How To Find if Triangles are Similar

Two triangles are similar if they have:

- all their angles equal
- corresponding sides are in the same ratio

But we don’t have to know all three sides and all three angles ...two or three out of the six is enough.

There are three ways to find if two triangles are similar: AA, SAS and SSS:

AA

AA stands for "angle, angle" and means that the triangles have two of their angles equal.
If two triangles have two of their angles equal, the triangles are similar.
For example, these two triangles are similar:

If two of their angles are equal, then the third angle must also be equal, because angles of a triangle always add to make 180°. In this case the missing angle is 180° - (72° + 35°) = 73°.
So AA could also be called AAA.

SAS

SAS stands for "side, angle, side" and means that we have two triangles where:
the ratio between two sides is the same as the ratio between another two sides and we also know the included angles are equal.
If two triangles have two pairs of sides in the same ratio and the included angles are also equal, then the triangles are similar.
For example:

In this example we can see that:

- one pair of sides is in the ratio of 21 : 14 = 3 : 2
- another pair of sides is in the ratio of 15 : 10 = 3 : 2
- there is a matching angle of 75° in between them

So there is enough information to tell us that the two triangles are similar. The geometric symbol for similarity in this case is:

$\triangle ABC \sim \triangle XYZ$
Special Properties of Midsegments

A midsegment is a line segment joining the midpoints of two sides of a triangle. A triangle has 3 possible midsegments.

BM = AM
BN = CN

These 2 triangles are similar ($\triangle ABC \sim \triangle MBN$) because the ratio of corresponding sides is constant ($2/1$) and the included angle of each triangle is congruent. Therefore, because of SAS. The ratio of the third side one triangle to the third side of the other triangle is also $2/1$. So the bigger triangle's third side is $2c$.

Since these triangles are similar, their corresponding angles are congruent.

So if $\angle BAC \cong \angle BMN$, then we are reminding of the following:

Converse of the Parallel Lines Conjecture If two lines are cut by a transversal to form a pair of congruent corresponding angles, congruent alternate interior angles, or congruent alternate exterior angles, then the lines are parallel.

Therefore, $MN \parallel AC$
Properties of Midsegments

In this lesson you will

- Discover properties of the midsegment of a triangle
- Discover properties of the midsegment of a trapezoid

In Chapter 3, you learned that a midsegment of a triangle is a segment connecting the midpoints of two sides. In this lesson you will investigate properties of midsegments.

Investigation 1: Triangle Midsegment Properties

Follow Steps 1–3 in your book. Your conclusions should lead to the following conjecture.

**Three Midsegments Conjecture** The three midsegments of a triangle divide it into four congruent triangles.

Mark all the congruent angles in your triangle as shown in this example.

Focus on one of the midsegments and the third side of the triangle (the side the midsegment doesn’t intersect). Look at the pairs of alternate interior angles and corresponding angles associated with these segments. What conclusion can you make? Look at the angles associated with each of the other midsegments and the corresponding third side.

Now compare the length of each midsegment to the length of the corresponding third side. How are the lengths related?

State your findings in the form of a conjecture.

**Triangle Midsegment Conjecture** A midsegment of a triangle is \( \frac{1}{2} \) the length of the third side.

The midsegment of a trapezoid is the segment connecting the midpoints of the two nonparallel sides.

Investigation 2: Trapezoid Midsegment Properties

Follow Steps 1–3 in your book. You should find that the trapezoid’s base angles are congruent to the corresponding angles at the midsegment. What can you conclude about the relationship of the midsegment to the bases?

Now follow Steps 5–7. You should find that the midsegment fits twice onto the segment representing the sum of the two bases. That is, the length of the
midsegment is half the sum of the lengths of the two bases. Another way to say this is: The length of the midsegment is the average of the lengths of the bases.

Use what you have learned about the midsegment of a trapezoid to complete this conjecture.

**Trapezoid Midsegment Conjecture** The midsegment of a trapezoid is ________________ to the bases and equal in length to ________________.

Read the text below the investigation on page 277 of your book and study the software construction. Make sure you understand the relationship between the Trapezoid and Triangle Midsegment Conjectures.

Work through the following example yourself before checking the solution.

**EXAMPLE** Find the lettered measures.

- **a.** 
  ![Diagram](image)
  
  13 cm
  
  72°
  
  x
  
  m
  
  By the Triangle Midsegment Conjecture, \(x = \frac{1}{2}(13\text{ cm}) = 6.5\text{ cm.}\)
  
  The Triangle Midsegment Conjecture also tells you that the midsegment is parallel to the third side. Therefore, the corresponding angles are congruent, so \(m = 72°\).

- **b.** 
  ![Diagram](image)
  
  9 cm
  
  12 cm
  
  58°
  
  y
  
  By the Trapezoid Midsegment Conjecture, \(\frac{1}{2}(12 + y) = 9\). Solving for \(y\) gives \(y = 6\).
  
  The Trapezoid Midsegment Conjecture also tells you that the midsegment is parallel to the bases. Therefore, the corresponding angles are congruent, so \(c = 58°\).
  
  By the Trapezoid Consecutive Angles Conjecture, \(b + 58° = 180°\), so \(b = 122°\).
Properties of Parallelograms

In this lesson you will

- Discover how the angles of a parallelogram are related
- Discover how the sides of a parallelogram are related
- Discover how the diagonals of a parallelogram are related

You have explored properties of kites and trapezoids and of the midsegments of triangles and trapezoids. In this lesson you will explore properties of parallelograms.

Investigation: Four Parallelogram Properties

Follow the directions in Step 1 in your book to construct and label a parallelogram.

Use patty paper or a protractor to compare the measures of the opposite angles. Then use your findings to complete this conjecture.

**Parallelogram Opposite Angles Conjecture**  The opposite angles of a parallelogram are ______________.

Consecutive angles are angles that share a common side. In parallelogram LOVE, \( \angle LOV \) and \( \angle EVO \) are one pair of consecutive angles. There are three other pairs. Find the sum of the measures of each pair of consecutive angles. You should find that the sum is the same for all four pairs. What is the sum? Complete this conjecture.

**Parallelogram Consecutive Angles Conjecture**  The consecutive angles of a parallelogram are ______________.

Suppose you are given the measure of one angle of a parallelogram. Describe how you can use the conjectures above to find the measures of the other three angles. If you don’t know, look at this particular figure. What are the values of \( a \), \( b \), and \( c \)? (Remember all your parallel lines conjectures.)

(continued)
Lesson 5.5 • Properties of Parallelograms (continued)

Use a compass or patty paper to compare the lengths of the opposite sides of your parallelogram. How are the lengths related? Complete this conjecture.

**Parallelogram Opposite Sides Conjecture** The opposite sides of a parallelogram are _________.

Now draw the diagonals of your parallelogram. Label the point where the diagonals intersect $M$. How do $LM$ and $VM$ compare? How do $EM$ and $OM$ compare? What does this tell you about the relationship between the diagonals? Complete this conjecture.

**Parallelogram Diagonals Conjecture** The diagonals of a parallelogram _________.

In your book, read the text about vectors that follows the investigation.

Here is an example using your new conjectures.

**EXAMPLE** In parts a and b, the figures are parallelograms. Find the lettered measures and state which conjectures you used.

a. 

![Diagram of parallelogram with sides labeled m and n, and a diagonal of 13 cm and another of 28 cm.]

b. 

![Diagram of parallelogram with a diagonal of 112°.]

**Solution**

a. By the Parallelogram Opposite Sides Conjecture, $m = 28$ cm.
   
   By the Parallelogram Diagonals Conjecture, $n = 13$ cm.

b. By the Parallelogram Opposite Angles Conjecture, $t = 112°$.

   By the Parallelogram Consecutive Angles Conjecture, $s = 180° - 112° = 68°$. 
Properties of Special Parallelograms

In this lesson you will

- Discover properties of rhombuses and their diagonals
- Discover properties of rectangles and their diagonals
- Discover properties of squares and their diagonals

In Lesson 5.5, you investigated parallelograms. In this lesson you focus on three special parallelograms—rhombuses, rectangles, and squares.

Investigation 1: What Can You Draw With the Double-Edged Straightedge?

Follow Steps 1–3 in your book. You should find that all the sides of the parallelogram you create are the same length. Use your findings to complete this conjecture.

**Double-Edged Straightedge Conjecture**

If two parallel lines are intersected by a second pair of parallel lines that are the same distance apart as the first pair, then the parallelogram formed is a ________________.

Now that you know a quick way to construct a rhombus, you will explore some special properties of rhombuses.

Investigation 2: Do Rhombus Diagonals Have Special Properties?

In this investigation you will look at the diagonals of a rhombus. Follow Steps 1 and 2 in your book. Then complete this conjecture.

**Rhombus Diagonals Conjecture**

The diagonals of a rhombus are ________________ and they ________________ each other.

Follow Step 3 to compare the two angles formed at each vertex by a diagonal and the sides. Then complete this conjecture.

**Rhombus Angles Conjecture**

The diagonals of a rhombus ________________ the angles of the rhombus.

You have just explored rhombuses, parallelograms with four congruent sides. Now you will look at rectangles, parallelograms with four congruent angles.

By the Quadrilateral Sum Conjecture, you know that the sum of the angle measures of a rectangle is $360^\circ$. Because all the angles have the same measures, each angle must have measure $90^\circ$. In other words, a rectangle has four right angles.

(continued)
Lesson 5.6 • Properties of Special Parallelograms (continued)

Investigation 3: Do Rectangle Diagonals Have Special Properties?
Follow Steps 1 and 2 in your book. What do you notice about the lengths of the two diagonals? Because a rectangle is a parallelogram, you also know that the diagonals bisect each other. You can use your compass to confirm this for your rectangle. Combine these two observations to complete the conjecture.

**Rectangle Diagonals Conjecture** The diagonals of a rectangle are ________ and ________ each other.

A square is a parallelogram that is both equiangular and equilateral. Here are two definitions of a square.

A **square** is an equiangular rhombus.

A **square** is an equilateral rectangle.

Because a square is a parallelogram, a rhombus, and a rectangle, all the properties of these quadrilaterals are also true for squares. Look back at what you know about the diagonals of each of these quadrilaterals, and use your findings to complete this conjecture.

**Square Diagonals Conjecture** The diagonals of a square are ________, ________, and ________.

**EXAMPLE** Find the lettered measures.

**Solution** The figure is a rhombus, so by the Rhombus Angles Conjecture, the diagonals bisect the angles. Therefore, $a = 23^\circ$.

By the Parallelogram Consecutive Angles Conjecture, $\angle WXY$ and $\angle XWZ$ are supplementary, so $m\angle XWZ + 46^\circ = 180^\circ$. Therefore, $m\angle XWZ = 134^\circ$. So, using the Rhombus Angles Conjecture, $b = \frac{1}{2}(134^\circ) = 67^\circ$.

By the Rhombus Diagonals Conjecture, the diagonals are perpendicular and bisect each other, so $c = 90^\circ$ and $d = 5\ cm$. 