to pay $326.64 in 8 equal payments. How much will each of the eight payments be?
   *Each payment will be in the amount of $40.83.*
1. A bicycle that was originally priced at 199.99 now costs only 124.75. By how much was the original price of the bicycle reduced?
   The price of the bicycle was reduced by $75.24.

2. Based on his current speed, it takes Tony 1.5 minutes to drive a mile. At that rate, how long will it take him to drive 13 miles?
   It will take Tony 19.5 minutes to drive 13 miles.

3. A square has sides of length 8.3 cm. What is the perimeter and area of the square?
   \[\text{Perimeter} = 33.2 \text{ cm} \quad \text{Area} = 68.89 \text{ cm}^2\]

4. A rectangle has a base measuring 9.25 ft and a height measuring 3.4 ft. What is the perimeter and area of the rectangle?
   \[\text{Perimeter} = 25.3 \text{ ft} \quad \text{Area} = 31.45 \text{ ft}^2\]

5. A long stick of plastic measures 15.84 meters. It needs to be cut into 4 equal pieces. How long will each piece be?
   Each piece of plastic will be 3.96 meters long.

6. Suppose you need to divide 4.5 ounces of cough syrup into 5 doses. How many ounces of cough syrup will each dose consist of?
   Each dose will consist of 0.9 ounces of cough syrup.

7. Diana works as an Administrative Assistant. She earned $856.32 working 40 hours this week. How much does she earn per hour? Round your answer to the nearest cent.
   Diana earns $21.41 per hour.

8. Carlos went to Sweeties, a chocolate store. He bought 4 pounds of chocolate and paid $15. How much does a pound of chocolate cost?
   A pound of chocolate costs $3.75
Chapter 5 Test

Answer the following questions for the number 457,830.915:

1. Determine which digit is in the following place:
   a. Hundredths place 1
   b. Tens place 3
   c. Ten thousands place 5
   d. Thousandths place 5
   e. Tenths place 9
   f. Ones 0
   g. Hundred thousands 4

2. Determine the place value of the following digits:
   a. 0 0
   b. 1 0.01
   c. 4 400,000
   d. 9 0.9
   e. 8 800
   f. 3 30
   g. 7 7,000

3. Round the number 84,367.185 to the specified place:
   a. Hundred thousands 100,000
   b. Tenths 84,367.2
   c. Ones 84,367
   d. Thousands 84,000
   e. Hundredths 84,367.19
   f. Ten thousands 80,000

4. Round the number $56,398.675 to the specified place:
   a. Nearest Dollar $56,399
   b. Nearest Cent $56,398.68
Perform each operation.

5. 
\[
\begin{array}{c}
9.47 \\
2.06 \\
7.51 \\
\hline
+ 0.89
\end{array}
\quad
\begin{array}{c}
532.7 \\
204.6 \\
74.0 \\
\hline
+ 19.8
\end{array}
\quad
\begin{array}{c}
8000.0 \\
6518.0 \\
0934.0 \\
\hline
+ 856
\end{array}
\]

6. 
\[
19.93
\quad
831.1
\quad
16.308
\]

7. 
\[
\begin{array}{c}
8.000
\end{array}
\quad
\begin{array}{c}
6.518
\end{array}
\quad
\begin{array}{c}
0.934
\end{array}
\quad
\begin{array}{c}
+ 8.56
\end{array}
\]

8. Add: $567 + 1,346.29 + 3,648.72$

$5,562.01$

9. 
\[
\begin{array}{c}
86.2 \\
- 45.2
\end{array}
\]

10. 
\[
\begin{array}{c}
5460
\end{array}
\quad
\begin{array}{c}
- 4581
\end{array}
\quad
\begin{array}{c}
6000
\end{array}
\quad
\begin{array}{c}
- 726
\end{array}
\]

11. 
\[
\begin{array}{c}
41
\end{array}
\quad
\begin{array}{c}
8.79
\end{array}
\quad
\begin{array}{c}
52.74
\end{array}
\]

12. Subtract $16,759.77$ from 20,000.

$3,240.23$

13. 
\[
\begin{array}{c}
9.3
\end{array}
\times
\begin{array}{c}
82
\end{array}
\]

14. 
\[
\begin{array}{c}
8364
\end{array}
\times
\begin{array}{c}
4.5
\end{array}
\]

15. 
\[
\begin{array}{c}
486
\end{array}
\times
\begin{array}{c}
7.5
\end{array}
\]

$762.6$

$376.38$

$3,645$
16. Multiply: \(9.45 \times 1000 = 9,450\)

17. \(\frac{4}{9.516} = 2.379\)

18. \(\sqrt{81.69} = 3.2676\)

19. \(47 \div 0.02 = 2,350\)

20. Divide: \(\frac{32.682}{0.003} = 10,894\)

21. Bob earns $38.60 per hour. If he worked 34 hours this week, how much did he earn?

   \(Bob\ earned\ \$1,312.40\ this\ week.\)

22. Jerry will divide $3690.96 evenly among his six workers. How much will each worker receive?

   \(Each\ worker\ will\ receive\ \$615.16.\)

23. Sonia has paid $1,567.52 of a $3,000 loan. If she wants to pay the rest of the amount that she owes in four equal payments, how much will each of the four payments be?

   \(Each\ of\ the\ four\ payments\ will\ be\ in\ the\ amount\ of\ \$358.12.\)

24. If Karina is driving at 65 miles per hour, at that rate how far will she travel in 1.6 hours?

   \(At\ that\ rate,\ Karina\ will\ travel\ 104\ miles\ in\ 1.6\ hours.\)

25. At a fundraiser, Britney sold cupcakes for $2.75 each. Her target was to raise $200. If she sold 60 cupcakes,

   - How much money did she raise? \($165\)
   - Did she meet her target of $200? \(No\)
   - If not, by how much did she fall short of raising $200? \($35\)
   - At least how many cupcakes in total did she need to sell to meet her target? \(Britney\ needed\ to\ sell\ at\ least\ 73\ cupcakes\ to\ meet\ her\ $200\ target.\)
Simplify completely. Use the order of operations.

1) \(\frac{\sqrt{144} + 90 \div 6}{6 \cdot \sqrt{25} + 3^3}\)
   \[= \frac{6 + 15}{60 + 27} = \frac{21}{87} = \frac{9}{29}\]

2) \(\frac{\sqrt{81} + 27}{2^3 - 2} + (28 \div 4)^2 - 200 \div 25\)
   \[= \frac{9 + 27}{8 - 2} + 9^2 - 8\]
   \[= \frac{36}{6} + 81 - 8\]
   \[= 6 + 73\]
   \[= 79\]

3) \(56 \div 0.04\)
   \[= 1,400\]

4) \(900 - 5.63\)
   \[= 894.37\]

5) \(19.1 \times 4.2\)
   \[= 80.22\]

6) \(654 + 23.56 + 75.012\)
   \[= 752.572\]

7) \((1.4)^2 + (50 - 36.1) \div 0.1\)
   \[= 1.96 + 13.9 \div 0.1\]
   \[= 1.96 + 139\]
   \[= 140.96\]

Write as a decimal:

8) \(\frac{4}{25}\)
   \[= 0.16\]

9) \(10 \frac{17}{100}\)
   \[= 10.17\]

Write as a fraction or mixed number. Simplify your answer completely.

10) \(0.36\)
    \[= \frac{9}{25}\]

11) \(27.068\)
    \[= \frac{27,17}{250}\]

12) Pat works as an administrative assistant. She earns $24.50 per hour. If she earned $980 this week, how many hours did Pat work this week? \(40\) hours

13) Daisy applied for, and was awarded, a graduate scholarship to conduct research at a university of her choice. If she will receive a monthly stipend of $2150.75, how much financial support will she receive in one year? \$25,809
Simplify completely. Write improper fractions as mixed numbers.

14) \[\frac{136}{4} + \frac{20}{5} - \frac{70}{10} = \frac{131}{10}\]

15) \[16\frac{7}{30} + 15\frac{8}{30} - 2\frac{2}{30} = 29\frac{13}{30}\]

16) \[\frac{2}{3} + \frac{1}{10} = \frac{23}{30}\]

17) \[\frac{47}{60} - \frac{1}{5} = \frac{7}{12}\]

18) \[5\frac{1}{3} \cdot 1\frac{1}{2} = 8\]

19) \[10\frac{2}{3} \div 1\frac{1}{3} = 8\]

20) Round $358.2648$ to the nearest cent (hundredth) $\approx 358.26$

Find the perimeter and area of each geometric figure. Show the appropriate units in your answers.

*Hint: Remember that perimeter is the total distance around a figure. Area is the total amount of “land” or “region” in square units inside a geometric figure.*

21) A square with sides equal to 9.4 feet.

Perimeter = $37.6\, ft$

Area = $88.36\, ft^2$

22) A rectangle with base of 8.7 meters and height of 3.9 meters.

Perimeter = $25.2\, m$

Area = $33.93\, m^2$

Indicate whether each of the following statements is true or false.

23) 5 is a factor of 30. *True*
24) 6,877 is divisible by 7.  \textit{False}

25) 642 is divisible by 3.  \textit{True}

Find the greatest common factor (GCF) of each set of numbers.

26) 900 and 540  \hspace{1cm} 27) 64, 48 and 120

\hspace{1cm} 180  \hspace{1cm} 8

Find the least common multiple (LCM) of each set of numbers.

28) 6, 24, and 30  \hspace{1cm} 29) 81, 27 and 90

\hspace{1cm} 120  \hspace{1cm} 810

Perform each operation.

30) 45.78 ÷ 1000  \hspace{1cm} 31) 0.092 ÷ 100  \hspace{1cm} 32) 3568 ÷ 10

\hspace{1cm} 0.04578  \hspace{1cm} 0.00092  \hspace{1cm} 356.8

33) 154.3 \times 1000  \hspace{1cm} 34) 67 \times 10  \hspace{1cm} 35) 0.006 \times 10^4

\hspace{1cm} 154,300  \hspace{1cm} 670  \hspace{1cm} 60

Reduce each fraction to lowest terms. Write improper fractions as mixed numbers.

36) \frac{3280}{4640}  \hspace{1cm} 37) \frac{150}{90}  \hspace{1cm} 38) \frac{6}{18}  \hspace{1cm} 39) \frac{531}{666}  \hspace{1cm} 40) \frac{40}{12}

\hspace{1cm} \frac{41}{58}  \hspace{1cm} \frac{1}{3}  \hspace{1cm} \frac{1}{3}  \hspace{1cm} \frac{59}{74}  \hspace{1cm} 3 \frac{1}{3}
Chapter 6

Ratios, Proportions and Percents
Chapter 6 Overview

By the end of this chapter, you will achieve mastery of the following concepts:

- **Ratios and Rates**
  
  A ratio is a comparison of two quantities which may or may not have the same units. There are three ways to write this comparison:

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Fractional Form</th>
<th>Word Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A : P$</td>
<td>$\frac{A}{P}$</td>
<td>“A to P”</td>
</tr>
<tr>
<td>The number of apples : The number of peaches</td>
<td>The number of apples / The number of peaches</td>
<td>“The number of apples to the number of peaches.”</td>
</tr>
<tr>
<td>45 apples : 53 peaches</td>
<td>45 apples / 53 peaches</td>
<td>45 apples to 53 peaches.</td>
</tr>
</tbody>
</table>

- **Proportions**
  
  A proportion is a statement saying that two ratios (fractions) are equivalent. To solve a proportion, we use the 2-step method known as “cross-multiply and divide.”

  To solve $\frac{28}{49} = \frac{4}{c}$, we first perform cross multiplication, and then we isolate the variable by division:

  $\frac{28}{49} = \frac{4}{c}$ → $28 \cdot c = 4 \cdot 49$

  $28 \cdot c = 196$ → $\frac{28 \cdot c}{28} = \frac{196}{28}$

  $1 \cdot c = 7$ → $c = 7$

- **Percent**
  
  A percent is a comparison of a quantity to 100. Thus, any fraction that has a denominator of 100 is a percent. Percents are useful because we can consider an original amount to represent 100%. We can then consider a quantity to represent some $x\%$ of the original amount.

  $\frac{34}{40} = \frac{85}{100}$

  34 out of 40 is equivalent to 85 out of 100. This means that 34 is 85% of 40.

- **Applications of Percent**
  
  In this chapter, you will use proportions to answer 3 types of questions involving percent. You will learn to identify the percent, the whole amount, and the amount representing a part of the whole.

  **Applications**
  - Percent discount
  - Sales tax
  - Interest rate
  - Value Appreciation / Depreciation
A ratio is a comparison between two quantities. For example, you could compare the total number of hours you worked last week (21hrs) to the total hours you worked this week (32hrs). You could also compare your current annual salary ($25,000) to the annual salary you had five years ago ($14,800). A ratio is also obtained, for example, when comparing the number of nursing courses to psychology courses in a community college. Let’s now illustrate what a ratio is with a more concrete example.

**Example 6.1.1**

Suppose that Pat and Karen will receive a total of $24,000 from their grandfather’s will. Of this amount, Pat and Karen will receive $12,000 each. We can compare the amount that Pat will receive to the amount Karen will receive by writing a ratio between the two amounts. A ratio can be written in several forms:

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>$P : K$</th>
<th>Pat’s amount : Karen’s amount</th>
<th>$12,000 : 12,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractional Form</td>
<td>$\frac{P}{K}$</td>
<td>$\frac{\text{Pat's amount}}{\text{Karen's amount}}$</td>
<td>$\frac{12,000}{12,000}$</td>
</tr>
<tr>
<td>Word Form</td>
<td>“P to K”</td>
<td>“Pat's amount to Karen's amount”</td>
<td>$12,000$ to $12,000$</td>
</tr>
</tbody>
</table>

Notice that a ratio can be written by using a colon : or by writing it as a fraction. Writing the ratio as a fraction is more useful because we may reduce the fraction to lowest terms, which gives a simpler ratio that is equivalent to the original ratio.

For the example above, the simplified ratio becomes $\frac{\text{Pat}}{\text{Karen}} = \frac{\$12,000}{\$12,000} = \frac{1}{1}$. This means that for every dollar that Pat will receive, Karen will also receive one dollar. This also means that for every dollar that Karen receives, Pat will also receive one.
dollar. Notice that we could have kept the units and left the ratio as \( \frac{1}{1} \). However, we usually eliminate the units in a ratio when we compare quantities that have the same units.

**Example 6.1.2**

What if Pat gets $16,000 and Karen gets $8,000 from their grandfather’s will? Then we have the following comparison between the amount that Pat will receive to the amount Karen will receive,

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>( P : K )</th>
<th>Pat’s amount : Karen’s amount</th>
<th>$16,000 : $8,000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fractional Form</strong></td>
<td>( \frac{P}{K} )</td>
<td>( \frac{\text{Pat’s amount}}{\text{Karen’s amount}} )</td>
<td>( \frac{$16,000}{$8,000} )</td>
</tr>
<tr>
<td><strong>Word Form</strong></td>
<td>P to K</td>
<td>Pat’s amount to Karen’s amount</td>
<td>$16,000 to $8,000</td>
</tr>
</tbody>
</table>

The simplified ratio in fractional form is \( \frac{\text{Pat}}{\text{Karen}} = \frac{\$16,000}{\$8,000} = \frac{2}{1} \). This means that Pat will receive two dollars for every dollar that Karen receives. This also means that \( \frac{\text{Karen}}{\text{Pat}} = \frac{\$8,000}{\$16,000} = \frac{1}{2} \), which represents the fact that Karen will receive one dollar for every two dollars that Pat receives.

**Example 6.1.3**

Suppose that Pat will receive $14,000 and Karen will get $10,000. If we want to compare the amount that Pat will receive to the amount that Karen will receive, we write the ratio as \( \frac{\text{Pat}}{\text{Karen}} = \frac{\$14,000}{\$10,000} = \frac{14}{10} = \frac{7}{5} \). On the other hand, if we want to compare the amount that Karen will receive to the amount that Pat will receive, we write the ratio as \( \frac{\text{Karen}}{\text{Pat}} = \frac{\$10,000}{\$14,000} = \frac{10}{14} = \frac{5}{7} \). Notice that we dropped the units.
because both quantities have the same units (dollars). This ratio says that Karen
will receive $5 for every $7 that Pat receives.

In real life applications, we often compare quantities that have different units. A
ratio between quantities having different units, or between different types of
items, is called a \textit{rate}. The following fractions are ratios. However, they \textit{are not rates}
because the numerator and denominator \textit{have the same units}. We can
reduce these ratios to lowest terms, and eliminate the units, if we want to.

\[
\frac{60 \text{ feet}}{40 \text{ feet}} = \frac{3}{2} \quad \frac{45 \text{ dollars}}{20 \text{ dollars}} = \frac{9}{4} \quad \frac{240 \text{ calories}}{300 \text{ calories}} = \frac{4}{5} \quad \frac{48 \text{ candies}}{60 \text{ candies}} = \frac{4}{5}
\]

On the other hand, all of the following ratios are also rates because the quantities
being compared have different units:

\[
\frac{350 \text{ miles}}{14 \text{ gallons}} = \frac{25 \text{ miles}}{1 \text{ gallon}} \quad \frac{2 \text{ grams of fat}}{10 \text{ ounces}} = \frac{1 \text{ gram of fat}}{5 \text{ ounces}}
\]

\[
\frac{40 \text{ feet}}{5 \text{ seconds}} = \frac{8 \text{ feet}}{1 \text{ second}} \quad \frac{48 \text{ chairs}}{8 \text{ tables}} = \frac{6 \text{ chairs}}{1 \text{ table}} \quad \frac{18 \text{ girls}}{12 \text{ boys}} = \frac{3 \text{ girls}}{2 \text{ boys}}
\]

Notice that we can still reduce these rates, but we cannot cancel the units in the
numerator and denominator because they are different. This is always the case
when we have rates.

A \textit{unit rate} is a rate in which the denominator has been simplified to 1 item. The
following rates are unit rates

\[
\frac{25 \text{ miles}}{1 \text{ gallon}} \quad \frac{0.2 \text{ grams of fat}}{1 \text{ ounce}} \quad \frac{8 \text{ feet}}{1 \text{ second}} \quad \frac{6 \text{ chairs}}{1 \text{ table}} \quad \frac{$2.50}{1 \text{ mile}}
\]
Note: It is common practice to drop the 1 in the denominator of unit rates. For example, we write \( \frac{25 \text{ miles}}{1 \text{ gallon}} \) as \( \frac{25 \text{ miles}}{\text{gallon}} \), we write \( \frac{8 \text{ feet}}{1 \text{ second}} \) as \( \frac{8 \text{ feet}}{\text{second}} \), and so on.

Also, keep in mind that it does not make sense to express some rates as a unit rate. For example, \( \frac{18 \text{ girls}}{12 \text{ boys}} \) can be simplified to \( \frac{3 \text{ girls}}{2 \text{ boys}} \). However, it does not make sense to write it as the unit rate \( \frac{15 \text{ girls}}{1 \text{ boy}} \).

To check whether two ratios or two rates are equal, we first perform cross-multiplication and check whether the two quantities we get are equal or not. If the two quantities are equal, then the two ratios or rates are equivalent.

**Example 6.1.4**

To determine whether the rates \( \frac{20 \text{ yards}}{5 \text{ seconds}} \) and \( \frac{14 \text{ yards}}{3 \text{ seconds}} \) are equivalent, we write the equation \( \frac{20}{5} = \frac{14}{3} \) and then cross multiply. To cross multiply means that we multiply \( 20 \times 3 \) and multiply \( 14 \times 5 \) as shown below:

\[
\begin{align*}
20 \times 3 &= 14 \times 5 \\
60 &\neq 70
\end{align*}
\]

Is \( 20 \times 3 = 14 \times 5 \)? No, because \( 60 \neq 70 \). This means that the rates \( \frac{20 \text{ yards}}{5 \text{ seconds}} \) and \( \frac{14 \text{ yards}}{3 \text{ seconds}} \) are not equivalent. This means that an object that is moving 20 yards every 5 seconds is not moving at the same speed as an object that travels 14 yards every 3 seconds.
To determine whether the rates \( \frac{68 \text{ miles}}{2.6 \text{ gallons}} \) and \( \frac{37 \text{ miles}}{1.2 \text{ gallons}} \) are equivalent, we first write the equation \( \frac{68}{2.6} = \frac{37}{1.2} \). To cross multiply, we multiply \( 68 \times 1.2 \) and \( 37 \times 2.6 \) as shown below:

\[
\begin{array}{c}
68 \times 1.2 \\
37 \times 2.6
\end{array}
\]

Is \( 68 \times 1.2 = 37 \times 2.6 \) ? No, because \( 81.6 \neq 96.2 \). This means that the rates \( \frac{68 \text{ miles}}{2.6 \text{ gallons}} \) and \( \frac{37 \text{ miles}}{1.2 \text{ gallons}} \) are not equivalent (not the same).

When we want to compare rates, it is much easier to compare them if we express each rate as a unit rate. The following example illustrates this.

**Example 6.1.6**

Suppose that you want to buy a car that is as fuel efficient as possible, and you have two choices. The car salesman tells you that car 1 drives \( \frac{40 \text{ miles}}{3.2 \text{ gallons}} \) and that car 2 drives \( \frac{34 \text{ miles}}{2.5 \text{ gallons}} \). Which car is more fuel efficient?

For car 1, we express the rate \( \frac{40 \text{ miles}}{3.2 \text{ gallons}} \) as a unit rate by dividing the numerator and denominator by 3.2, which gives \( \frac{12.5 \text{ miles}}{1 \text{ gallon}} \).

For car 2, we express the rate \( \frac{34 \text{ miles}}{2.5 \text{ gallons}} \) as a unit rate by dividing the numerator and denominator by 2.5, which gives \( \frac{13.6 \text{ miles}}{1 \text{ gallon}} \).

**Car 1:** \( \frac{40 \text{ miles}}{3.2 \text{ gallons}} = \frac{12.5 \text{ miles}}{1 \text{ gallon}} \)  
**Car 2:** \( \frac{34 \text{ miles}}{2.5 \text{ gallons}} = \frac{13.6 \text{ miles}}{1 \text{ gallon}} \)

By writing the original rates as unit rates, it is clear that Car 2 is more fuel efficient because it can drive a longer distance for the same amount of gas. Remember that it is common to drop the 1 from the denominator of unit rates.
This video describes what a ratio is and how to simplify fractional ratios to lowest terms:

http://www.youtube.com/watch?v=mQsXmnqd774

Here is a video that shows how a unit rate can be used to determine the better deal when going grocery shopping.

http://www.youtube.com/watch?v=x4j9_lQq8dU

Classwork 6.1

1. Tiger hit a golf ball 324 yards that unfortunately, landed in a sand trap. The ball took 6 full seconds to travel that far. Write how fast the golf ball traveled as a unit rate (number of yards per second).

   The golf ball traveled 54 yards per second.

2. Melissa paid $550 to fly 2,200 miles from Los Angeles, California to Washington D.C. Write the cost of her flight as a unit rate (dollars per mile).

   The cost of the flight from Los Angeles to Washington D.C. was $0.25/mile.

3. A bag of 18 candy bars costs $3.78, and a bag of 25 candy bars costs $5.50. Which is the better buy?

   The better buy is the bag with 18 condies because the unit price is $0.21, compared to the bag of 25 candies which has a unit price of $0.22.

4. A 12-pack of an energy drink costs $13.80, and an 18-pack of the same energy drink costs $19.80. Which is the better buy?

   The better buy is the 18 — pack because each energy drink costs $1.10, whereas for the 12 — pack each energy drink costs $1.15.
Determine whether each of the following is a ratio, a rate, or both, and reduce it completely. Leave your answer in fractional form.

5. \( \frac{60 \text{ miles}}{2 \text{ gallons}} \)  
6. \( \frac{100 \text{ students}}{4 \text{ teachers}} \)  
7. \( \frac{10 \text{ inches}}{2 \text{ inches}} \)  
8. \( \frac{7 \text{ oranges}}{35 \text{ oranges}} \)

5. both, \( \frac{30 \text{ miles}}{\text{gallon}} \)  
6. both, \( \frac{25 \text{ students}}{\text{teacher}} \)  
7. ratio, \( \frac{5}{1} \)  
8. ratio, \( \frac{1}{5} \)

---

**Homework 6.1**

1. Tiffany can type 110 words in two minutes. Write how fast she can type as a unit rate (number of words per minute).

   Tiffany can type 55 words per minute.

2. Albert bought 6 notebooks for $16.50. Write the cost of each notebook as a unit rate (dollars per notebook).

   Each notebook costs $2.75.

3. A box of 20 power bars costs $16, and a box of 10 power bars costs $9.50. Which is the best buy?

   The best buy is the box of 20 power bars because the unit price is $0.80, whereas for the box of 10 power bars the unit price is $0.95.

4. Car 1 drives 95 miles on 2.5 gallons of gasoline. Car 2 drives 45 miles on 1.5 gallons of gasoline. Which car is more fuel efficient?

   Car 1 drives 38 miles per gallon, whereas car 2 drives 30 miles per gallon. Therefore, car 1 is more fuel efficient.

Determine whether each of the following is a ratio, a rate, or both, and reduce it completely. Leave your answer in fractional form.

5. \( \frac{90 \text{ meters}}{120 \text{ meters}} \)  
6. \( \frac{\$56}{7 \text{ notebooks}} \)  
7. \( \frac{48 \text{ tourists}}{2 \text{ buses}} \)  
8. \( \frac{\$600}{\$1800} \)

5. ratio, \( \frac{3}{4} \)  
6. both, \( \frac{\$8}{\text{notebook}} \)  
7. both, \( \frac{24 \text{ tourists}}{\text{bus}} \)  
8. ratio, \( \frac{1}{3} \)
Section 6.2 – Proportions

A proportion is a statement saying that two ratios or rates are equal. For example, when we simplify a fraction to its lowest terms, \( \frac{12}{20} = \frac{3}{5} \), we are stating that the two fractions are equivalent. A statement of equality of two fractions is called a proportion. Here are some examples of proportions involving rates:

<table>
<thead>
<tr>
<th>Rate</th>
<th>Proportion</th>
<th>Is the Proportion True or False?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{36 \text{ apples}}{63 \text{ oranges}} )</td>
<td>( \frac{36 \text{ apples}}{63 \text{ oranges}} = \frac{4 \text{ apples}}{7 \text{ oranges}} )</td>
<td>True: ( 36 \times 7 = 4 \times 63 )</td>
</tr>
<tr>
<td>( \frac{100}{4 \text{ hours}} )</td>
<td>( \frac{100}{4 \text{ hours}} = \frac{25}{1 \text{ hour}} )</td>
<td>True: ( 100 \times 1 = 25 \times 4 )</td>
</tr>
<tr>
<td>( \frac{160 \text{ miles}}{4 \text{ hours}} )</td>
<td>( \frac{160 \text{ miles}}{4 \text{ hours}} = \frac{20 \text{ miles}}{1 \text{ hours}} )</td>
<td>False: ( 160 \neq 80 )</td>
</tr>
<tr>
<td>( \frac{2400 \text{ students}}{80 \text{ teachers}} )</td>
<td>( \frac{2400 \text{ students}}{80 \text{ teachers}} = \frac{30 \text{ students}}{1 \text{ teacher}} )</td>
<td>True: ( 2400 \times 1 = 30 \times 80 )</td>
</tr>
<tr>
<td>( \frac{2}{5 \text{ ounces}} )</td>
<td>( \frac{2}{5 \text{ ounces}} = \frac{0.40}{1 \text{ ounce}} )</td>
<td>True: ( 2 \times 1 = 0.40 \times 5 )</td>
</tr>
<tr>
<td>( \frac{2.5 \text{ inches}}{50 \text{ miles}} )</td>
<td>( \frac{2.5 \text{ inches}}{50 \text{ miles}} = \frac{6 \text{ inches}}{120 \text{ miles}} )</td>
<td>True: ( 2.5 \times 120 = 6 \times 50 )</td>
</tr>
<tr>
<td>( \frac{35 \text{ miles}}{1.2 \text{ gallons}} )</td>
<td>( \frac{35 \text{ miles}}{1.2 \text{ gallons}} = \frac{68 \text{ miles}}{2.3 \text{ gallons}} )</td>
<td>False: ( 80.5 \neq 81.6 )</td>
</tr>
</tbody>
</table>

Notice that some of the proportions above are false because the cross product gives different values. When we write a proportion, we are stating that two fractions are equal. But our statement of equality may be true or false. A
statement of equality of fractions is still a proportion regardless of whether it is true or false.

Suppose that we have the proportion

\[
\frac{30}{60} = \frac{n}{2}
\]

and would like to determine the value of \( n \) that makes it a true statement. By now, you already know that when you reduce \( \frac{30}{60} \) to lowest terms you get \( \frac{30}{60} = \frac{1}{2} \). Therefore, \( n = 1 \) is the value that makes the proportion true.

Suppose now that we have the proportion

\[
\frac{7}{12} = \frac{r}{60}
\]

and would like to determine the value of \( r \) that makes it a true statement. You may have noticed that \( 12 \times 5 = 60 \). This means that we must multiply \( 7 \times 5 \), which gives 35 and allows us to conclude that \( r = 35 \). Hence, we are claiming that \( \frac{7}{12} = \frac{35}{60} \), which is true.

Let’s now consider the following proportion

\[
\frac{54}{117} = \frac{k}{39}
\]

As you can see, sometimes it gets a little difficult to find the missing value that will make the proportion true. However, there is good news! There is a quick and easy method to solve proportions. This method is outlined below.
Method to Solve Proportions

**Step 1:** Perform cross multiplication. You will then have an equation that has an unknown variable on one side.

\[
\frac{54}{39} = \frac{k}{117}
\]

\[54 \times 39 = k \cdot 117\]
\[2106 = k \cdot 117\]

**Step 2:** We must now isolate the variable to determine its value. To do so, divide both sides of the equation by the number that is on the same side as the variable (here we divide both sides by 117).

This leaves the variable alone on one side, which allows us to determine its value.

\[
\frac{2106}{117} = k \cdot \frac{117}{117}
\]
\[18 = k \cdot 1\]
\[18 = k \quad \leftrightarrow \quad k = 18\]

**Step 3:** Check your answer by substituting the value of the variable into the proportion and perform cross multiplication.

\[
\frac{54}{39} = \frac{18}{39}
\]

\[54 \times 39 = 18 \times 117\]
\[2106 = 2106\]

**True!**

Hence, the proportion is also true.

Let’s now practice solving proportions using this method by going over some examples.
**Example 6.2.1**

Find the value of $c$ that makes this proportion true: \[ \frac{28}{49} = \frac{4}{c} \]

Answer:

First perform cross multiplication, and then isolate the variable by division to find its value.

\[
\frac{28}{49} = \frac{4}{c} \quad \rightarrow \quad 28 \cdot c = 4 \cdot 49 \\
28 \cdot c = 196 \\
\frac{28}{28} \cdot c = \frac{196}{28} \\
1 \cdot c = 7 \\
c = 7
\]

**Example 6.2.2**

Find the value of $w$ that makes this proportion true: \[ \frac{15}{w} = \frac{36}{48} \]

Answer:

First perform cross multiplication, and then isolate the variable by division to find its value.

\[
\frac{15}{w} = \frac{36}{48} \quad \rightarrow \quad 15 \times 48 = 36 \cdot w \\
720 = 36 \cdot w \\
\frac{720}{36} = \frac{36}{36} \cdot w \\
20 = 1 \cdot w \\
w = 20
\]
Example 6.2.3

Find the value of $p$ that makes this proportion true: \[
\frac{p}{20} = \frac{12.4}{50}
\]

Answer:

First perform cross multiplication, and then isolate the variable by division to find its value.

\[
\frac{p}{20} = \frac{12.4}{50} \quad \rightarrow \quad p \times 50 = 12.4 \times 20
\]

\[
p \cdot 50 = 248
\]

\[
p \cdot \frac{50}{50} = \frac{248}{50}
\]

\[
p \cdot 1 = 4.96
\]

\[
p = 4.96
\]

Example 6.2.4

Find the value of $e$ that makes this proportion true: \[
\frac{e}{100} = \frac{28}{80}
\]

Answer:

First perform cross multiplication, and then isolate the variable by division to find its value.

\[
\frac{e}{100} = \frac{28}{80} \quad \rightarrow \quad e \cdot 80 = 28 \times 100
\]

\[
e \cdot 80 = 2800
\]

\[
e \cdot \frac{80}{80} = \frac{2800}{80}
\]

\[
e \cdot 1 = 35
\]

\[
e = 35
\]
Example 6.2.5

Find the value of \( r \) that makes this proportion true: \( \frac{64}{100} = \frac{36}{r} \)

Answer:
First perform cross multiplication, and then isolate the variable by division to find its value.

\[
\frac{64}{100} = \frac{36}{r} \quad \Rightarrow \quad 64 \cdot r = 36 \times 100 \\
64 \cdot r = 3600 \\
\frac{64}{64} \cdot r = \frac{3600}{64} \\
1 \cdot r = 56.25 \\
r = 56.25
\]

Rates occur in many real-life applications, and setting up a proportion is often the best approach to solve word problems involving rates. The strategy consists in setting up a proportion in a way that the numerator and denominator on each side of the proportion have the same units. For example, if the rate involves the units “words” and “minutes,” we can set up a proportion either as

\[
\frac{a \text{ words}}{b \text{ minutes}} = \frac{c \text{ words}}{d \text{ minutes}} \quad \text{or} \quad \frac{b \text{ minutes}}{a \text{ words}} = \frac{d \text{ minutes}}{c \text{ words}}
\]

Notice how in the proportion on the left, both numerators have the units “words” and both denominators have units of “minutes.” Alternatively, you could set up the proportion as shown on the right, where both numerators have units of “minutes” and both denominators have units of “words.” You are free to decide how you want to set up the proportion, as long as the units are consistent on both side of the proportion.

We will now illustrate the use of proportions to solve problems involving rates.
Example 6.2.6

Jasmine can type 45 words per minute. At that rate, how many words can she type in 5 minutes?

\[
\begin{align*}
\frac{45 \text{ words}}{1 \text{ minute}} &= \frac{c \text{ words}}{5 \text{ minutes}} \\
\frac{45}{1} &= \frac{c}{5} \\
45 \times 5 &= c \cdot 1 \\
225 &= c
\end{align*}
\]

Jasmine can type 225 words in 5 minutes.

Example 6.2.7

Armando jogged 4 miles in 42 minutes. On average, how long does it take Armando to jog 1.5 miles?

\[
\begin{align*}
\frac{4 \text{ miles}}{42 \text{ minutes}} &= \frac{1.5 \text{ miles}}{d \text{ minutes}} \\
\frac{4}{42} &= \frac{1.5}{d} \\
4 \cdot d &= 1.5 \times 42 \\
4 \cdot d &= 63 \\
\frac{4}{4} \cdot d &= \frac{63}{4} \\
1 \cdot d &= 15.75 \quad \rightarrow \quad d = 15.75
\end{align*}
\]

Armando jogs 1.5 miles in 15.75 minutes.
Elsie works as an administrative assistant. She earned $650 this week working 40 hours. What is her hourly rate?

Answer:

\[
\frac{\$650}{40 \text{ hours}} = \frac{c}{1 \text{ hour}}
\]

\[
\frac{650}{40} = \frac{c}{1}
\]

\[
650 \times 1 = c \cdot 40
\]

\[
650 = c \cdot 40
\]

\[
\frac{650}{40} = \frac{c \cdot 40}{40}
\]

\[
16.25 = c \cdot 1
\]

\[
16.25 = c
\]

Elsie earns $16.25 per hour.
Example 6.2.9

At Las Calabasas Community College, there are 8 math classes for every 12 English classes. If there is actually a total of 32 math classes being offered at Las Calabasas Community College now, how many English classes are being offered?

Answer:

\[
\frac{8 \text{ math}}{12 \text{ English}} = \frac{32 \text{ math}}{d \text{ English}}
\]

\[
\frac{8}{12} = \frac{32}{d}
\]

\[
8 \cdot d = 32 \times 12
\]

\[
8 \cdot d = 384
\]

\[
\frac{8}{8} \cdot d = \frac{384}{8}
\]

\[
1 \cdot d = 48
\]

\[
d = 48
\]

Las Calabasas Community College is offering 48 English classes this semester.
Example 6.2.10
At a hardware store, 5 feet of rope cost $2.35. How much does 4 feet of rope cost?

Answer:

\[
\frac{5 \text{ feet}}{\$2.35} = \frac{4 \text{ feet}}{d}
\]

\[
\frac{5}{2.35} = \frac{4}{d}
\]

\[
5 \cdot d = 4 \times 2.35
\]

\[
5 \cdot d = 9.4
\]

\[
\frac{5 \cdot d}{5} = \frac{9.4}{5}
\]

\[
1 \cdot d = 1.88
\]

\[
d = 1.88
\]

4 feet of rope cost $1.88.

Here are some videos that show how to solve a proportion:

http://www.youtube.com/watch?v=Ss3fp51ghMo
http://www.youtube.com/watch?v=votbrVXAsr4
http://www.youtube.com/watch?v=j-va9qeYyOI

This video shows some applications of proportions:

http://www.youtube.com/watch?v=8bPmP06f-rE
Classwork 6.2

Solve each proportion.

1) \( \frac{50}{a} = \frac{20}{12} \)
2) \( \frac{32}{24} = \frac{5}{b} \)
3) \( \frac{200}{h} = \frac{3}{12} \)
4) \( \frac{42}{15} = \frac{n}{10} \)
5) \( \frac{3}{4} = \frac{6.6}{x} \)
6) \( \frac{25}{100} = \frac{y}{12} \)

7) Martin’s car can travel 108 miles on 2.5 gallons of gas. How many miles can the car travel on 6.5 gallons of gas?

8) In a school election, Victor received 10 votes for every 15 votes that Daniel received. If Daniel received a total of 624 votes, how many votes did Victor get?

1) \( a = 30 \)  2) \( b = 3.75 \)  3) \( h = 800 \)  4) \( n = 28 \)  5) \( x = 8.8 \)  6) \( y = 3 \)
7) 280.8 miles  8) Victor received a total of 416 votes.

Homework 6.2

Solve each proportion.

1) \( \frac{8}{a} = \frac{5}{76} \)
2) \( \frac{3}{4} = \frac{21}{b} \)
3) \( \frac{98}{x} = \frac{2.5}{20} \)
4) \( \frac{30.8}{4} = \frac{n}{10.5} \)
5) \( \frac{12}{100} = \frac{p}{40} \)
6) \( \frac{m}{100} = \frac{8.5}{20} \)

7) A company sells 7 ft\(^2\) of synthetic turf for $58.52. At that rate, how much would 52 ft\(^2\) of synthetic turf cost?

8) A store sells 5 lb of salmon for $56.40. How much would 3 lb of salmon cost?

1) \( a = 121.6 \)  2) \( b = 28 \)  3) \( x = 784 \)  4) \( n = 80.85 \)  5) \( p = 4.8 \)  6) \( m = 42.5 \)
7) $434.72  8) $33.84.

The following website has additional word problems with solutions involving applications of proportions:

http://cnx.org/content/m34982/latest/?collection=col10615/latest
Section 6.3 – Percents

The concept of “percent” appears in so many real life applications that it is likely you have encountered percents before. The concept of percent makes comparisons between numbers easier. The objective of this section is to help you understand what percents are, to make you more comfortable working with them, and to prepare you for the applications of percents that we will encounter in Section 6.5.

A **percent** is simply a ratio in which the denominator is 100. In other words, any fraction that has a 100 in the denominator is called a percent. The word *percent* means “per hundred” or “out of every 100.”

<table>
<thead>
<tr>
<th>Fraction</th>
<th>How to Write Fractions and Percents</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{100})</td>
<td>“One hundredth”</td>
<td>“One percent”</td>
</tr>
<tr>
<td>(\frac{5}{100})</td>
<td>“Five hundredths”</td>
<td>“Five percent”</td>
</tr>
<tr>
<td>(\frac{25}{100})</td>
<td>“Twenty-five hundredths”</td>
<td>“Twenty-five percent”</td>
</tr>
<tr>
<td>(\frac{50}{100})</td>
<td>“Fifty hundredths”</td>
<td>“Fifty percent”</td>
</tr>
<tr>
<td>(\frac{75}{100})</td>
<td>“Seventy-five hundredths”</td>
<td>“Seventy-five percent”</td>
</tr>
<tr>
<td>(\frac{100}{100})</td>
<td>“One hundred hundredths”</td>
<td>“One hundred percent”</td>
</tr>
<tr>
<td>(\frac{150}{100})</td>
<td>“One hundred fifty hundredths”</td>
<td>“One hundred fifty percent”</td>
</tr>
<tr>
<td>(\frac{200}{100})</td>
<td>“Two hundred hundredths”</td>
<td>“Two hundred percent”</td>
</tr>
<tr>
<td>(\frac{500}{100})</td>
<td>“Five hundred hundredths”</td>
<td>“Five hundred percent”</td>
</tr>
</tbody>
</table>
**Note:** 100% is used to represent a whole amount, a whole set, all the members or parts that make up a whole, or the original price of an item. 100% is also used to represent any amount or item that we define to be the whole or complete item or amount. A percent less than 100% represents less than the whole amount, or a fraction of a whole item. A percent more than 100% represents more than the original amount or more than one whole item.

The following site has an interactive applet that you can use to practice identifying a given percent.

http://nlvm.usu.edu/en/nav/frames_asid_333_g_2_t_1.html?from=topic_t_1.html

**Example 6.3.1**

1. List 5 examples of things that you consider represent 100%.
2. List 5 examples of things that you consider to be less than 100%.

<table>
<thead>
<tr>
<th><strong>100%</strong></th>
<th><strong>Less than 100%</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>all the students enrolled in a class</td>
<td>15 students attending class out of 20 enrolled</td>
</tr>
<tr>
<td>a whole pizza</td>
<td>3 out of 4 members of a rock band</td>
</tr>
<tr>
<td>a new roll of masking tape</td>
<td>one slice of pizza</td>
</tr>
<tr>
<td>an unopened apple juice can</td>
<td>an empty/less than full can of soda</td>
</tr>
<tr>
<td>the original price of a shirt before a discount</td>
<td>the price of a dress after a discount</td>
</tr>
</tbody>
</table>

**Example 6.3.2**

List 5 examples of things that you consider to be more than 100%.

<table>
<thead>
<tr>
<th><strong>More than 100%</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pizza + ⅔ of another pizza of the same kind,</td>
</tr>
<tr>
<td>the price of gasoline today compared to 20 years ago,</td>
</tr>
<tr>
<td>Joe’s current annual salary compared to his salary 5 years ago,</td>
</tr>
<tr>
<td>the price of a scalped ticket to a concert.</td>
</tr>
</tbody>
</table>
### Example 6.3.3
1. List 3 examples of a decrease of 50%.
2. List 3 examples of a decrease of 25%.

<table>
<thead>
<tr>
<th>Decrease of 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A decrease in the enrollment in a physics course from 48 students last semester to 24 this semester.</td>
</tr>
<tr>
<td>2. One-half of a pizza is eaten, and (\frac{1}{2}) is left over.</td>
</tr>
<tr>
<td>3. A decrease in the price of a new computer from $900 to $450.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decrease of 25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A $25 decrease in the price of a DVR from $100 to $75.</td>
</tr>
<tr>
<td>2. Cutting a pizza into 4 pieces and eating 1 slice. There is (\frac{3}{4}) pizza left.</td>
</tr>
<tr>
<td>3. Eating 6 oranges from a bag that originally had 24 oranges. Now there are only 18 oranges.</td>
</tr>
</tbody>
</table>

### Example 6.3.4
1. List 3 examples of an increase of 50%.
2. List 3 examples of an increase of 25%.

<table>
<thead>
<tr>
<th>Increase of 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. An increase in the enrollment in a math course from 20 students last semester to 30 this semester.</td>
</tr>
<tr>
<td>2. An increase in the price of a cell phone from $600 to $900 due to high demand.</td>
</tr>
<tr>
<td>3. An increase in Tom’s weekly salary from $240 to $360.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Increase of 25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. An increase in Eli’s weight from 160 lb five years ago to 200 lb now.</td>
</tr>
<tr>
<td>2. Having 4 sweaters and getting another one for my birthday.</td>
</tr>
<tr>
<td>3. An increase in Bertha’s weekly salary from $320 to $400.</td>
</tr>
</tbody>
</table>
It should be clear that a quantity that represents 100% is the original amount to which gains and losses are compared.

*Please ask your instructor for assistance if, after going over the examples above, you still do not understand the concept of percent.*

The following table lists some fractions along with their decimal and percent representations.

<table>
<thead>
<tr>
<th>Equivalent Fractions, Decimals and Percents</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Chapters 3-4)</td>
</tr>
<tr>
<td>Fraction</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$\frac{1}{7}$</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>Fraction</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1/10</td>
</tr>
<tr>
<td>1/20</td>
</tr>
<tr>
<td>1/40</td>
</tr>
<tr>
<td>1/50</td>
</tr>
<tr>
<td>1/100</td>
</tr>
<tr>
<td>1/200</td>
</tr>
<tr>
<td>1/500</td>
</tr>
<tr>
<td>1/800</td>
</tr>
<tr>
<td>1/1000</td>
</tr>
</tbody>
</table>

You do not need to memorize this list. It is given to you as a reference. However, you should know how to express any fraction, decimal or percent into the other two forms. Here is a graphic that tells you what you need to do:

To write a fraction as a decimal, divide the numerator by the denominator.

To write a decimal as a fraction, simply read the decimal out loud and write the corresponding fraction. Reduce the fraction, if possible.
The steps above can be summarized as follows:

\[ D \xrightarrow{\times 100\%} P \xleftarrow{\div 100\%} \]

To write a decimal as a percent, multiply the decimal by 100%. Doing this moves the decimal point 2 places to the right and introduces a % sign.

To write a percent as a decimal, divide the percent by 100%. Doing this moves the decimal point 2 places to the left and removes the % sign.

**Example 6.3.5**

Write the fraction \( \frac{3}{8} \) as a decimal and then back into a fraction in lowest terms.

\[ 3 \div 8 = 0.375 \quad \text{"three hundred seventy five thousandths"} \]

\[
\frac{375 \div 5}{1000 \div 5} = \frac{75 \div 25}{200 \div 25} = \frac{3}{8}
\]
Write the decimal 0.87 as a percent, and then write the percent back as a decimal.

\[ 0.87 \times 100\% = 87\% \quad (a \% \text{ sign appears}) \quad 87\% \div 100\% = 0.87 \quad (\text{the } \% \text{ sign disappears}) \]

Write each decimal as a percent.

\[ 0.01, \quad 0.09, \quad 8.15, \quad 0.046, \quad 0.73, \quad 1.25, \quad 3, \quad 12.5, \]

**Example 6.3.6**

\[
\begin{align*}
d & \quad \times 100\% \quad \rightarrow \quad p \\
0.01 &= 0.01 \times 100\% = 1\% \\
0.09 &= 0.09 \times 100\% = 9\% \\
8.15 &= 8.15 \times 100\% = 815\% \\
0.046 &= 0.046 \times 100\% = 4.6\% \\
0.73 &= 0.73 \times 100\% = 73\% \\
1.25 &= 1.25 \times 100\% = 125\% \\
3 &= 3 \times 100\% = 300\% \\
12.5 &= 12.5 \times 100\% = 1250\%
\end{align*}
\]

**Note:** To express a decimal as a percent, multiply the decimal by 100%. The quick way to do this is to simply move the decimal point to the right exactly two **spaces** and attach the % symbol.
Write each percent as a decimal or whole number.

29%, 85%, 250%, 15.6%, 45.62%, 600%, 0.57%, 80%

Answers: $p \div 100\% \rightarrow d$

- 29% = 29% ÷ 100% = 0.29
- 85% = 85% ÷ 100% = 0.85
- 250% = 250% ÷ 100% = 2.50 = 2.5
- 15.6% = 15.6% ÷ 100% = 0.156
- 45.62% = 45.62% ÷ 100% = 0.4562
- 600% = 600% ÷ 100% = 6.00 = 6
- 0.57% = 0.57% ÷ 100% = 0.0057
- 80% = 80% ÷ 100% = 0.80 = 0.8

Note: To express a percent as a decimal, divide the percent by 100%. The quick way to do this is to simply move the decimal point to the left exactly two spaces and remove the % symbol.
Write each fraction as a percent:

\[
\frac{7}{20}, \quad \frac{1}{4}, \quad \frac{36}{40}, \quad \frac{124}{5}, \quad \frac{6}{100}, \quad \frac{12}{25}, \quad \frac{5}{18}, \quad \frac{2}{3}
\]

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{7}{20})</td>
<td>0.35</td>
<td>35%</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>(\frac{36}{40})</td>
<td>0.9</td>
<td>90%</td>
</tr>
<tr>
<td>(\frac{124}{5})</td>
<td>12.8</td>
<td>1280%</td>
</tr>
<tr>
<td>(\frac{6\frac{3}{100}}{1})</td>
<td>6.03</td>
<td>603%</td>
</tr>
<tr>
<td>(\frac{12}{25})</td>
<td>0.48</td>
<td>48%</td>
</tr>
<tr>
<td>(\frac{5}{18})</td>
<td>0.27777...</td>
<td>27.7%</td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
<td>0.66666...</td>
<td>66.7%</td>
</tr>
</tbody>
</table>

**Note:** To express a fraction as a percent, first express the fraction as a decimal by dividing the numerator by the denominator. Then move the decimal point to the right exactly two spaces and attach the % symbol.
Example 6.3.10

Write each percent as a fraction or mixed number and simplify your answer:

33%, 75%, 60%, 145%, 12.5%, 97.5%, 300%, 450%

Answers: \( p \div 100\% \rightarrow d \) read and write \( f \)

\[
33\% = 0.33 = \frac{33}{100}
\]

\[
75\% = 0.75 = \frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}
\]

\[
60\% = 0.60 = \frac{60}{100} = \frac{60 \div 10}{100 \div 10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}
\]

\[
145\% = 1.45 = 1 \frac{45}{100} = \frac{1 + \frac{45}{100}}{1 + \frac{5}{100}} = 1 \frac{9}{20}
\]

\[
12.5\% = 0.125 = \frac{125}{1000} = \frac{125 \div 5}{1000 \div 5} = \frac{25 \div 25}{200 \div 25} = \frac{1}{8}
\]

\[
97.5\% = 0.975 = \frac{975}{1000} = \frac{975 \div 5}{1000 \div 5} = \frac{195 \div 5}{200 \div 5} = \frac{39}{40}
\]

\[
300\% = 3.00 = 3
\]

\[
450\% = 4.50 = 4 \frac{50}{100} = 4 \frac{50 \div 50}{100 \div 50} = 4 \frac{1}{2}
\]

Note: To express a percent as a fraction, first express the percent as a decimal by moving the decimal point to the left exactly two spaces and remove the % symbol. Then read the decimal out loud and write the corresponding fraction. Reduce the fraction to lowest terms, if possible.
The following applet provides a visual relationship between fractions and percents. You may use this applet to check your work when converting fractions to percents.

http://nlvm.usu.edu/en/nav/frames_asid_183_g_2_t_1.html?open=activities&from=topic_t_1.html

This video shows how to change a fraction to a decimal:

http://www.youtube.com/watch?v=jnWtoeDIQPg

This video shows how to convert a decimal to a percent:

http://www.youtube.com/watch?v=90r0NSZNCQ

http://www.youtube.com/watch?v=YDg_hksxhQM

This video shows how to convert a fraction to a percent:

http://www.youtube.com/watch?v=LXIS-7rQc74

This video shows how to change a percent to a decimal:

http://www.youtube.com/watch?v=KBtjkblzbrw

---

**Classwork 6.3**

1. A full can of soda has 12 ounces. Someone drinks 25% of the soda. How many ounces are left?  
   There are 9 ounces of soda left.

2. If in a large class there are 50 students and 20% were absent today, how many students were absent and how many were present?  
   Today, there were 10 students absent and 40 students were present.

3. Write 0.65 as a percent.  
   65%
4. Write 1.72 as a percent. 172%

5. Write 55% as a decimal. 0.55

6. Write 420% as a decimal. 4.20 or 4.2

7. Write $\frac{2}{5}$ as a decimal. Then write it as a percent.
   
   Decimal: 0.4  Percent: 40%

8. Write $1 \frac{1}{4}$ as a decimal. Then write it as a percent.
   
   Decimal: 1.25  Percent: 125%

9. Write 88% as a fraction in reduced form. $\frac{22}{25}$

10. Write 650% as a mixed number in reduced form. $6 \frac{1}{2}$

---

**Homework 6.3**

1. A pair of jeans was originally priced at $46. It is now on sale at a discount of 50%. What is the discounted price of the pair of jeans? $23

2. Assume that a full box of apples contained 28 apples. Then, if 25% of the 28 apples were removed from the box, how many apples were removed and how many remained? 7 apples were removed and 21 remained in the box.
3. Write 0.976 as a percent. 97.6%

4. Write 2.35 as a percent. 235%

5. Write 2.7% as a decimal. 0.027

6. Write 155% as a decimal. 1.55

7. Write \( \frac{13}{20} \) as a decimal. Then write it as a percent.

   Decimal: 0.65   Percent: 65%

8. Write \( 2 \frac{7}{8} \) as a decimal. Then write it as a percent.

   Decimal: 2.875   Percent: 287.5%

9. Write 70% as a fraction in reduced form. \( \frac{7}{10} \)

10. Write 545% as a mixed number in reduced form. \( 5 \frac{9}{20} \)

The following website has additional exercises with solutions involving writing fractions and decimals as a percent, and vice versa:

http://cnx.org/content/m34983/latest/?collection=col10615/latest
Section 6.4 – Proportions Involving Percents

In this section, we use proportions to answer questions involving percents. The examples covered in this section illustrate how proportions can be used to solve each of the following types of questions:

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>“Find 15% of 160.” “15% of 160 is what number?” “What number is 15% of 160?”</td>
</tr>
<tr>
<td>2</td>
<td>“What percent is 72 of 200?” “What percent of 200 is 72?” “72 is what percent of 200?”</td>
</tr>
<tr>
<td>3</td>
<td>“8 is 25% of what number?”</td>
</tr>
</tbody>
</table>

The purpose of this table is twofold: 1) to point out that there are 3 different types of questions involving percent that you may be asked, and 2) to make you aware of the fact that sometimes there is more than one way to ask the same question. For example, the phases in the green row are asking the same type of question, the phrases in the blue row are asking the same question, and those in the purple row are asking the same question.

An approach that can be used to answer any type of question involving a percent is to set up the following proportion

\[
\frac{\text{Percent}}{b} = \frac{c}{d}
\]

Number 1

Number 2
If you are given a percent, say 15%, you could write it as \( \frac{15}{100} \) on the left side of the proportion. If you are not given the percent, then you may write the unknown percent as \( \frac{x}{100} \). The number that follows the word “of” must be written in the denominator on the right side of the proportion, since this number represents the original or whole amount. Remember that the original or whole amount corresponds to 100%. Finally, the remaining number must be placed on the last remaining spot, which is the numerator of the right side of the proportion. The following table shows how to set up proportions for questions of type 1, type 2 and type 3.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Examples</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>“Find 15% of 160.” “15% of 160 is what number?” “What number is 15% of 160?”</td>
<td>( \frac{15}{100} = \frac{x}{160} )</td>
</tr>
<tr>
<td>2</td>
<td>“What percent is 72 of 200?” “What percent of 200 is 72?” “72 is what percent of 200?”</td>
<td>( \frac{y}{100} = \frac{72}{200} )</td>
</tr>
<tr>
<td>3</td>
<td>“8 is 25% of what number?”</td>
<td>( \frac{25}{100} = \frac{8}{z} )</td>
</tr>
</tbody>
</table>

Notice that there are 4 possible locations to write numbers and variables in any proportion. Two of those locations are always reserved for the percent, namely, the fraction on the left side. Thus, there are only 2 positions left where you must put the other 2 numbers. Since the number after the word “of” must be placed in the denominator of the fraction on the right side, the remaining number must be placed on the only place that is left, which is the numerator on the right side.

Therefore, there should not be any confusion as to “what goes where” in a proportion that involves a percent. If a number or percent is not given, simply
write any variable you like in its place. Then cross multiply and divide to obtain
the number or percent you are trying to find.

Example 6.4.1

Find 15% of 160.

This is a Type 1 question because you are asked to find some percent of a given
number.

Answer:

\[
\frac{15}{100} = \frac{m}{160}
\]

\[
15 \times 160 = m \cdot 100
\]

\[
2400 = m \cdot 100
\]

\[
\frac{2400}{100} = m \cdot \frac{100}{100}
\]

\[
24 = m \cdot 1
\]

\[
24 = m
\]

24 is 15% of 60.
Example 6.4.2

Find 120.5% of 400.

This is a Type 1 question because you are asked to find some percent of a given number.

Answer:

\[
\frac{120.5}{100} = \frac{r}{400}
\]

\[
120.5 \times 400 = r \cdot 100
\]

\[
48200 = r \cdot 100
\]

\[
\frac{48200}{100} = r \cdot \frac{100}{100}
\]

\[
482 = r \cdot 1
\]

\[
482 = r
\]

482 is 120.5% of 400.

Example 6.4.3

What percent is 72 of 200?

This is a Type 2 question because you are asked to find the percent. You are given a number and the whole amount.

Answer:

\[
\frac{p}{100} = \frac{72}{200}
\]

\[
p \cdot 200 = 72 \times 100
\]

\[
p \cdot 200 = 7200
\]

\[
p \cdot \frac{200}{200} = \frac{7200}{200}
\]

\[
p \cdot 1 = 36
\]

\[
p = 36
\]

72 is 36% of 200.
Example 6.4.4

What percent is 59 of 40?

This is a Type 2 question because you are asked to find the percent. You are given a number and the whole amount.

Answer:

\[
\frac{a}{100} = \frac{59}{40}
\]

\[
a \cdot 40 = 59 \times 100
\]

\[
a \cdot 40 = 5900
\]

\[
a \cdot \frac{40}{40} = \frac{5900}{40}
\]

\[
a \cdot 1 = 147.5 \quad \rightarrow \quad a = 147.5
\]

59 is 147.5% of 40.

Example 6.4.5

8 is 25% of what number?

This is a Type 3 question because you are asked to find the original/whole amount.

Answer:

\[
\frac{25}{100} = \frac{8}{d}
\]

\[
25 \cdot d = 8 \times 100
\]

\[
25 \cdot d = 800
\]

\[
\frac{25 \cdot d}{25} = \frac{800}{25}
\]

\[
1 \cdot d = \frac{800}{25} \quad \rightarrow \quad d = 32
\]

8 is 25% of 32.
Example 6.4.6

325 is 130% of what number?

This is a Type 3 question because you are asked to find the original/whole amount.

Answer:

\[
\frac{130}{100} = \frac{325}{w}
\]

\[130 \cdot w = 325 \times 100\]

\[130 \cdot w = 32500\]

\[\frac{130 \cdot w}{130} = \frac{32500}{130}\]

\[1 \cdot w = 250 \quad \rightarrow \quad w = 250\]

325 is 130% of 250.

We end this section by noting that although all questions involving percent may be solved by setting up a proportion, there may be faster ways to obtain an answer depending on the numbers involved. For example, based on your understanding of percent, you should be able to obtain the following answers without resorting to any paper and pencil calculations:

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is 50% of 12?</td>
<td>6</td>
</tr>
<tr>
<td>What is 50% of 30.8</td>
<td>15.4</td>
</tr>
<tr>
<td>What is 100% of 67?</td>
<td>67</td>
</tr>
<tr>
<td>What is 200% of 19?</td>
<td>38</td>
</tr>
<tr>
<td>What is 400% of 2?</td>
<td>8</td>
</tr>
<tr>
<td>5 is 50% of what number?</td>
<td>10</td>
</tr>
<tr>
<td>30 is 25% of what number?</td>
<td>120</td>
</tr>
<tr>
<td>70 is 200% of what number?</td>
<td>35</td>
</tr>
<tr>
<td>8 is what percent of 16?</td>
<td>50%</td>
</tr>
<tr>
<td>24 is what percent of 12?</td>
<td>200%</td>
</tr>
</tbody>
</table>
You should try the following applet, which will allow you to check your work when using a proportion to answer a Type 1, Type 2 or Type 3 question involving a percent:

http://nlvm.usu.edu/en/nav/frames_asid_160_g_2_t_1.html?open=activities&from=topic_t_1.html

**Note:** If you are asked to find a percent of a number (a Type 1 question), there is an additional method that does not involve setting up a proportion. It involves writing the percent as a decimal and performing a multiplication. For example, if you want to find 12.5% of 46, you would multiply $0.125 \times 46$ which gives 5.75. This means that the number 5.75 is 12.5% of 46. As an additional example, let’s find 30% of 824. We write 30% as a decimal and then multiply $824 \times 0.30$, which gives 247.2. This means that 247.2 is 30% of 824.

These videos show you how to set up a proportion to solve problems involving a percent:

http://www.youtube.com/watch?v=QIbIdikcqVA
http://www.youtube.com/watch?v=GwNv4FI9VwU

**Classwork 6.4**

1. 100 is 20% of what number? 500
2. Find 8.2% of 70. 5.74
3. 62 is what percent of 80? 77.5%
4. What number is 150% of 46? 69
5. 318 is 63.6% of what number? 500
6. 2 out of 16 is what percent? 12.5%

7. 25 is 160% of what number? 15.625

8. Find 15% of 75.2. 11.28

9. 95 out of 300 is what percent? 31.6%

10. 12.5% of 56 is what number? 7

**Homework 6.4**

1. 82 is 40% of what number? 205

2. Find 6.5% of 130. 8.45

3. 52 is what percent of 80? 65%

4. What number is 230% of 70? 161

5. 3 is 5% of what number? 60

6. 7 out of 10 is what percent? 70%

7. 10 is 200% of what number? 5

8. Find 25% of 52. 13

9. 480 out of 250 is what percent? 192%

10. 650% of 24 is what number? 156
Section 6.5 – Applications Involving Percents

In this section, we present applications of proportions involving percent. We hope that by presenting real-life applications, the student will come to appreciate the importance of understanding this concept. The focus of this section is on showing the student how to set up a proportion to solve a given percent word problem. If the student needs a review on proportions and percent, Sections 6.2 and 6.3 should be studied thoroughly before continuing reading this section.

Note: For problems involving a percent, we set up a proportion where one side is reserved for the percent, and the other side is reserved for the two numbers. We let a variable represent the percent or amount we are trying to find.

\[
\frac{a}{b} = \frac{c}{d}
\]

Since \( \frac{a}{b} \) represents the fractional form of the percent, \( b \) will always equal 100. We only need to assign values to the variables \( a, c \) and \( d \). This is illustrated in the following examples.
A jacket was originally priced at $145. It is now on sale at 30% of the original price. How much does the jacket cost now?

First, we will set up a proportion to find 30% of $145, and then we will subtract this amount from $145. You should notice that the original price of $145 represents the whole or initial amount, and should be written in the denominator of the right side of the proportion (it’s the \(d\) in the figure above). We also know the percent, which we write as \(\frac{30}{100}\) (it’s the \(\frac{a}{b}\) in the figure above). There is only one spot remaining (the \(c\)) which in this case is the amount that will be subtracted from the original price. This gives the following proportion:

\[
\frac{30}{100} = \frac{c}{145}
\]

Answer:

\[
\frac{30}{100} = \frac{c}{145} \\
30 \times 145 = c \times 100 \\
4350 = 100c \\
43.50 = c
\]

The original price of the jacket will be reduced by $43.50. Thus, its discounted price is $145 - $43.50 = $101.50.
Wendy went to a store and bought an LED TV for $1,590. If there is a 6% sales tax, how much will she pay in total?

First, we will set up a proportion to find 6% of $1,590, and then we will add this amount to $1,590. You should notice that the original price of $1,590 represents the whole or initial amount, and should be written in the denominator of the right side of the proportion (it’s the \( d \)). We also know the percent, which we write as \( \frac{6}{100} \) (it’s the \( \frac{a}{b} \)). The is only one spot remaining (the \( c \)) which in this case is the amount in taxes that Wendy will have to pay. This gives the following proportion:

\[
\frac{6}{100} = \frac{c}{1590}
\]

Answer:

\[
\frac{6}{100} = \frac{c}{1590}
\]

\[6 \times 1590 = c \cdot 100\]

\[9540 = c \cdot 100\]

\[\frac{9540}{100} = \frac{c \cdot 100}{100}\]

\[95.40 = c \cdot 1\]

\[95.40 = c\]

The cost of the LED TV, including the 6% tax, will be $1,590 + $95.40 = $1685.40.
A professional basketball team won 56 out of 82 games this season. What was the winning percentage of this team? In other words, what percentage of the total numbers of games played did the team win? Also, what percent of the 82 games did the team lose? Round your answer to the nearest tenth.

In this question, 82 represents the *entire* or *total* number of games played, and should be written in the denominator of the right side of the proportion (it’s the \(d\)). We also know the number of games won (the \(c\)) out of the total games played. We do not know the percent won, which we write as \(\frac{a}{100}\) (it’s the \(\frac{a}{b}\)). This gives the following proportion:

\[
\frac{a}{100} = \frac{56}{82}
\]

Answer:

\[
\frac{a}{100} = \frac{56}{82}
\]

\[
a \cdot 82 = 56 \times 100
\]

\[
a \cdot 82 = 5600
\]

\[
a \cdot \frac{82}{82} = \frac{5600}{82}
\]

\[
a \cdot 1 = 68.29268
\]

\[
a = 68.3
\]

Rounding the answer to the nearest tenth, the winning percentage of this basketball team was 68.3%. This means that the team lost 100% – 68.3% = 31.7% of the 82 games they played.
Example 6.5.4

A car dealer sold 1150 vehicles last year. Of this number, 38% were trucks. How many vehicles sold last year were trucks?

In this question, 1150 represents the entire or total number of vehicles sold, and should be written in the denominator of the right side of the proportion (it’s the \(d\)). We also know the percent of trucks sold, which we write as \(\frac{38}{100}\) (it’s the \(\frac{a}{b}\)). We do not know the number of trucks sold (the \(c\)) out of the total number of vehicles sold. This gives the following proportion:

\[
\frac{38}{100} = \frac{c}{1150}
\]

Answer:

\[
\frac{38}{100} = \frac{c}{1150}
\]

\[
38 \times 1150 = c \cdot 100
\]

\[
43700 = c \cdot 100
\]

\[
\frac{43700}{100} = \frac{c \cdot 100}{100}
\]

\[
437 = c \cdot 1
\]

\[
437 = c
\]

Last year, the car dealer sold 437 trucks.
Example 6.5.5

In a company, there are 632 employees, 280 of which are female.

A. What percent of the employees are female?

B. What percent of the employees are male?

In this question, the total number of employees is 632 and should be written in the denominator of the right side of the proportion (it’s the \( d \)). We also know the number of female employees (the \( c \)). We do not know the percent of trucks sold, which we write as \( \frac{a}{100} \) (it’s the \( \frac{a}{b} \)). This gives the following proportion:

\[
\frac{a}{100} \cdot 632 = \frac{280 \times 100}{632}
\]

Answer:

\[
\frac{a}{100} = \frac{280}{632}
\]

\[
a \cdot 632 = 280 \times 100
\]

\[
a \cdot 632 = 28000
\]

\[
a \cdot \frac{632}{632} = \frac{28000}{632}
\]

\[
a \cdot 1 = 44.3
\]

\[
a = 44.3
\]

Of the 632 workers that the company employs, 44.3% are female. Since 100% − 44.3% = 55.7%, this means that 55.7% of the employees are male.
In a small district, 2,530 people turned out to vote for proposition A. Forty percent of the voters voted against it.

A. What percent of the electorate voted in favor of Proposition A?

B. How many people voted in favor of Proposition A?

In this question, the total number of voters was 2,530 and should be written in the denominator of the right side of the proportion (it’s the \( d \)). We also know that the percent of the electorate who voted against Proposition A was 40\%. This means that 100\% - 40\% = 60\% voted in favor of it. We write this percent as \( \frac{60}{100} \) (it’s the \( \frac{a}{b} \)). We do not know the number of people who voted in favor of Proposition A (the \( c \)). This gives the following proportion:

\[
\frac{60}{100} = \frac{c}{2530}
\]

Answer:

\[
\frac{60}{100} = \frac{c}{2530} \\
60 \times 2530 = c \cdot 100 \\
151800 = c \cdot 100 \\
\frac{151800}{100} = c \cdot \frac{100}{100} \\
1518 = c \cdot 1 \\
1518 = c
\]

Of the 2530 people who voted, 1518 voted in favor of Proposition A.
At Tech-O University there are currently 360 male faculty. If this number represents 45% of the total number of faculty, how many faculty currently work at Tech-O University?

In this question, we know the percent of male faculty working at the university, 45%, which we write as \( \frac{45}{100} \) (it’s the \( \frac{a}{b} \)). We also know that the number of male faculty is 360 (this is the \( c \)). We do not know the total number of faculty working at Techo University (the \( d \)). Therefore, the proportion can be set up as:

\[
\frac{45}{100} = \frac{360}{d}
\]

**Example 6.5.7**

\[
\frac{45}{100} = \frac{360}{d}
\]

\[
45 \cdot d = 360 \times 100
\]

\[
45 \cdot d = 36000
\]

\[
\frac{45}{45} \cdot d = \frac{36000}{45}
\]

\[
1 \cdot d = 800
\]

\[
d = 800
\]

Answer:

Tech-O University currently employs 800 faculty in total.
Example 6.5.8

The current selling price of a 3-bedroom house is $218,000. If the price of the house decreased by 12% from a year ago, what was the price of the house a year ago? Round your answer to the nearest dollar.

Since the price of the house decreased by 12% compared to the price it had a year ago, this means that its current price of $218,000 represents $100\% - 12\% = 88\%$ of the price it had a year ago. Hence, we know the percent, 88\%, which we write as \(\frac{88}{100}\) (it’s the \(\frac{a}{b}\)). We also know the current price which is $218,000 (this is the \(c\)). We do not know the original price of the house a year ago (the \(d\)). Therefore, the proportion can be set up as:

\[
\frac{88}{100} = \frac{218000}{d}
\]

Answer:

\[
\frac{88}{100} = \frac{218000}{d}
\]

\[
88 \cdot d = 218000 \times 100
\]

\[
88 \cdot d = 21,800,000
\]

\[
\frac{88}{88} \cdot d = \frac{21,800,000}{88}
\]

\[
1 \cdot d = 247,727.2727
\]

\[
d = 247,727.2727
\]

Rounded to the nearest dollar, the price of the house a year ago was $247,727.

This video summarizes the use of proportions to solve real-life applications involving percent:  
http://www.youtube.com/watch?v=yl0Rb6T09VM
1. In January of this year, in a northeastern city, it snowed a total of 6 days. What percent of the total days of January did it snow in this city? Round your answer to the nearest whole percent.

   Of the 31 days of January, it snowed approximately 19% of the days.

2. If 40% of all the students in a history class are males and there are 14 male students enrolled, how many students in total are enrolled in the history class.

   There is a total of 35 students enrolled in the history class (14 males and 21 females).

3. A laptop was originally priced at $799. After a promotional discount, it is now on sale for $599. What was the % discount? Round your answer to the nearest whole percent.

   The promotional percent discount was 25%.

4. The value of a classic car has increased by 20% in the last ten years. If it was valued at $25,000 ten years ago, what is its current value?

   The classic car is now valued at $30,000.

5. Five years ago, George used to earn $12,000. He now earns $15,000 annually. What is the percent increase in his salary compared to five years ago?

   The percent increase in George's salary compared to five years ago is 25%.

6. Carmen and her family had dinner at a restaurant. The bill, without the tip, was $83.40. If she leaves a tip of 10% of the bill, how much will she pay in total?

   Carmen will pay $91.74 in total.

7. A baseball team lost 70 of its 162 games. What was its winning percentage? Round you answer to the nearest tenth of a percent.

   The team won 92 out of the 162 games. Its winning percentage was 56.8%.
8. Marleen answered 41 out of 50 questions correctly. What percent of all the questions did Marleen answer correctly? What percent of all the questions did she get wrong?

Marleen answered 82% of the questions correctly and she answered 18% wrong.

---

**Homework 6.5**

1. If Roxanne gives a tip at a restaurant of 8% of the bill and the bill without the tip is 42.75, how much will she end up paying in total?

Roxanne will pay $46.17 in total.

2. You bought a painting for $15,000 three years ago. It is now valued at $20,000. What was the percent increase in its value during the last three years?

The percent increase in the value of the painting was 33%.

3. A car stereo was originally priced at $200. After a promotional discount, it is now on sale for $125. What was the % discount? 37.5%.

4. At a community college, 45 math courses are being offered this semester. If 20% of all the courses offered by the college are math courses, how many courses in total are being offered this semester?

This semester 225 courses in total are being offered by the college.

5. Marie used to earn $18,000 annually five years ago. She now earns $36,000 annually. What was the percent increase in her salary compared to five years ago? 100%.
6. Michael took a psychology exam and he answered 37 out of the 40 questions correctly. What percent of the total number of questions did Michael got correct? 92.5%.

7. Kimberly earned $300 this week working at a bookstore. Of the $300, she put $65 in her savings account. What percent of her salary did she save? Round your answer to the nearest tenth of a percent. Kimberly put 21.7% of the $300 she earned this week in her savings account.

8. A professional basketball player made 11 out of 16 free throws in a game. What percent of all the free throw shots he attempted did he make? Round the percent to the nearest whole number. The basketball player made 69% of his free throws.

The following website has additional word problems with solutions involving percent. You may set up a proportion to solve these problems, just like we did in this section. You can then compare your answers to the posted solutions.

http://cnx.org/content/m35007/latest/?collection=col10615/latest
Chapter 6 Test

1. Rick drove 128 miles in 2 hours. Write this rate as a fractional unit rate.

   \[
   \frac{64 \text{ miles}}{1 \text{ hour}} \quad \text{or} \quad \frac{64 \text{ miles}}{2 \text{ hours}}
   \]

2. Of the 48 students enrolled in a business administration course, 28 of the students are male.

   a. Write the ratio of male students to female students in fractional form and reduce it to lowest terms.

   \[
   \frac{7 \text{ males}}{5 \text{ females}}
   \]

   b. Write the ratio of female students to the total number of students enrolled in the course in fractional form and reduce it.

   \[
   \frac{5 \text{ females}}{12 \text{ students}}
   \]

3. One bag of 40 Tangy candies costs $5.20. Another bag has 30 Tangy candies but costs $4.20. Which is the better deal?

   The bag of 40 Tangy candies is the better deal because each candy costs 13 cents. This is one cent less that the cost of each candy in the 30-candy bag.

Solve each proportion.

4. \[\frac{40}{16} = \frac{u}{5}\]  
   \[u = 12.5\]

5. \[\frac{12}{y} = \frac{8}{50}\]  
   \[y = 75\]

6. \[\frac{m}{6.6} = \frac{9.8}{2.4}\]  
   \[m = 26.95\]

Solve each word problem by setting a proportion.

7. Marie can repair 3 bicycles in 1.5 hours. At that rate, how many bicycles can she repair in 4.5 hours?

   \[\text{At that rate, Marie can repair 9 bicycles in 4.5 hours.}\]

8. In a supermarket, if 4 lb of pears cost $3 how much would 10.5 lb of pears cost? Round your answer to the nearest cent.

   \[\text{In this supermarket, 10.5 lb of pears would cost $7.88.}\]

9. Write \(\frac{12}{80}\) as a decimal.
   \[0.15\]

10. Write 0.0385 as a percent.
    \[3.85\%\]
11. Write 534% as a decimal: 5.34

12. Write $9.037$ as a mixed number: $9 \frac{37}{1000}$

13. Write $\frac{17}{20}$ as a percent: 85%

14. Find 165% of 94: 155.1

15. What number is 5% of 300?: 15

16. 72 is what % of 200?: 36%

17. Deborah always puts 15% of each paycheck into her savings account. If her last check was for $2,680, how much did she put in her savings account?

   Deborah put $402 out of the $2,680 into her savings account.

18. Andy went shopping and bought a jacket that was on sale for $84 after a 30% discount. What was the original price of the jacket?

   The original price of the jacket was $120.

19. Vicky originally bought her house for $250,000. It is now valued at $340,000. What is the percent increase in the value of the house?

   The percent increase in the value of the house is 36%.

20. Abraham gets a 2.5% commission on each car he sells. If he sold a car for $35,000, how much did he receive in commission?

   Abraham received $875 in commission for selling this car.
Cumulative Review - Chapters 1-6

Write the following ratio as a fraction and simplify completely:

1) 100 inches : 350 inches

\[
\frac{2}{7}
\]

2) 240 dollars : 8 hours

\[
\frac{30 \text{ dollars}}{1 \text{ hour}}
\]

Solve by finding the value of \( n \) that makes the equation true:

3) \( \frac{14}{15} = \frac{n}{105} \)

\( n = 98 \)

4) A roofing company sells roofing material for $20 per square yard. How many square yards of roofing material can be purchased for $1400?

\( 70 \text{ square yards of roofing material can be purchased}. \)

5) Karen received 18 votes for every 24 people who voted. If she received a total of 360 votes, how many people voted?

\( 480 \text{ people voted}. \)

6) Sophia was able to drive 400 miles on 30 gallons of gasoline. At that rate, how many miles will she be able to drive on 45 gallons?

\( \text{Sophia will be able to drive 600 miles on 45 gallons of fuel}. \)

7) 50% of 48 is what number? \( 24 \)

8) 25% of what number is 32? \( 128 \)

9) 45% of 160 = \( 72 \)

Evaluate each expression by following the order of operations.

10) \( (6 - 4)^3 \cdot (48 \div 12)^3 = \)

\( 512 \)

11) \( 30 \cdot \sqrt{27} \cdot 3 + \sqrt{100} - 25 \cdot 8 = \)

\( 80 \)
12) If Wendy averages 60 miles per hour on her trip, how far will she be able to travel in 3.5 hours? \(210\) miles

Multiply or divide. Simplify each answer completely.

\[13) \frac{2}{7} \cdot \frac{14}{30} = \frac{2}{15} \quad 14) 8 \cdot \frac{1}{4} = 2 \quad 15) 1\frac{2}{3} \cdot 2\frac{2}{5} = 4\]

\[16) \frac{5}{8} \div \frac{12}{15} = \frac{25}{32} \quad 17) 6\frac{2}{3} \div 2\frac{2}{5} = 2\frac{7}{9}\]

18) Nate’s dog Rosey, a golden retriever, eats \(\frac{2}{3}\) cups of dog food each day. How many cups of dog food does Rosey eat in a 30-day month?

\(20\) cups of dog food

Add or Subtract. Simplify your answer if possible.

\[19) \frac{5}{8} + \frac{2}{8} = \frac{7}{8} \quad 20) \frac{4}{7} + \frac{2}{21} = \frac{2}{3} \quad 21) 12\frac{3}{4} - 5\frac{1}{4} = 7\frac{1}{2}\]

\[22) \frac{5}{9} - \frac{12}{27} = \frac{1}{9} \quad 23) 4\frac{1}{8} - 1\frac{1}{4} = 2\frac{7}{8}\]

Round each of these numbers to the nearest hundredth.

\[24) 5.0345 \quad 25) 12.99521 \]

\(5.03 \quad 13.00\)
Perform each operation. Use the order of operations.

26) Subtract 27.8 from 136.9 = 109.1

27) Subtract 62.74 from 500 = 437.26

28) \((2.5 + 7.346) – 3.14 = \)

\[ \frac{2.16}{0.6} = 3.6 \]

29) \(26 – (6.38 + 5.65) = \)

30) \(12 \times (0.14)^2 = \)

31) \((0.7)^3 = \)

32) \(5.32 \times 2.5 = \)

33) \(10.6 \times 0.002 = \)

34) \(0.426 \div 6^2 = \)

35) Monica has a hybrid car that runs on gasoline and electricity. Her car averages 32.5 miles per gallon. How far can Monica drive on 6.5 gallons of gas?

\[ 211.25 \text{ miles} \]

36) Find the Greatest Common Factor (GCF) of each pair of numbers. If the GCF is 1, write “relatively prime.”

37) 20 and 48

\[ \text{relatively prime} \]

38) 128 and 125

\[ \text{relatively prime} \]

39) 45, 180 and 27

\[ 540 \]

40) 6, 33 and 12

\[ 132 \]
Write the first five multiples of each number

41) 7  
    7, 14, 21, 28, 35

42) 12  
    12, 24, 36, 48, 60

43) 18  
    18, 36, 54, 72, 90

Round 3,672.982 to the nearest

44) tenth  
    3,673.0

45) thousand  
    4,000

46) whole number  
    3,673

47) Find the area and perimeter of a triangle with base = 32 cm and height = 24.6 cm.
   
   \[ \text{Area} = 393.6 \text{ cm}^2 \quad \text{Perimeter} = 113.2 \text{ cm} \]

48) Find the area and perimeter of the following parallelogram:

   \[ \begin{align*}
   \text{Area} & = 44 \text{ yd}^2 \\
   \text{Perimeter} & = 32 \text{ yd}
   \end{align*} \]

49) Find the area and perimeter of a rectangle with base = 62 yd and height = 45 yd.

   \[ \begin{align*}
   \text{Area} & = 2790 \text{ yd}^2 \\
   \text{Perimeter} & = 214 \text{ yd}
   \end{align*} \]

50) Find the area and perimeter of a square with sides measuring \(6 \frac{3}{5}\) meters long.

   \[ \begin{align*}
   \text{Area} & = 43 \frac{14}{25} \text{ m}^2 \\
   \text{Perimeter} & = 26 \frac{2}{5} \text{ m}
   \end{align*} \]
Chapter 7 Overview
By the end of this chapter, you will achieve mastery of the following concepts:

- Addition and Subtraction of Signed Numbers
  - Positive numbers usually represent an increase or a gain, whereas negative numbers often represent a decrease, a debt or a loss.
  - $730 An amount earned this week.
  - $1,341 An amount Ron owes to the bank.
  - $5 An increase in temperature.

- Multiplication and Division of Signed Numbers

- Solving Linear Equations

- The Laws of Exponents

Using an analogy involving a monetary transaction often helps to determine the sign of the result when adding or subtracting positive and negative numbers.

When multiplying or dividing numbers, the answer will be positive if the two numbers have the same sign, and will be negative if the signs are different.

An equation is a statement of equality. Solving an equation requires finding the value of the variable that leads to a true statement. To solve an equation, we perform a series of opposite operations until the variable is isolated on one side of the equation.

Solve: $6x - 15 = 4x - 42$
- Subtract 4x from each side: $2x = -27$
- $\frac{2x}{2} = -\frac{27}{2}$
- $x = -13.5$

<table>
<thead>
<tr>
<th>Rules for Multiplication and Division of Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Number</td>
</tr>
<tr>
<td>+</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>+</td>
</tr>
<tr>
<td>-</td>
</tr>
</tbody>
</table>

- $-32 \cdot 5 = -160$
- $(-8)(-3) = 24$
- $7(-9.5) = -66.5$
- $81 + (-3) = -27$
- $(-20.5) + 2 = -10.25$
- $-42 + (-3) = 14$

| Negative Exponent Law | $y^{-a} = \frac{1}{y^a}$ | $6^{-5} = \frac{1}{6^5}$ |
| Multiplication Law | $y^a \cdot y^b = y^{a+b}$ | $10^{-5} \cdot 10^8 = 10^{-5+8} = 10^3$ |
| Division Law | $\frac{y^a}{y^b} = y^{a-b}$ | $\frac{13^9}{13^4} = 13^{9-4} = 13^5$ |
| Power Law | $(y^a)^b = y^{a\cdot b}$ | $(2^{-6})^{-3} = 2(-6)(-3) = 2^{18}$ |
| Fractional Exponent Law | $y^{\frac{a}{b}} = \left(\sqrt[y]{y}\right)^a = \sqrt[y]{y^a}$ | $\frac{3}{64x} = \left(\sqrt[64]{64}\right)^3 = \frac{3}{\sqrt[64]{64^3}} = 512$ |
Section 7.1 – Addition and Subtraction of Positive and Negative Numbers

So far in this book, we have performed operations on whole numbers, fractions, and decimals. In doing so, we have considered numbers equal to or greater than zero. That is, we have dealt only with applications involving the right side of the number line and zero. These are the nonnegative numbers.

Recall that we have performed operations such as

\[ 453.27 + 16.20 \quad 4 \frac{3}{5} - 1 \frac{7}{8} \quad 356 \times 25 \quad 4,578 \div 12 \]

In this chapter, we introduce the notion of negative numbers and present some of their real-life applications. This will allow us to consider the entire number line:

To write a negative number, we place a negative sign to the left of the number, for example \(-5\). The following table lists some interpretations and applications of positive numbers, negative numbers, and zero (which is neither positive nor negative).
<table>
<thead>
<tr>
<th>Number</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 lb</td>
<td>A gain of 10 pounds.</td>
</tr>
<tr>
<td>8</td>
<td>An increase in the number of courses offered by 8.</td>
</tr>
<tr>
<td>$3,760.51</td>
<td>The amount I have in my checking account.</td>
</tr>
<tr>
<td>$–5,000</td>
<td>A loss of $5,000 through investments.</td>
</tr>
<tr>
<td>−10 yd</td>
<td>In football, a loss of 10 yards due to a penalty.</td>
</tr>
<tr>
<td>$–1,340.56</td>
<td>The amount I owe the bank.</td>
</tr>
<tr>
<td>$–78.56</td>
<td>The amount I spent at the supermarket today.</td>
</tr>
<tr>
<td>0</td>
<td>No net gain or loss in the value of a property.</td>
</tr>
</tbody>
</table>

A positive number may be thought of as representing a gain, an increase in value or an amount of money that is yours, whereas a negative number often represents a loss or an amount you owe. We may interpret the addition sign " + " as having the same meaning as the word “and.” With this interpretation of the plus sign, and the meaning of positive and negative numbers, an equation such as

$$1000 + –375 = 625$$

can represent a real-life situation. For example, this equation may be interpreted as saying that “James initially had $1,000 in his checking account, but after spending $375 at a department store he is now left with only $625.” Notice that the answer is positive because the $625 belongs to James. It is not money that he owes.

A second example would be  

$$–45.67 + 50 = 4.33$$

This equation may represent a situation where “Sue spent $45.67 in groceries. She paid with a $50 and received $4.33 in change.”

A third example is  

$$–565.78 + 500 = –65.78$$

This equation may represent a situation where “Mallory bought a new refrigerator that cost $565.78. She paid $500 using her checking account and used a credit card to pay the rest. She owes the bank $65.78.” The reason the answer is
negative is because after the transaction, the $65.78 represents an amount that Mallory owes to the bank.

Finally, consider the following equation

\[-240 + -300 = -540\]

This equation may represent a situation where “Kim borrowed $240 from her brother and $300 from a friend. She now owes $540.”

When you need to add positive and negative numbers, a strategy that will help you figure out the answer is to think of a real-life situation involving money. If you have enough money to pay the amount you owe or spent, the answer will be positive. However, if you spend or owe more than the amount that you have, the answer will be negative.

**Example 7.1.1**

By thinking of a scenario involving a monetary transaction, you will be able to figure out the sign of the result:

465 + 350 = 815  
“I earned $465 last week and $350 this week. I now have $815.”

734 + -145 = 589  
“I had $734 and then spent $145. I now have $589.”

820 + -75 = 745  
“I had $820 and then spent $75. I now have $745.”

342 + -342 = 0  
“I had $342 and then spent the $342. I have no money left.”

-45.78 + 32.61 = -13.17  
“I spent $45.78 and had $32.61. I now owe $13.17 to the bank.”

-126 + 85 = -41  
“I spent $126 and had $85. I now owe $41 to the bank.”

-29 + 29 = 0  
“I spent $29 and had exactly $29. I have no money left.”

-14 + -6 = -20  
“I spent $14 in store 1 and spent $6 in store 2. I spent $20 in total.”

-89.32 + -130 = -219.32  
“I borrowed $89.32 from a friend and $130 from another. I owe $219.32 in total.”
In mathematics, the minus sign " − " represents subtraction. However, it can also be thought of as asking for the *opposite value* of a number. For example, the following expressions and phrases have the same meaning:

−1  “the opposite of 1”  −1
−32  “the opposite of 32”  −32
−956  “the opposite of 956”  −956
−78.54  “the opposite of 78.54”  −78.54
−− 14  “the opposite of −14”  14
−− 56.3  “the opposite of −56.3”  56.3
−− 1  “the opposite of −1”  1
−(−2)  “the opposite of −2”  2

**Note:** The opposite of a positive number is a negative number (−(6) = −6), and the opposite of a negative number is a positive number (−− 13 = 13). However, zero is its own opposite (−0 = 0).

**Example 7.1.2**

Simplify the following expression: 45 − 70

We rewrite the original expression as 45 + −70

This expression can be read as “I have $45 but I owe $70 to a friend. After I give her the $45 I have, I will still owe her $25.” Thus, the answer is negative:

45 + −70 = −25
Example 7.1.3

Simplify the following expression: $-93 - 157 + 1000$

We rewrite the original expression as

$$-93 + (-157) + 1000$$

$$-250 + 1000 = 750$$

This expression can be read as “I spent $93 at the supermarket and I spent $157 at an apparel store. Therefore, I spent a total of $250. I had $1000 in my checking account. I am now left with $750 after making these purchases.”

Example 7.1.4

Simplify the following expression: $-846.41 + 1956.37 - 430$

We rewrite the original expression as

$$-846.41 + 1956.37 + (-430)$$

After thinking of a real-life scenario to help simplify the expression above, you should obtain the following result:

$$1109.96 + (-430) = 679.96$$
Example 7.1.5
Simplify the following expression: \(45 - 70 + 100 - 35\)

Answer:

\[45 - 70 + 100 - 35\]
\[45 + (-70 + 100 + (-35)\]

Adding the numbers from left to right gives

\[-25 + 100 + (-35)\]
\[75 + (-35)\]
\[40\]

Example 7.1.6
Simplify the following expression: \(-58.3 - (-67.8) + 470 - 180\)

Answer:

\[-58.3 - (-67.8) + 470 - 180\]
\[-58.3 + 67.8 + 470 + (-180)\]

Adding the numbers from left to right gives

\[9.5 + 470 + (-180)\]
\[479.5 + (-180)\]
\[299.5\]
Example 7.1.7
Simplify the following expression: \(161.49 - 200 - 37.46 - (-50)\)

Answer:

\[
161.49 - 200 - 37.46 - (-50) \\
161.49 + -200 + -37.46 + 50
\]

Adding the numbers from left to right gives

\[
-38.51 + -37.46 + 50 \\
-75.97 + 50 \\
-25.97
\]

The following videos show you how to add and subtract positive and negative numbers:

http://www.youtube.com/watch?v=HnqdC21Rcvo
http://www.youtube.com/watch?v=bl9TrThTqc
http://www.youtube.com/watch?v=M_lWN282LYY
http://www.youtube.com/watch?v=hQy6v7QFZCU
http://www.youtube.com/watch?v=PqYkONujjnU

Classwork 7.1
Simplify the following expressions completely. Try to think of a real-life scenario involving money to help you figure out the answer.

A) \(68 + 328\)  
B) \(28 + -80\)

F) \(-13 + -20\)  
G) \(-90 + -61.87\)

K) \(-6.4 + -9.5\)  
L) \(156 - -61.87\)
Simplify the following expressions completely. Try to think of a real-life scenario involving money to help you figure out the answer.

A) \(99.6 + 436.67\)  
B) \(67 + -123\)  
C) \(345 + -17\)  
D) \(-89.56 + 62\)  
E) \(-116.5 + 400\)  
F) \(-16 + -33\)  
G) \(-156.87 + -45\)  
H) \(-12 + -9\)  
I) \(-54 + 59\)  
J) \(-3.26 + 4.56\)  
K) \(-2.6 + -14\)  
L) \(200 - -34.98\)  
M) \(-61.3 - -61.3\)  
N) \(-(-6) - -2\)  
O) \(-88 - -87\)

The following websites have additional exercises with solutions involving addition and subtraction of signed numbers:

http://cnx.org/content/m35031/latest/?collection=col10615/latest

http://cnx.org/content/m35032/latest/?collection=col10615/latest
Section 7.2 – Multiplication and Division of Positive and Negative Numbers

In this section, we present the rules for multiplying and dividing positive and negative numbers. We then present examples that illustrate how to simplify expressions that involve several operations with positive and negative numbers.

<table>
<thead>
<tr>
<th>Rules for Multiplication and Division of Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of First Number</td>
</tr>
<tr>
<td>+</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>+</td>
</tr>
<tr>
<td>-</td>
</tr>
</tbody>
</table>

The above rules can be easily remembered by noting that

<table>
<thead>
<tr>
<th>Rules for Multiplication and Division of Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of First Number</td>
</tr>
<tr>
<td>A good thing</td>
</tr>
<tr>
<td>A bad thing</td>
</tr>
<tr>
<td>A good thing</td>
</tr>
<tr>
<td>A bad thing</td>
</tr>
</tbody>
</table>

In general, you should follow the order of operations to simplify expressions involving multiple operations, as you did in previous chapters.
Example 7.2.1
Simplify the following expression: $-14 \cdot 5$

Remembering that a negative number times a positive number gives a negative answer, we conclude that $-14 \cdot 5 = -70$.

Example 7.2.2
Simplify the following expression: $-36.8 \times -2.5$

Remembering that a negative number times a negative number gives a positive answer. Thus, $-36.8 \times -2.5 = 92$.

Example 7.2.3
Simplify the following expression: $423 \times -8$

Since a positive number times a negative number gives a negative answer, we have $423 \times -8 = -3,384$.

In some textbooks, a negative number such as $-3,384$ is written as $\neg 3,384$. This should not confuse you, since we have already mentioned that the negative sign "−" asks for the opposite value. This means that $-3,384 = \neg 3,384$ because the opposite of $3,384$ is $\neg 3,384$ or simply $-3,384$. Therefore, both representations of a negative number are correct.

Note: In this book, we will use the conventional notation $-543$ to represent a negative number. The notation $-543$ will also be interpreted as asking for the opposite of $543$, which is $-543$. 
**Example 7.2.4**

Simplify the following expression: \(75.83 - 42 \times 3\)

Applying the order of operations, we will first perform the multiplication and then the subtraction. After noting that \(42 \times 3 = 126\), the minus sign "\(-\)" can be interpreted as asking for the opposite of 126, which is \(-126\). Finally, we must add 75.83 and \(-126\), which gives \(-50.17\).

**Answer:**

\[
\begin{align*}
75.83 - 42 \times 3 &= 75.83 - 126 \\
75.83 + (-126) &= -50.17
\end{align*}
\]

**Example 7.2.5**

Simplify the following expression: \(16 - (5 - 18) \times -2\)

Applying the order of operations, we first simplify the expression inside the parentheses and then perform the multiplication.

**Answer:**

\[
\begin{align*}
16 - (5 - 18) \times -2 &= 16 - (-13) \times -2 \\
16 - (-13) \times -2 &= 16 - 26 \\
16 + (-26) &= -10
\end{align*}
\]
**Example 7.2.6**

Simplify the following expression: \(-360 ÷ 2 + 5 \times (-42 ÷ 6) + 100\)

\[
\begin{align*}
-360 ÷ 2 &+ 5 \times (-42 ÷ 6) + 100 \\
-360 ÷ 2 &+ 5 \times (-7) + 100 \\
-180 &+ 5 \times (-7) + 100 \\
-180 &+ (-35) + 100 \\
-115 &
\end{align*}
\]

**Answer:** 
-115

---

**Example 7.2.7**

Simplify the following expression: \(60 ÷ -3 + (31 - 36)^2\)

\[
\begin{align*}
60 ÷ -3 &+ (31 - 36)^2 \\
60 ÷ -3 &+ (31 - 36)^2 \\
60 ÷ -3 &+ (-5)^2 \\
60 ÷ -3 &+ 25 \\
-20 &+ 25 \\
5 &
\end{align*}
\]

**Answer:** 5
Example 7.2.8

Simplify the following expression: \(150 - (-7)^3 + (-2)^4\)

Answer:
\[
150 - (-7)^3 + (-2)^4 \\
150 - (-7 \times -7 \times -7) + (-2 \times -2 \times -2 \times -2) \\
150 - (-343) + 16 \\
150 + 343 + 16 \\
509
\]

Example 7.2.9

Simplify the following expression: \(-10 \times \sqrt{169} - 45 \times -2 + (7 - 8)^3\)

Answer:
\[
-10 \times \sqrt{169} - 45 \times -2 + (7 - 8)^3 \\
-10 \times 13 + -45 \times -2 + (7 - 8)^3 \\
-10 \times 13 + -45 \times -2 + (-1)^3 \\
-10 \times 13 + -45 \times -2 + -1 \\
-130 + 90 + -1 \\
-40 + -1 \\
-41
\]
**Note:** Raising a negative number to an even exponent gives a positive answer, whereas raising a negative number to an odd exponent gives a negative answer.

\[
(-1)^0 = 1 \quad (-2)^0 = 1 \quad (-3)^0 = 1 \\
(-1)^1 = -1 \quad (-2)^1 = -2 \quad (-3)^1 = -3 \\
(-1)^2 = 1 \quad (-2)^2 = 4 \quad (-3)^2 = 9 \\
(-1)^3 = -1 \quad (-2)^3 = -8 \quad (-3)^3 = -27 \\
(-1)^4 = 1 \quad (-2)^4 = 16 \quad (-3)^4 = 81 \\
(-1)^5 = -1 \quad (-2)^5 = -32 \quad (-3)^5 = -243 \\
\vdots \quad \vdots \quad \vdots
\]

The following videos illustrate how to multiply and divide signed numbers:

http://www.youtube.com/watch?v=57XEV2DieDQ
http://www.youtube.com/watch?v=tD5cD7frDCw

**Classwork 7.2**

Use the order of operations to simplify the following expressions completely.

1) \(-13.5 \times 9\) \(-121.5\)

2) \(-48 \div -16\) \(3\)

3) \(100 - 450 + (-70.4 \times -2)\) \(-209.2\)

4) \(84 + 24 \div -6 + (-5)^3\) \(-45\)

5) \(-4 \times \sqrt{64} - 52 \times 4 + \frac{-20}{-5}\) \(-236\)

6) \(\frac{160}{-4} \div 5 + 680 \div 4 \times -3\) \(-518\)

7) \(16.25 - 40 \times 3 + (-1)^{10}\) \(-102.75\)

8) \(8.6 \times -2.4 - 9.15 + 25 \div -5\) \(-34.79\)
Homework 7.2

Use the order of operations to simplify the following expressions completely.

1) \(-61.8 \times -4.5\)  2) \(-80 \div 5\)  3) \(970 - 1350 + (-12.6 \times -4)\)  4) \(-44 + 17 \times -6 + (-2)^5\)

5) \(\sqrt{36} + -9 \times 2 + \frac{20.4}{-2}\)  6) \(700 \div -5 + 124 \div 4 \times -2\)

7) \(47 - (-1)^{15} + (9 - 12)^2\)  8) \(1.32 \times -8 - 66.2 \div 5 + \frac{49}{7}\)

The following website has additional exercises with solutions involving multiplication and division of signed numbers:

http://cnx.org/content/m35033/latest/?collection=col10615/latest
Section 7.3 – The Order of Operations Revisited

In Section 2.5, we discussed the importance of having a set of rules to follow when simplifying mathematical expressions. If an order of operations is not established, this would lead to different people obtaining different answers. The first objective of this section is to introduce notation used in algebra. The second objective is to apply the order of operations to simplify algebraic expressions involving variables, which are letters that are used to represent numbers. You will also learn two algebraic properties that may be used to simplify expressions, even though their application would not follow the order of operations. For your convenience, the order of operations is once again presented below.

<table>
<thead>
<tr>
<th>The Order of Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong></td>
</tr>
<tr>
<td><strong>Step 2</strong></td>
</tr>
<tr>
<td><strong>Step 3</strong></td>
</tr>
<tr>
<td><strong>Step 4</strong></td>
</tr>
</tbody>
</table>

The following table lists some algebraic expressions and the operation each expression represents.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Algebraic Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>x + y 7 + c</td>
</tr>
<tr>
<td>Subtraction</td>
<td>x − y 7 − c</td>
</tr>
<tr>
<td>Multiplication</td>
<td>xy  x · y (x)(y) x(y) x * y</td>
</tr>
<tr>
<td></td>
<td>9y  9 · y (9)(y) 9(y) 9 * y</td>
</tr>
<tr>
<td>Division</td>
<td>x ÷ y x y</td>
</tr>
<tr>
<td>Square Root</td>
<td>√x</td>
</tr>
<tr>
<td>Exponents</td>
<td>x^2 y^5</td>
</tr>
</tbody>
</table>
Let’s practice applying the order of operations to simplify algebraic expressions.

**Example 7.3.1**

Suppose that \( n = 10, p = 8, r = 5, k = 1, x = 3 \).
Evaluate each of the following expressions:

A) \( 7p + (x - 1)^4 + \sqrt{k + x} \)

**Answer:**

\[
7p + (x - 1)^4 + \sqrt{k + x} \\
7 \cdot 8 + (3 - 1)^4 + \sqrt{1 + 3} \\
7 \cdot 8 + (2)^4 + \sqrt{4} \\
7 \cdot 8 + 16 + 2 \\
56 + 16 + 2 = 74
\]

B) \( \frac{np}{r} + 12(p^2 - r^2) - 15\sqrt{20r} \)

**Answer:**

\[
\frac{np}{r} + 12(p^2 - r^2) - 15\sqrt{20r} \\
\frac{10 \cdot 8}{5} + 12(8^2 - 5^2) - 15\sqrt{20 \cdot 5} \\
\frac{10 \cdot 8}{5} + 12(8^2 - 5^2) - 15\sqrt{20 \cdot 5} \\
\frac{10 \cdot 8}{5} + 12(64 - 25) - 15\sqrt{100} \\
\frac{80}{5} + 12(39) - 15 \cdot 10 \\
16 + 468 - 150 = 334
\]
C) \[ 4x^2 + 2r^3 - (2x)^3 \]

Answer:
\[
4x^2 + 2r^3 - (2x)^3 \\
4 \cdot 3^2 + 2 \cdot 5^3 - (2 \cdot 3)^3 \\
4 \cdot 3^2 + 2 \cdot 5^3 - 6^3 \\
4 \cdot 9 + 2 \cdot 125 - 216 \\
36 + 250 - 216 \\
70
\]

D) \[ 4p^2 - 2n^3 + (-4x)^2 \]

Answer:
\[
4p^2 - 2n^3 + (-4x)^2 \\
4 \cdot 8^2 - 2 \cdot 10^3 + (-4 \cdot 3)^2 \\
4 \cdot 8^2 - 2 \cdot 10^3 + (-12)^2 \\
4 \cdot 64 - 2 \cdot 1000 + 144 \\
256 - 2000 + 144 \\
-1600
\]
In algebra, there are two properties that are often used to simplify expressions. They are valid even though they do not necessarily follow the order of operations. You may use these properties at any step where it is appropriate to use them. The first property is called the **distributive property** and the second one is called “**combining like terms**.” Combining like terms refers to the addition or subtraction of similar terms. **Like terms** have the same variable raised to the same exponent. These properties are defined below and examples are given to illustrate their use.

<table>
<thead>
<tr>
<th>Property</th>
<th>Algebraic Notation</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distributive Property</strong></td>
<td>$a \cdot (b + c) = a \cdot b + a \cdot c$</td>
<td>$6 \cdot (b + 4) = 6 \cdot b + 6 \cdot 4$</td>
</tr>
<tr>
<td></td>
<td>$a \cdot (b - c) = a \cdot b - a \cdot c$</td>
<td>$6 \cdot (b - 4) = 6 \cdot b - 6 \cdot 4$</td>
</tr>
<tr>
<td>“Combining Like Terms” Property</td>
<td>$5z + 3z + z = 9z$</td>
<td>$5 \cdot 4 + 3 \cdot 4 + 1 \cdot 4 = 9 \cdot 4$</td>
</tr>
<tr>
<td></td>
<td>$12m^3 + 6m^3 - m^3 = 17m^3$</td>
<td>$5m^5 + 4m^2 - 2m^5 + 8m^2 + m^5 = 4m^5 + 12m^2$</td>
</tr>
</tbody>
</table>

You may use either of these two properties when you need to simplify an expression. There are cases when combining like terms first, and then applying the order of operations, leads to a faster simplification process. On the other hand, although the distributive property may not necessarily speed up the process of simplifying an expression, it will be very useful when we solve linear equations later in the chapter.

Note: The distributive property $a \cdot (b - c) = a \cdot b - a \cdot c$ can be applied only if the term $(b - c)$ is raised to the first power. For example, the following distribution of $a$ is **not valid**:

$$a \cdot (b - c)^2 = (a \cdot b - a \cdot c)^2$$
Example 7.3.2

Suppose that $a = 9$, $b = 6$, $c = 4$.

Simplify the following expression: $3a + 8b + 12c + 4a - b - 8c + 9a$

Answer:

*We notice there are like terms and combine them:*

$$3a + 8b + 12c + 4a - b - 8c + 9a$$

$$3a + 4a + 9a + 8b - b + 12c - 8c = 16a + 7b + 4c$$

*We substitute the value of each variable:*

$$16 \cdot 9 + 7 \cdot 6 + 4 \cdot 4$$

*We then apply the order of operations:*

$$144 + 42 + 16 = 202$$

**Warning:** Make sure you understand that when you combine like terms, the exponents remain the same. Do not add or subtract the exponents when you combine like terms. For example, it is correct to say that

$$32k^6 + 15k^6 = 47k^6$$

$$m + m + m + m + m = 5m$$

$$4c^3 + c^3 + c^3 + c^3 + c^3 = 8c^3$$

However, the following simplifications are incorrect:

$$32k^6 + 15k^6 = 47k^{12}$$

$$m + m + m + m + m = m^5$$

$$m + m + m + m + m = 5m^5$$

$$4c^3 + c^3 + c^3 + c^3 + c^3 = 4c^3$$
Example 7.3.3
Suppose that \(x = 1, y = 4, z = 9\).
Simplify the following expression: \(6(3x + 2y) - 2(z + 5) + 8(z + x)\)

Answer:

*We apply the distributive property:*

\[
6(3x + 2y) - 2(z + 5) + 8(z + x) = 18x + 12y - 2z - 10 + 8z + 8x
\]

*We combine the like terms:*

\[
26x + 12y + 6z - 10
\]

*We substitute the value of each variable:*

\[
26 \cdot 1 + 12 \cdot 4 + 6 \cdot 9 - 10
\]

*We then apply the order of operations:*

\[
26 + 48 + 54 - 10 = 118
\]

Example 7.3.4
Suppose that \(w = -3, f = 2, h = -5\).
Simplify the following expression: \(5w^3 + 60f - 4h^3 - 100f + 2w + 5h^3\)

Answer:

*We notice there are like terms and combine them:*

\[
5w^3 + 60f - 4h^3 - 100f + 2w + 5h^3 = 5w^3 + 2w - 40f + h^3
\]

*We substitute the value of each variable:*

\[
5(-3)^3 + 2(-3) - 40(2) + (-5)^3
\]

*We then apply the order of operations:*

\[
5(-27) + 2(-3) - 40(2) + -125 = -135 - 6 - 80 - 125 = -346
\]
Example 7.3.5

Suppose that \(a = -8, d = 2\).
Simplify the following expression: \(4a^2 + 14d - 9a - 4a^2 - 10d - 4d + 2a\)

**Answer:**

*We notice there are like terms and combine them:*

\[
4a^2 + 14d - 9a - 4a^2 - 10d - 4d + 2a = -7a
\]

*We substitute the value of the variable and simplify:*

\[
-7 \cdot -8 = -56 = 56
\]

This video shows how to apply the distributive property and combine like terms:
http://www.youtube.com/watch?v=P2VJKFxXdXU

These videos illustrate the order of operations with signed numbers:
http://www.youtube.com/watch?v=5P0tup2ZT78
http://www.youtube.com/watch?v=ZPRTboH6Yng

These videos illustrate how to evaluate algebraic expressions:
http://www.youtube.com/watch?v=32Pix_FRw2E
http://www.youtube.com/watch?v=u1Y7Xb2serU
http://www.youtube.com/watch?v=H9iUy07QKXQ

Classwork 7.3

Given that \(r = -1, t = 5, m = 9, a = -2.3\) and \(c = 12.8\), simplify each of the following expressions completely.

1) \(t^2 - m^2 + \sqrt{m}\)

\(-53\)

2) \((c - 15)^2 - 6ar\)

\(-8.96\)

3) \(10a + 25r - 9a - 18r\)

\(-9.3\)

4) \(2m^2 - 5m^2 + at\)

\(-254.5\)
5) \[ \frac{10(0.2+c+2t+2)}{2t-60} \]

6) \[ \frac{4m+76}{4r^5} \]

7) \[ -9c + 4t + 9c - 4t \]

8) \[ 6r^{10} + 6ar^{13} \]

9) \[ 16(t + r) - 3(m + t)^2 \]

10) \[ 46 + 5\sqrt{tm} + 4 - 3c \]

**Homework 7.3**

Given that \( x = 4, \ y = 5, \ z = -16, \ w = 2 \) and \( d = 6.5 \), simplify each of the following expressions completely.

1) \[ yz - xy^2 - 8d \]

2) \[ 70 - dy + 2(z - y) \]

3) \[ 3z + 9y - 8x - 3z + 8x \]

4) \[ 4y^2 - 5d^2 + zw \]

5) \[ \frac{z+xy}{12w} \]

6) \[ \frac{8\sqrt{x+y}+6z}{xyz} \]

7) \[ 3z + 4(x + y)^2 \]

8) \[ 3w^5 - 2dw \]

9) \[ -9(xy + z) + 3(-y + 2)^2 \]

10) \[ \frac{z^2}{w} + 15\sqrt{z} + 20 - d^2 \]

The following website has additional exercises with solutions involving the simplification of algebraic expressions by applying the order of operations:

[http://cnx.org/content/m35038/latest/?collection=col10615/latest](http://cnx.org/content/m35038/latest/?collection=col10615/latest)
Section 7.4 – Solving Linear Equations

In this section, you will learn how to solve linear equations. An equation is a statement saying that two expressions are equal. Solving an equation means determining the value of the variable that makes this equality statement true. The following are examples of the types of equations you will learn to solve:

\[
\begin{align*}
x &= 13 - 9 \\
g - 43 &= 7 \\
2m &= 30 \\
5k + 3 &= 23 \\
-56 + z &= -57 \\
3a + 8 &= -1 \\
2(x + 6) - 5x &= 42 \\
7.5h + 9 &= 6.3h - 12 \\
2.7(3y - 10) + 8 &= -5(y + 2) + 20
\end{align*}
\]

All of the equations on the left are \textit{linear} because the variable has an exponent of 1.

For example, the equations below are \textit{not linear}:

\[
\begin{align*}
x^2 + 12 &= 48 \\
y^2 + 5y &= -6 \\
k^3 &= 64
\end{align*}
\]

First, let’s look at some results that will be useful for solving linear equations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Notation</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additive Inverse</td>
<td>(-8 + 8 = 0) \hspace{1cm} (-45.27 + 45.27 = 0) \hspace{1cm} (7.6 - 7.6 = 0) \hspace{1cm} (19 + (-19) = 0)</td>
<td>Opposite numbers add up to zero. They “cancel” each other when added.</td>
</tr>
<tr>
<td>Multiplicative Inverse</td>
<td>(4 \times \frac{1}{4} = 1) \hspace{1cm} (\frac{1}{15} \times 15 = 1) \hspace{1cm} (-9 \times -\frac{1}{9} = 1) \hspace{1cm} (-\frac{1}{2} \times -2 = 1)</td>
<td>When you multiply reciprocals, the answer is always 1.</td>
</tr>
</tbody>
</table>
The additive and multiplicative inverse are often used to isolate the variable on one side of the equation. This allows us to determine the value of the variable. The following examples illustrate how we make use of the additive inverse and the multiplicative inverse to isolate the variable and, thus, solve the given equation.

\[
\begin{array}{c|c|c}
\frac{6}{13} \times \frac{13}{6} &= 1 \\
\frac{-25}{36} \times \frac{-36}{25} &= 1
\end{array}
\]

Example 7.4.1

Solve the following equation by finding the value of \( k \) that makes the equation true. Check your answer by substituting it into the original equation.

\[
k + 9 = 48
\]

To solve this equation, we must isolate the variable \( k \). Therefore, we need to eliminate the 9 that appears on the left side. To do so, we subtract 9 from each side of the equation to preserve the equality. This gives

\[
k + 9 - 9 = 48 - 9 \\
k + 0 = 39 \\
k = 39
\]

We can check the answer by replacing the variable \( k \) in the original equation with the value that was obtained. If this substitution leads to a true statement, then the answer is correct. When we substitute \( k = 39 \) in the equation we get

\[
39 + 9 = 48 \\
48 = 48
\]

which is true.
Example 7.4.2

Solve the following equation by finding the value of \( m \) that makes the equation true. Check your answer by substituting it into the original equation.

\[ m - 73 = 28 \]

To solve this equation, we must isolate the variable \( m \) on the left side. To do so, we need to eliminate the \(-73\) that appears on the left side. This can be accomplished by adding 73 to each side of the equation to preserve the equality. This gives

\[ m - 73 + 73 = 28 + 73 \]

\[ m + 0 = 101 \]

\[ m = 101 \]

We can check the answer by replacing the \( m \) in the original equation with the value that was obtained. If this substitution leads to a true statement, then the answer is correct. When we substitute \( m = 101 \) in the equation we get

\[ 101 - 73 = 28 \]

\[ 28 = 28 \]

which is true.
Solve the following equation by finding the value of \( y \) that makes the equation true. Check your answer.

\[ y + 67.21 = 95.22 \]

To solve this equation, we must isolate the variable \( y \) on the left side of the equation. To do so, we eliminate the 67.21 by subtracting 67.21 from each side of the equation to preserve the equality. This gives

\[ y + 67.21 - 67.21 = 95.22 - 67.21 \]

\[ y + 0 = 28.01 \]

\[ y = 28.01 \]

We can check the answer by replacing the \( y \) in the original equation with the value that was obtained. If this substitution leads to a true statement, then the answer is correct. When we substitute \( y = 28.01 \) in the original equation we get

\[ 28.01 + 67.21 = 95.22 \]

\[ 95.22 = 95.22 \]

which is true.

---

**Example 7.4.4**

Solve the following equation by finding the value of \( z \) that makes the equation true. Check your answer.

\[ 2z + 40 = 72 \]
To solve this equation, we must isolate the variable \( z \) on the left side. This is accomplished by eliminating the 40 and the 2 that appear on the left side. To do so, we first subtract 40 from each side of the equation to preserve the equality. This gives

\[
2z + 40 - 40 = 72 - 40 \\
2z + 0 = 32 \\
2z = 32
\]

Next, we can either multiply both sides by \( \frac{1}{2} \) or divide both sides by 2 to isolate \( z \). Doing the latter gives

\[
\frac{2}{2} z = \frac{32}{2} \\
1 \cdot z = 16 \\
z = 16
\]

We can check the answer by replacing the \( z \) in the original equation with the value that was obtained. If this substitution leads to a true statement, then the answer is correct. When we substitute \( z = 16 \) in the original equation we get

\[
2(16) + 40 = 72 \\
32 + 40 = 72 \\
72 = 72
\]

which is true.

---

**Example 7.4.5**

Solve the following equation by finding the value of \( a \) that makes the equation true. Check your answer.

\[
3.5a - 9.5 = -14.4
\]
To solve this equation, we must isolate the variable $a$ on the left side. This is accomplished by eliminating the $-9.5$ and the $3.5$ that appear on the left side. To do so, we first add $9.5$ to each side of the equation to preserve the equality. This gives

$$3.5a - 9.5 + 9.5 = -14.4 + 9.5$$

$$3.5a + 0 = -4.9$$

$$3.5a = -4.9$$

Next, we can either multiply both sides by $\frac{1}{3.5}$ or divide both sides by $3.5$ to isolate $a$. Doing the latter gives

$$\frac{3.5}{3.5} a = \frac{-4.9}{3.5}$$

$$1 \cdot a = \frac{-4.9}{3.5}$$

$$a = -1.4$$

We can check the answer by replacing the $a$ in the original equation with the value that was obtained. If this substitution leads to a true statement, then the answer is correct. When we substitute $a = -1.4$ in the original equation we get

$$3.5(-1.4) - 9.5 = -14.4$$
$$-4.9 - 9.5 = -14.4$$
$$-14.4 = -14.4$$

which is true.

---

Example 7.4.6

Solve the following equation by finding the value of $x$ that makes the equation true. Check your answer.

$$-2.5x + 43.5 = 10$$
To solve this equation, we must isolate the variable \( x \) on the left side. This is accomplished by eliminating the \( 43.5 \) and the \( -2.5 \) that appear on the left side. To do so, we first subtract \( 43.5 \) from each side of the equation to preserve the equality. This gives

\[
-2.5x + 43.5 - 43.5 = 10 - 43.5 \\
-2.5x + 0 = -33.5 \\
-2.5x = -33.5
\]

Next, we can either multiply both sides by \( \frac{1}{-2.5} \) or divide both sides by \( -2.5 \) to isolate \( x \). Doing the latter gives

\[
\frac{-2.5}{-2.5} x = \frac{-33.5}{-2.5} \\
1 \cdot x = \frac{-33.5}{-2.5} \\
x = 13.4
\]

We can check the answer by replacing the variable \( x \) in the original equation with the value that was obtained above. If this substitution leads to a true statement, then the answer is correct. When we substitute \( x = 13.4 \) in the original equation we get

\[
-2.5(13.4) + 43.5 = 10 \\
-33.5 + 43.5 = 10
\]

which is true.

In the next few examples, we will skip some of the detailed explanation but we will perform similar steps. Basically, to solve a linear equation we must isolate the variable on one side by taking advantage of the property of the additive inverse and/or the property of the multiplicative inverse, as needed.
Example 7.4.7

Solve the following equation:

\[ \frac{4}{5}x = 80 \]

Answer:

\[ \frac{4}{5}x = 80 \]

To isolate the variable \( x \), we multiply both sides by \( \frac{5}{4} \) which is the multiplicative inverse (reciprocal) of \( \frac{4}{5} \)

\[ \frac{5}{4} \cdot \frac{4}{5}x = \frac{5}{4} \cdot 80 \]

\[ 1 \cdot x = \frac{5}{4} \cdot \frac{80}{1} \]

We cross-cancel before multiplying the fractions

\[ x = \frac{5 \cdot 20}{4 \div 4} \]

\[ x = \frac{5}{1} \cdot \frac{20}{1} = \frac{100}{1} \]

\[ x = 100 \]
Example 7.4.8

Solve the following equation:
\[- \frac{4}{9} y = 53\]

Answer:

\[- \frac{4}{9} y = 53\]

To isolate the variable \( y \), we multiply both sides by \(- \frac{9}{4}\) which is the multiplicative inverse of \(- \frac{4}{9}\)

\[- \frac{9}{4} \cdot \frac{4}{9} y = - \frac{9}{4} \cdot 53\]

\[1 \cdot y = - \frac{9}{4} \cdot \frac{53}{1}\]

\[y = - \frac{477}{4}\]

Dividing \(-477\) by 4 and expressing the answer as a decimal gives

\[y = -119.25\]

We may also express the answer as a mixed number

\[y = -119 \frac{1}{4}\]
Example 7.4.9

Solve the following equation:
\[-\frac{2}{3}m - 12 = 16\]

Answer:
\[-\frac{2}{3}m - 12 = 16\]

To isolate the variable \(m\), we first add 12 to each side

\[-\frac{2}{3}m - 12 + 12 = 16 + 12\]

\[-\frac{2}{3}m + 0 = 28\]

\[-\frac{2}{3}m = 28\]

We then multiply both sides by \(-\frac{3}{2}\) which is the multiplicative inverse (reciprocal) of \(-\frac{2}{3}\)

\[-\frac{3}{2} \cdot -\frac{2}{3}m = -\frac{3}{2} \cdot 28\]

\[1 \cdot m = -\frac{3}{2} \cdot \frac{28}{1}\]

\[m = -\frac{3}{2} \cdot \frac{28}{1}\]

We cross cancel before multiplying the fractions

\[m = -\frac{3 \cdot 28}{2 \div 2} \cdot \frac{1}{1}\]

\[m = -\frac{3 \cdot 14}{1 \div 1} = -42\]
There are instances when we are asked to solve a linear equation that has a variable on both sides of the equation. To solve this type of equation, we must first eliminate the variable from one side of the equation. To do this, we must add to both sides the additive inverse of the term we want to eliminate. For example, if we wanted to eliminate a term such as $3x$ from one side of the equation, we would add $-3x$ to both sides of the equation to preserve the equality. This is equivalent to subtracting $3x$ from each side of the equation. On the other hand, to eliminate a term such as $-7.4z$ from one side of the equation, we would need to add $7.4z$ to both sides of the equation to preserve the equality. We end this section by presenting some examples that illustrate this process.

**Example 7.4.10**

Solve the following equation:

$$9c + 45 = 4c + 120$$

**Answer:**

$$9c + 45 = 4c + 120$$

*To eliminate the term $4c$, we begin by subtracting $4c$ from each side of the equation*

$$9c - 4c + 45 = 4c - 4c + 120$$

$$5c + 45 = 0 + 120$$

$$5c + 45 = 120$$

*We then subtract 45 from each side of the equation*

$$5c + 45 - 45 = 120 - 45$$

$$5c + 0 = 75$$

$$5c = 75$$

*Finally, we divide both sides by 5*

$$\frac{5c}{5} = \frac{75}{5} \rightarrow c = 15$$
Example 7.4.11

Solve the following equation:

\[-4w + 32 = -6w - 68\]

Answer:

\[-4w + 32 = -6w - 68\]

To eliminate the term \(-6w\), we begin by adding \(6w\) to each side of the equation:

\[-4w + 6w + 32 = -6w + 6w - 68\]

\[2w + 32 = 0 - 68\]

\[2w + 32 = -68\]

We then subtract 32 from each side of the equation:

\[2w + 32 - 32 = -68 - 32\]

\[2w + 0 = -100\]

\[2w = -100\]

Finally, we divide both sides by 2:

\[\frac{2}{2} \cdot w = \frac{-100}{2}\]

\[1 \cdot w = -50\]

\[w = -50\]

Note: You should always check your answer to any equation to verify that the answer leads to a true statement.
In the next two examples, we make use of the distributive property to solve the given equation.

**Example 7.4.12**

Solve the following equation:

\[ 3(x + 12) - 8 = 10 \]

Answer:

\[
\begin{align*}
3(x + 12) - 8 &= 10 \\
We apply the distributive property and combine like terms \\
3x + 36 - 8 &= 10 \\
3x + 28 &= 10 \\
We then subtract 28 from each side of the equation \\
3x + 28 - 28 &= 10 - 28 \\
3x &= -18 \\
Finally, we divide both sides by 3 \\
\frac{3}{3}x &= \frac{-18}{3} \\
1 \cdot x &= -6 \\
x &= -6
\end{align*}
\]
**Example 7.4.13**

Solve the following equation:

\[-5(p - 42) + 9p = 2(p + 3) - 20\]

<table>
<thead>
<tr>
<th>Answer:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-5(p - 42) + 9p = 2(p + 3) - 20]</td>
</tr>
<tr>
<td><em>We apply the distributive property and combine like terms</em></td>
</tr>
<tr>
<td>[-5p + 210 + 9p = 2p + 6 - 20]</td>
</tr>
<tr>
<td>[4p + 210 = 2p - 14]</td>
</tr>
<tr>
<td><em>We then subtract 2p from each side of the equation</em></td>
</tr>
<tr>
<td>[4p - 2p + 210 = 2p - 2p - 14]</td>
</tr>
<tr>
<td>[2p + 210 = 0 - 14]</td>
</tr>
<tr>
<td>[2p + 210 = -14]</td>
</tr>
<tr>
<td><em>We subtract 210 from each side of the equation</em></td>
</tr>
<tr>
<td>[2p + 210 - 210 = -14 - 210]</td>
</tr>
<tr>
<td>[2p + 0 = -224]</td>
</tr>
<tr>
<td>[2p = -224]</td>
</tr>
<tr>
<td><em>Finally, we divide each side by 2</em></td>
</tr>
<tr>
<td>[\frac{2}{2} \cdot p = \frac{-224}{2}]</td>
</tr>
<tr>
<td>[1 \cdot p = -112]</td>
</tr>
<tr>
<td>[p = -112]</td>
</tr>
</tbody>
</table>
Example 7.4.14

Solve the following equation:

\[ \frac{x - 2}{6} = \frac{3}{4} \]

Notice that this is a proportion that has a variable. It can be solved by cross multiplication and then division.

Answer:

\[ \frac{x - 2}{6} = \frac{3}{4} \]

We cross-multiply

\[ 4 \cdot (x - 2) = 6 \cdot 3 \]

We apply the distributive property

\[ 4x - 8 = 18 \]

We then add 8 to each side of the equation

\[ 4x - 8 + 8 = 18 + 8 \]

\[ 4x + 0 = 26 \]

\[ 4x = 26 \]

We divide each side of the equation by 4

\[ \frac{4}{4} \cdot x = \frac{26}{4} \]

\[ 1 \cdot x = 6.5 \]

\[ x = 6.5 \]
These videos show you how to solve certain linear equations:

http://www.youtube.com/watch?v=3Iaitp5xc10
http://www.youtube.com/watch?v=ObtE0lnusiM
http://www.youtube.com/watch?v=PI0HJySPWBU
http://www.youtube.com/watch?v=PTZpTg7ZVC4
http://www.youtube.com/watch?v=pNBrhfl-A-iA
http://www.youtube.com/watch?v=ZxVACV_iDC
http://www.youtube.com/watch?v=XHPkaHVJDRc
http://www.youtube.com/watch?v=IHS8V4Y8Dyc
http://www.youtube.com/watch?v=jqgBKSvTLQc
http://www.youtube.com/watch?v=JIDRfQlTm48

This video shows how to solve a proportion that involves a variable:

http://www.youtube.com/watch?v=hF1c4MRVWmk

**Classwork 7.4**

Solve each equation. Check your work by substituting your answer into the equation.

1) \( h + 23.75 = 14 \)  
   \[ h = -9.75 \]

2) \( 8t + 96 = -64 \)  
   \[ t = -20 \]

3) \( 2.5a + 60.4 = -40.6 \)  
   \[ a = -40.4 \]

4) \( \frac{12}{25} d - 24 = -12 \)  
   \[ d = 25 \]

5) \( -\frac{1}{5}z + 4.2 = 30.6 \)  
   \[ z = -132 \]

6) \( 77k - 59 - 72k - 6 = -40 \)  
   \[ k = 5 \]

7) \( 16k - 21 - 19k + 6 = -k + 56 \)  
   \[ k = -35.5 \]

8) \( 4(7z - 11) = 20z \)  
   \[ z = 5.5 \]

9) \( -6(4x - 7) - 32 = 4(-5x - 18) \)  
   \[ x = 20.5 \]

10) \( -6r + 2.8 + 3.5r - 60 = -2(r - 45.2) \)  
    \[ r = -295.2 \]
**Homework 7.4**

Solve each equation. Check your work by substituting your answer into the equation.

1) \( c - 10 = 28.7 \)
   \( c = 38.7 \)

2) \( -3f + 42 = 57 \)
   \( f = -5 \)

3) \( 6.2m + 30.2 = 8.5 \)
   \( m = -3.5 \)

4) \( \frac{4}{5}t - 48 = -60 \)
   \( t = -15 \)

5) \( \frac{-8}{15}z - 24 = 16 \)
   \( z = -75 \)

6) \( 7k - 12 + 3k - 73 = 68 \)
   \( k = 153 \)

7) \( -8m + 20.2 + 6m = 2m + 8 \)
   \( m = 3.05 \)

8) \( 6(3x - 8) = 10x \)
   \( x = 6 \)

9) \( -2(5x - 16) + 7 = -4(2x - 9) \)
   \( x = 1.5 \)

10) \( -r + 27 + 7r - 42 = 3(r - 6) \)
    \( r = -1 \)

The following websites have additional exercises with solutions on techniques to solve certain kinds of linear equations:

[http://cnx.org/content/m35044/latest/?collection=col10615/latest](http://cnx.org/content/m35044/latest/?collection=col10615/latest)
[http://cnx.org/content/m35045/latest/?collection=col10615/latest](http://cnx.org/content/m35045/latest/?collection=col10615/latest)
Section 7.5 – The Laws of Exponents

In this section you will learn to simplify expressions involving exponents. Exponential notation is a useful way to write complex expressions in a more compact form. In this section we will present the relationship between exponents and square roots, cubic roots, etc. We begin by recalling that

\[ k^1 = k \quad \quad 2^1 = 2 \]

\[ k^2 = k \cdot k \quad \quad 2^2 = 2 \cdot 2 = 4 \]

\[ k^3 = k \cdot k \cdot k \quad \quad 2^3 = 2 \cdot 2 \cdot 2 = 8 \]

\[ k^4 = k \cdot k \cdot k \cdot k \quad \quad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16 \]

\[ k^5 = k \cdot k \cdot k \cdot k \cdot k \quad \quad 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 \]

\[ k^6 = k \cdot k \cdot k \cdot k \cdot k \cdot k \quad \quad 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64 \]

\[ \vdots \quad \quad \vdots \]

Moreover, by definition for any \( k \neq 0 \) we have

\[ k^0 = 1 \quad 2^0 = 1 \quad 14^0 = 1 \quad (-5)^0 = 1 \quad \left(\frac{3}{4}\right)^0 = 1 \quad (9.7)^0 = 1 \]

In the following examples, we will make use of the fact that for any \( k \neq 0 \), we have \( \frac{k}{k} = 1 \). For example, \( \frac{15}{15} = 1 \), \( \frac{2}{2} = 1 \), \( \frac{7.4}{7.4} = 1 \) and so on. This means that whenever the numerator and denominator are the same, the fraction reduces to 1. We will use this fact to simplify the expressions in the following examples.
Example 7.5.1

Simplify \[ \frac{3 \cdot 5 \cdot 5 \cdot 5 \cdot 3 \cdot 7 \cdot 2}{5 \cdot 3 \cdot 5 \cdot 7 \cdot 2 \cdot 2} \]
and leave your answer in exponential form.

Answer:

*We first rearrange the numbers in ascending order for convenience*

\[
\frac{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7}
\]

Dividing gives

\[
\frac{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7}
\]

\[
1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{5}{2} = \frac{5}{2}
\]

Therefore,

\[
\frac{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7} = \frac{2^{1}3^{2}5^{3}7^{1}}{2^{2}3^{2}5^{2}7^{1}} = \frac{5}{2}
\]

Notice that since there was one more 2 in the denominator than in the numerator, the answer has one 2 in the denominator. Similarly, since there was one more 5 in the numerator than in the denominator, the answer has one 5 in the numerator. Moreover, since there was an equal amount of 3’s in the numerator and the denominator, all of the 3’s became 1’s upon division. This left no 3’s appearing in the simplified answer. Finally, since there was an equal amount of 7’s in the numerator and the denominator, all of the 7’s became 1’s upon division. This left no 7’s appearing in the answer.
Example 7.5.2

Simplify \( \frac{3 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 2 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 2 \cdot 2 \cdot 7 \cdot 5 \cdot 5 \cdot 5} \) and leave your answer in exponential form.

Answer:

*We first rearrange the numbers in ascending order for convenience*

\[
\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7}
\]

*Dividing gives*

\[
\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7}
\]

\[
1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{2 \cdot 2 \cdot 3 \cdot 3}{5 \cdot 5 \cdot 7} = \frac{2^2 \cdot 3^2}{5^2 \cdot 7}
\]

*Therefore,*

\[
\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7} = \frac{2^5 \cdot 3^4 \cdot 5^2}{2^3 \cdot 3^2 \cdot 5^4 \cdot 7^1} = \frac{2^2 \cdot 3^2}{5^2 \cdot 7}
\]

Notice that since there were two more 2’s in the numerator than in the denominator, the answer has two 2’s in the numerator. Since there were two more 3’s in the numerator than in the denominator, the answer has two 3’s in the numerator. Since there were two more 5’s in the denominator than in the numerator, the answer has two 5’s in the denominator. The lone 7 appearing in the denominator remained in the denominator.
Example 7.5.3

Simplify each of the following expressions and leave your answer in exponential form. Try to determine a rule or law of exponents by comparing your answer to the original expression.

A) \( \frac{19^7}{19^4} \)  
B) \( \frac{19^7}{19^6} \)  
C) \( \frac{19^7}{19^7} \)  
D) \( \frac{19^7}{19^8} \)  
E) \( \frac{19^{16}}{19^{21}} \)

Answer:

There is no need to expand each numerator and denominator. Instead, based on the number of 19’s appearing in the numerator and denominator we see that after dividing the 19’s in the numerator and denominator we get the following results:

A) \( \frac{19^7}{19^4} = 19^3 \)  
B) \( \frac{19^7}{19^6} = 19^1 = 19 \)  
C) \( \frac{19^7}{19^7} = 1 \)

D) \( \frac{19^7}{19^8} = \frac{1}{19} \)  
E) \( \frac{19^{16}}{19^{21}} = \frac{1}{19^5} \)

The exponents in the answers can be obtained by subtracting the original exponents. If the exponent in the numerator is larger than the exponent in the denominator, the answer will be the same whole number (base) raised to an exponent equal to the difference of the larger exponent minus the smaller exponent. If the exponent in the denominator is larger than the exponent in the numerator, the answer will be a fraction with a denominator having the base raised to the difference of the larger exponent minus the smaller exponent. If the original exponents happen to be equal, the simplified answer will be 1.

The results of Example 7.5.3 can be used to deduce some of the **Laws of Exponents**, which allow us to simplify expressions involving multiplication and division of numbers that have the same base. The following table lists the main results.
### The Laws of Exponents

#### Negative Exponent Law

Another way to express any fraction of the form $\frac{1}{y^a}$ is to use negative exponents.

- $\frac{1}{y^a} = y^{-a}$
- $y^{-a} = \frac{1}{y^a}$
- $\frac{1}{y^{-a}} = y^a$
- $\frac{x^{-b}}{y^{-a}} = \frac{y^a}{x^b}$

#### Multiplication Law

When we multiply expressions having the same base, we add their exponents.

- $y^a \cdot y^b = y^{a+b}$
- $4^7 \cdot 4^{11} = 4^{7+11} = 4^{18}$
- $15 \cdot 15^3 = 15^{1+3} = 15^4$
- $9^{10} \cdot 9^5 = 9^{10+5} = 9^{15}$
- $7^{-2} \cdot 7^8 = 7^{-2+8} = 7^6$
- $12^{-3} \cdot 12^3 = 12^{-3+3} = 12^0 = 1$
- $12^{-4} \cdot 12^{-6} = 12^{-4+(-6)} = 12^{-10} = \frac{1}{12^{10}}$

#### Division Law

When we divide expressions having the same base, we subtract their exponents (top minus bottom).

- $\frac{y^a}{y^b} = y^{a-b}$
- $\frac{5^{12}}{5^9} = 5^{12-9} = 5^3$
- $\frac{2^{17}}{2^{16}} = 2^{17-16} = 2^1 = 2$
- $\frac{3^8}{3^8} = 3^{8-8} = 3^0 = 1$
### Power Law

When an expression that already has an exponent is raised to another exponent, we multiply the exponents.

\[(y^a)^b = y^{a \cdot b}\]

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3^{10}}{3^{14}})</td>
<td>(3^{10-14} = 3^{-4} = \frac{1}{3^4})</td>
</tr>
<tr>
<td>(\frac{20^{-3}}{20^{-5}})</td>
<td>(20^{-3-(-5)} = 20^{-3+5} = 20^2)</td>
</tr>
<tr>
<td>((7^3)^2)</td>
<td>(7^{3 \cdot 2} = 7^6)</td>
</tr>
<tr>
<td>((12^4)^5)</td>
<td>(12^{4 \cdot 5} = 12^{20})</td>
</tr>
<tr>
<td>((8^9)^0)</td>
<td>(8^{9 \cdot 0} = 8^0 = 1)</td>
</tr>
<tr>
<td>((6^{-2})^7)</td>
<td>(6^{-2 \cdot 7} = 6^{-14} = \frac{1}{6^{14}})</td>
</tr>
<tr>
<td>((2^5)^{-4})</td>
<td>(25^{-4-4} = 2^{-20} = \frac{1}{2^{20}})</td>
</tr>
<tr>
<td>((3^{-6})^{-2})</td>
<td>(3^{-6 \cdot -2} = 3^{12})</td>
</tr>
</tbody>
</table>

### Fractional Exponent Law

Fractional exponents have the following interpretation:

\[y^{\frac{1}{n}} = \sqrt[n]{y} = \frac{\sqrt[n]{y^1}}{\sqrt[n]{1}} = \sqrt[n]{y}\]

\(\sqrt[n]{}\) is the “nth root” symbol.

Moreover,

\[y^{\frac{a}{b}} = (\sqrt[b]{y})^a = \sqrt[b]{y^a}\]

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{9^2})</td>
<td>(\frac{1}{2\sqrt{91}} = \sqrt{9} = 3)</td>
</tr>
<tr>
<td>(\frac{1}{8^3})</td>
<td>(\frac{1}{3\sqrt{81}} = \frac{3}{\sqrt{81}} = \frac{3}{9} = \frac{1}{3})</td>
</tr>
<tr>
<td>(\frac{2}{8^3})</td>
<td>(\frac{2}{3\sqrt{82}} = \frac{3}{\sqrt{82}} = \frac{3}{8} = \frac{1}{8})</td>
</tr>
<tr>
<td>(81^\frac{1}{4})</td>
<td>(\frac{4}{\sqrt{81}} = \frac{4}{9} = \frac{1}{3})</td>
</tr>
<tr>
<td>(125^\frac{2}{3})</td>
<td>((\sqrt[3]{125})^2 = 5^2 = 25)</td>
</tr>
<tr>
<td>(32^\frac{-3}{5})</td>
<td>(\frac{1}{\sqrt[5]{32^3}} = \frac{1}{2^3} = \frac{1}{8})</td>
</tr>
</tbody>
</table>

We will now illustrate the Laws of Exponents through some examples. As we do so, please note that there may be more than one way to obtain the same answer. In simplifying an expression, there is sometimes an easier and faster way to obtain an answer, as well as a more involved process. It does not matter which
method is used to simplify an expression, as long as the laws of exponents are applied correctly in each step.

**Example 7.5.4**

Simplify \( \frac{5^4 \cdot 3^{-8} \cdot 2^{-9}}{3^4 \cdot 5^7 \cdot 2^{-7}} \). Express your answer using positive exponents only.

\[
\frac{5^4 \cdot 3^{-8} \cdot 2^{-9}}{3^4 \cdot 5^7 \cdot 2^{-7}} = \frac{2^{-9} \cdot 3^{-8} \cdot 5^4}{2^{-7} \cdot 3^4 \cdot 5^7}
\]

\[
= 2^{-9+7} \cdot 3^{-8-4} \cdot 5^{4-7}
\]

\[
= 2^{-2} \cdot 3^{-12} \cdot 5^{-3}
\]

\[
= \frac{1}{2^2} \cdot \frac{1}{3^{12}} \cdot \frac{1}{5^3} = \frac{1}{2^2 \cdot 3^{12} \cdot 5^3}
\]

When dividing expressions with the same base, we subtract the exponent in the numerator minus the exponent in the denominator.

**Example 7.5.5**

Simplify \( 13^6 \cdot 13^{-10} \cdot 13^7 \). Express your answer using positive exponents only.

**Answer:**

\[
13^6 \cdot 13^{-10} \cdot 13^7 = 13^{6+(-10)+7} = 13^{-4+7} = 13^3
\]

When multiplying expressions with the same base, we add their exponents.
Example 7.5.6

Simplify $20 \cdot 20^{-8} \cdot 20^3$. Express your answer using positive exponents only.

When multiplying expressions with the same base, we add their exponents.

Answer:

$20 \cdot 20^{-8} \cdot 20^3$

$20^{1+(-8)+3}$

$20^{-7+3}$

$20^{-4} = \frac{1}{20^4}$

Example 7.5.7

Simplify $\frac{2^8 \cdot 7^5}{2^2 \cdot 7^5}$. Express your answer using positive exponents only.

When dividing expressions with the same base, we subtract the exponent in the numerator minus the exponent in the denominator.

Answer:

$\frac{2^8 \cdot 7^5}{2^2 \cdot 7^5}$

$\frac{2^8}{2^2}$

$\frac{7^5}{7^5}$

$2^{8-2} \cdot 7^{5-5}$

$2^6 \cdot 7^0$

$2^6$
Example 7.5.8

Simplify \( \frac{12^{-6} \cdot 7^{-6}}{5^{-1} \cdot 13^{-2}} \). Express your answer using positive exponents only.

Answer:

\[
\frac{12^{-6} \cdot 7^{-6}}{5^{-1} \cdot 13^{-2}} = \frac{5^1 \cdot 13^2}{12^6 \cdot 7^6}
\]

Example 7.5.9

Simplify \( \frac{43^{-1} \cdot 19^3 \cdot 7^{-3}}{7^{-4} \cdot 19^5 \cdot 43^{-5}} \). Express your answer using positive exponents only.

Answer:

When dividing expressions with the same base, we subtract the exponent in the numerator minus the exponent in the denominator.

\[
\frac{43^{-1} \cdot 19^3 \cdot 7^{-3}}{7^{-4} \cdot 19^5 \cdot 43^{-5}} = \frac{7^{-3} \cdot 19^3 \cdot 43^{-1}}{7^{-4} \cdot 19^5 \cdot 43^{-5}}
\]

\[
\frac{7^{-3} \cdot 19^3 \cdot 43^{-1}}{7^{-4} \cdot 19^5 \cdot 43^{-5}} = \frac{7^{3-4} \cdot 19^{3-5} \cdot 43^{-1+5}}{7^{4} \cdot 19^{5} \cdot 43^{1+5}}
\]

\[
\frac{7^{3-4} \cdot 19^{3-5} \cdot 43^{-1+5}}{7^{4} \cdot 19^{5} \cdot 43^{1+5}} = \frac{7^{-1} \cdot 19^{2} \cdot 43^{4}}{7^{1} \cdot 19^{2} \cdot 43^{4}}
\]

\[
\frac{7^{-1} \cdot 19^{2} \cdot 43^{4}}{7^{1} \cdot 19^{2} \cdot 43^{4}} = \frac{7^{1} \cdot 43^{4}}{19^{2}}
\]
Example 7.5.10

Simplify \( \frac{2^7 \cdot 3 \cdot 11^{-4} \cdot 13^{-1}}{11^{-4} \cdot 13^{-2} \cdot 2^7 \cdot 3^5} \). Express your answer using positive exponents only.

Answer:

\[
\frac{2^7 \cdot 3 \cdot 11^{-4} \cdot 13^{-1}}{11^{-4} \cdot 13^{-2} \cdot 2^7 \cdot 3^5} = \frac{2^7 \cdot 3 \cdot 11^{-4} \cdot 13^{-1}}{2^7 \cdot 3^5 \cdot 11^{-4} \cdot 13^{-2}}
\]

\[
= \frac{2^7 \cdot 3}{2^7 \cdot 3^5} \cdot \frac{11^{-4} \cdot 13^{-1}}{11^{-4} \cdot 13^{-2}}
\]

\[
= 2^{7-7} \cdot 3^{1-5} \cdot 11^{-4-(-4)} \cdot 13^{-1-(-2)}
\]

\[
= 2^0 \cdot 3^{-4} \cdot 11^0 \cdot 13^1
\]

\[
= 1 \cdot \frac{1}{3^4} \cdot 1 \cdot 13^1
\]

\[
= \frac{13}{3^4}
\]
Example 7.5.11
Simplify each of the following expressions.

A) \(\sqrt[3]{216}\)  
B) \(\sqrt[4]{625}\)  
C) \(\sqrt{1}\)  
D) \(\sqrt[4]{16}\)  
E) \(\sqrt[5]{1024}\)

Answer:

A) \(\sqrt[3]{216} = 6\)  
   because  
   \(6 \cdot 6 \cdot 6 = 216\)
B) \(\sqrt[4]{625} = 5\)  
   because  
   \(5 \cdot 5 \cdot 5 \cdot 5 = 625\)
C) \(\sqrt{1} = 1\)  
   because  
   \(1 \cdot 1 \cdot 1 = 1\)
D) \(\sqrt[4]{16} = 2\)  
   because  
   \(2 \cdot 2 \cdot 2 \cdot 2 = 16\)
E) \(\sqrt[5]{1024} = 4\)  
   because  
   \(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024\)

Example 7.5.12
Simplify each of the following expressions.

A) \(8^{\frac{7}{3}}\)  
B) \(49^{\frac{3}{2}}\)  
C) \(1000^{\frac{2}{3}}\)  
D) \(36^{\frac{5}{2}}\)  
E) \(169^{\frac{-5}{2}}\)

Answer:

A) \(8^{\frac{7}{3}} = (\sqrt[3]{8})^7 = 2^7 = 128\)
B) \(49^{\frac{3}{2}} = (\sqrt{49})^3 = 7^3 = 343\)
C) \(1000^{\frac{2}{3}} = \frac{1}{1000^{\frac{1}{3}}} = \frac{1}{(\sqrt[3]{1000})^2} = \frac{1}{10^2} = \frac{1}{100}\)
D) \(36^{\frac{5}{2}} = (\sqrt[2]{36})^5 = 6^5 = 7776\)
E) \(169^{\frac{-5}{2}} = \frac{1}{169^{\frac{5}{2}}} = \frac{1}{(\sqrt[2]{169})^5} = \frac{1}{13^5} = \frac{1}{371,293}\)
Note: The following are common errors that you must avoid when trying to apply the Laws of Exponents. These simplifications are incorrect:

\[
\begin{align*}
5^4 \cdot 5^3 &= 5^{12} & 5^4 + 5^3 &= 5^7 & 5^4 - 5^3 &= 5^1 \\
\frac{5^{20}}{5^4} &= 5^5 & (7 + 3)^4 &= 7^4 + 3^4 & (7 - 3)^4 &= 7^4 - 3^4 \\
\sqrt{9 + 16} &= 3 + 4 = 7 & \sqrt{100 - 64} &= 10 - 8 = 2 & 5^4 \cdot 3^7 &= 15^{11}
\end{align*}
\]

We end this section with an example that shows that we may be able to simplify certain expressions even when the \(n^{th}\) root of a number is not a whole number.

**Example 7.5.13**

Simplify each of the following expressions.

A) \(\sqrt{50}\)  
B) \(\sqrt{18}\)  
C) \(\sqrt{500}\)  
D) \(\sqrt[3]{16}\)  
E) \(\sqrt[3]{128}\)

Answer:

A) \(\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5 \cdot \sqrt{2} = 5\sqrt{2}\)

B) \(\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3 \cdot \sqrt{2} = 3\sqrt{2}\)

C) \(\sqrt{500} = \sqrt{100 \cdot 5} = \sqrt{100} \cdot \sqrt{5} = 10 \cdot \sqrt{5} = 10\sqrt{5}\)

D) \(\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2 \cdot \sqrt[3]{2} = 2\sqrt[3]{2}\)

E) \(\sqrt[3]{128} = \sqrt[3]{64 \cdot 2} = \sqrt[3]{64} \cdot \sqrt[3]{2} = 4 \cdot \sqrt[3]{2} = 4\sqrt[3]{2}\)
Since variables represent numbers, the Laws of Exponents can also be used to simplify expressions involving variables. The following examples illustrate how we can apply the Laws of Exponents to simplify algebraic expressions.

Example 7.5.14

Assuming that all variables have positive values, simplify \[ \frac{20 \cdot x^3 \cdot z \cdot y^{-4} \cdot w^{-4}}{4 \cdot y^{-4} \cdot w^{-3} \cdot x^{-2} \cdot z^8} \].

Express your answer using positive exponents only.

Answer:

\[
\begin{align*}
&\frac{20 \cdot x^3 \cdot z \cdot y^{-4} \cdot w^{-4}}{4 \cdot y^{-4} \cdot w^{-3} \cdot x^{-2} \cdot z^8} \\
&= \frac{20 \cdot w^{-4} \cdot x^3 \cdot y^{-4} \cdot z}{4 \cdot w^{-3} \cdot x^{-2} \cdot y^{-4} \cdot z^8} \\
&= 5 \cdot w^{-4+3} \cdot x^{3+2} \cdot y^{-4+4} \cdot z^{1-8} \\
&= 5 \cdot w^{-1} \cdot x^5 \cdot y^0 \cdot z^{-7} \\
&= \frac{5x^5}{w^1z^7} \\
&= \frac{5x^5}{wz^7}
\end{align*}
\]
Example 7.5.15

Assuming that all variables have positive values, simplify

\[(2m^3k^{-5})^4 \cdot (3m^{-6}k^8)^2.\]

Express your answer using positive exponents only.

Answer:

\[\frac{144}{k^4}\]

When raising an exponent to another exponent, we multiply the exponents. This is an application of the Power Law.

When multiplying expressions with the same base, we add their exponents.

The following videos show you how to apply the Laws of Exponents:

Multiplication Law:  http://www.youtube.com/watch?v=FhlqiVkmwzo

Division Law:  http://www.youtube.com/watch?v=rNC4LJpQt70

Power Law:  http://www.youtube.com/watch?v=8SjN22qdldo

Negative Exponent Law:  http://www.youtube.com/watch?v=OxGi39EZt8s

Applying several laws:  http://www.youtube.com/watch?v=itvK0ggj-LE
Simplify each expression. Express your answer using positive exponents.
Assume that all the variables represent positive numbers.

1) \(6^{-7} \cdot 11^4 \cdot 6^2 \cdot 11^{-1} = \frac{11^3}{6^5}\)

2) \(7^{-8} \cdot 2^{-5} \cdot 2^6 \cdot 7^{-4} = \frac{2^1}{7^{12}}\)

3) \(5^9 \cdot 13^{-1} \cdot 7^{-4} \cdot 5^{-5} = \frac{5^4}{74 \cdot 13^1}\)

4) \(\frac{5^6 \cdot 23^{-2} \cdot 3^{-4}}{23^{-2} \cdot 3^{-1} \cdot 5^{-2}} = \frac{5^8}{3^5}\)

5) \(\frac{1^{13} \cdot 2^{10} \cdot 3^{-7}}{3^{-6} \cdot 2^4} = \frac{2^6}{3^1}\)

6) \(3\sqrt{26} = 4\)

7) \(225^3 = 3,375\)

8) \(9^{-\frac{5}{2}} = \frac{1}{243}\)

9) \(\sqrt{4000} = 20\sqrt{10}\)

10) \(\sqrt[4]{405} = 3\sqrt[5]{5}\)

11) \(\frac{60t^4p^{-2}r^{-7}}{84p^{6r^{-4}}t^6} = \frac{5}{7 \cdot t^2 \cdot p^8 \cdot r^3}\)

12) \((7w^{-6}x^7yz^{-2})^3 = \frac{343 \cdot x^{21} \cdot y^3}{w^{18} \cdot z^6}\)

13) \((2p^4k^{-4}y^4(3p^{-2}k^{-2}y^{-1})^3 = \frac{432 \cdot p^6 \cdot y^1}{k^{22}}\)

14) \((34m^{-8} \cdot 5w^{-3})^0 = 1\)

### Homework 7.5

Simplify each expression. Express your answer using positive exponents.
Assume that all the variables represent positive numbers.

1) \(2^{-3} \cdot 13^2 \cdot 2^2 \cdot 13^{-6} = \frac{1}{21 \cdot 13^4}\)

2) \(5^{-9} \cdot 11^3 \cdot 11^{-1} \cdot 5^7 = \frac{11^2}{5^2}\)

3) \(17^{-3} \cdot 13^{-5} \cdot 15^6 \cdot 2^{-8} = \frac{15^6}{28 \cdot 13^5 \cdot 17^3}\)

4) \(\frac{25 \cdot 19^{-4} \cdot 5^6}{5^{-4} \cdot 7^{-2} \cdot 2^2} = \frac{2^3 \cdot 5^{10} \cdot 7^2}{19^4}\)

5) \(\frac{7^0 \cdot 13^{-3} \cdot 3^8}{3^{8} \cdot 13^{-7}} = 13^4\)

6) \(\sqrt[3]{24} = 4\)

7) \(400^3 = 8,000\)

8) \(36^{-\frac{3}{2}} = \frac{1}{216}\)

9) \(\sqrt{72} = 6\sqrt{2}\)

10) \(\sqrt[3]{3000} = 10\sqrt[3]{3}\)

11) \(\frac{27t^{-8}p^5r^{-1}}{63p^5rt^{-4}} = \frac{3}{7 \cdot t^4 \cdot r^2}\)

12) \((5w^{-2}x^7z^{-4})^{-2} = \frac{w^{4} \cdot z^8}{5^2 \cdot x^{14}}\)

13) \((6h^4g^{-3}n)^{-2} (5h^{-1}g^6n^{-3})^2 = \frac{5^2 \cdot g^{18}}{6^2 \cdot h^{10} \cdot n^6}\)

14) \((7a^3b^{-4}c^{-14})^1 = \frac{7 \cdot a^3}{b^4 \cdot c^{14}}\)
Chapter 7 Test

Simplify each expression completely.

1. \(34.82 - 46.75\)  
   \(-11.93\)

2. \(98 - 120 + 42\)  
   \(20\)

3. \(-9.75 - 6.63\)  
   \(-16.38\)

4. \(-(-56) + (-8)\)  
   \(48\)

5. \((-6.4)(3.5)\)  
   \(-22.4\)

6. \(-4 \cdot -9\)  
   \(36\)

7. \(-574 ÷ -2\)  
   \(287\)

8. \(9.21 ÷ -0.3\)  
   \(-30.7\)

Simplify each expression completely.

9. \(6(-3.5) + (-7)(4) - (-2)^3\)  
   \(-41\)

10. \(\frac{48}{-6} - 9(-3) + (-5)^2\)  
    \(44\)

11. \(\frac{-3+48}{-4-1} - (12)(-5) + (-4)^2\)  
    \(67\)

12. \(\frac{\sqrt{144-68}}{(-2)^3+30}\)  
    \(28\)

13. \(-4(6 - 8) + 12(-9 - 4)\)  
    \(-148\)

14. \(-5\sqrt{64} + 3\sqrt{25} - 6 \cdot 4\)  
    \(-49\)

Solve each equation.

15. \(-3p + 8 = 92\)  
    \(p = -28\)

16. \(3.5x + 30 - 1.5x - 42 = -100\)  
    \(x = -44\)

17. \(4(a - 9) - 3(-2a + 14) = 12.8\)  
    \(a = 9.08\)

18. \(\frac{5}{8}w - 20 = 45\)  
    \(w = 104\)

19. \(7x + 36 = -5x - 144\)  
    \(x = -15\)

20. \(6(m - 3) + 42 = -2(8 + m) + 56\)  
    \(m = 2\)
Simplify completely. Express your answer using only positive exponents.

21. \( \frac{5^{12} \cdot 7^{-9}}{5^{8} \cdot 7^{-7} \cdot 3^{-2}} \)

22. \( (2^4 \cdot 3^{-5})(2^{-4} \cdot 3^{-4}) \)

23. \( (5^4 \cdot 13^{-2} \cdot 2^5)^{-2} \)

24. \( (25)^{-\frac{3}{2}} \)

25. \( (4)^{-\frac{5}{2}} + (8)^{-\frac{4}{3}} \)

26. \( (400)^{\frac{1}{2}} + (27)^{\frac{2}{3}} \)

27. \( \sqrt{320} \)

28. \( \sqrt{8400} \)

29. \( \sqrt{45} \)

30. \( \sqrt{2925} \)

\( \frac{32}{3} \)

\( \frac{1}{125} \)

\( \frac{3}{32} \)

\( 8\sqrt{5} \)

\( 20\sqrt{21} \)

\( 3\sqrt{5} \)

\( 15\sqrt{13} \)
Cumulative Review - Chapters 1-7

Simplify completely. Use the order of operations.

1) \( \frac{\sqrt{169} - 3 \cdot 2}{\sqrt{64} - 1^{0}} + \frac{4^{2} + 2}{8} \)
2) \( 20 + 68 \div 4 - 3 \cdot 5 + (48 \div 12)^{2} \)

Reduce each fraction to lowest terms. Write improper fractions as mixed numbers.

3) \( \frac{20}{45} = \frac{4}{9} \)
4) \( \frac{480}{360} = \frac{1}{3} \)
5) \( \frac{88}{55} = \frac{3}{5} \)

6) There are 2500 students at Garfield High School of which 1200 students are male.
   a) Write the ratio of male students to female students as a fractional ratio and simplify the ratio.
      \( \frac{12}{13} \)
   b) Write the ratio of male students to the total number of students as a fractional ratio and simplify the ratio.
      \( \frac{12}{25} \)
   c) Write the ratio of female students to the total number of students as a fractional ratio and simplify the ratio.
      \( \frac{13}{25} \)

Simplify

7) \( 40.8 \div 0.04 \)
8) \( \$908.4 - \$131.27 \)
9) \( 18.3 \times 5.2 \)
10) \( 1432.008 + 45 - 6.3 \times 10^{2} \)
11) \( (16.2 - 10.4) \div 0.2 + (9.4)^{2} \)
Write as a decimal:

12) \( \frac{24}{18} \) \( = \) 1.3
13) \( \frac{25}{10} \) \( = \) 25.7
14) \( \frac{67}{1000} \) \( = \) 0.067
15) \( \frac{8}{11} \) \( = \) 0.7272727272727273

Write as a fraction or mixed number. Simplify your answer completely.

16) \( 0.64 \) \( = \) \( \frac{16}{25} \)
17) \( 9.025 \) \( = \) \( 9 \frac{1}{40} \)
18) \( 20.35 \) \( = \) \( 20 \frac{7}{20} \)
19) \( 0.8 \) \( = \) \( \frac{4}{5} \)

Find the value of the variable that makes the proportion true.

20) \( \frac{c}{12} = \frac{20}{4} \);
21) \( \frac{17}{20} = \frac{t}{45} \);

Simplify using the order of operations.

22) \( -430 + 115 \div 5 \cdot 2 + (-6)^3 \);
23) \( 8\sqrt{25} - 16\sqrt{81} + (-4)(-3)^2 \);

24) \( \frac{-75 \div 15 - 7 \cdot 10}{-23 + 9 \cdot 2} \);

Simplify. Express your answer using positive exponents.

25) \( (3^2 \cdot 5^{-6} \cdot 11^3)^4 \cdot (3^4 \cdot 5^{-8} \cdot 11^7)^{-2} \);

26) Miley received 8 votes for every 10 people who voted. If she received a total of 520 votes, how many people voted? Hint: write a proportion and find the missing value that makes the proportion true.
27) Mark earned $300 working 15 hours. At that rate, how much will he earn if he works 20 hours? *Hint: write a proportion and find the missing value that makes the proportion true.*  
Mark will earn $400 if he works 20 hours.

28) If the price of 5 notebooks is $37.50, what would be the price of 12 notebooks? Express your answer with the dollar $ symbol. *Hint: write a proportion and find the missing value that makes the proportion true.*  
The price of 12 notebooks will be $90.

29) What percent is 108 out of 120? 90%

30) 145% of 80 is what number? 116

31) 40% of what number is 26? 65

32) What percent of 120 is 300? 250%

33) Find the value of K if \( K = \frac{B}{C} + D \) and \( B = 144 \), \( C = 6 \), \( D = 40 \).  
\[ K = 64 \]

34) 60% of the voters at a local district voted Democrat. If 2,400 people voted Democrat, how many voted Republican?  
1,600 out of the 4,000 people who voted in the district voted Republican.

Simplify completely. Write final answers that have improper fractions as mixed numbers.

35) \[ \frac{320}{10} + \frac{96}{4} - \frac{33}{11} \]

36) \[ 15 \cdot \frac{9}{35} + \frac{4}{35} \cdot 8 \cdot \frac{4}{35} \]

37) \[ \frac{7}{12} + \frac{1}{3} \]

38) \[ \frac{8}{9} - \frac{7}{45} \]

39) \[ \frac{2}{15} \cdot \frac{3}{12} \]

40) \[ 4\frac{1}{3} - 1\frac{1}{2} \]

41) \[ 6\frac{2}{5} \div 1\frac{1}{7} \]

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42) Round 156.2839 to the thousandths place  
   \[156.284\]

43) Round 345.79 to the tens place  
   \[350\]

Find the perimeter and area of each geometric figure. Show the appropriate units in your answers.

44) A square with sides equal to 13.8 feet.

Perimeter = \[55.2\text{ ft}\]

Area = \[190.44\text{ ft}^2\]

45) A rectangle with base of \(8 \frac{3}{4}\) meters and height of \(3 \frac{2}{5}\) meters.

Perimeter = \[24 \frac{3}{10}\text{ m}\]

Area = \[29 \frac{3}{4}\text{ m}^2\]

Solve each equation.

46) \(4x - 73.5 = -42.6\)  \(x = 7.725\)

47) \(3.2m + 12.6 = 6.4\)  \(m = -1.9375\)

48) \(-\frac{2}{3}x + 12 = -36\)  \(x = 72\)

49) \(-9y + 148 + 2y - 36 = -2y + 20\)  \(y = 18.4\)

Simplify.

50) \(\sqrt{800}\)  \(20\sqrt{2}\)