Factoring Special Case Polynomials

Two terms:
Is it a *difference of squares*? Factor by using: \( a^2-b^2 = (a+b)(a-b) \)

Example 1: \( 36x^2 - 49 = (6x)^2 - 7^2 = (6x-7)(6x+7) \)

Example 2: Factor \( 9x^6 - y^8 \)
Since 6 and 8 are both even exponents, and 9 is a perfect square, this can be rewritten as

\[ 9x^6 - y^8 = \]

Is it a *difference of cubes*? Factor by using \( a^3-b^3 = (a - b)(a^2 + ab + b^2) \)

\[ x^3 - 64 \]
\[ (x - 4)(x^2 + 4x + 16) \]

Is it a *sum of cubes*? Factor by using \( a^3 + b^3 = (a + b)(a^2 - ab + b^2) \)

\[ x^3 + 125 \]
\[ (x + 5)(x^2 - 5x + 25) \]

Example: Factor \( 8x^3 - 27 \)

\[ 8x = (2x)^3 \quad 27 = 3^3 \]
\[ (2x)^3 - 3^2 = (2x - 3)((2x)^2 + (2x)(3) + (3)^2) \]
\[ = (2x - 3)(4x^2 + 6x + 9) \]

If the polynomial can’t be factored, it is PRIME.

Three terms: Is it a *perfect square trinomial*? 
If it is it would be in the form \( a^2x^2 + 2abx + b^2 \), which is factored as \( (a+b)^2 \) or \( a^2x^2 + 2abx + b^2 \) which is factored as \( (a-b)^2 \)

Example:
\[ 4x^2 + 12x + 9 \]
\[ = (2x)^2 + 2(2)(3)x + 3^2 = (2x + 3)^2 \]
Three Terms:

Factoring $ax^2 + bx + c$

Try factoring by the Grouping Method (or $a*c$ Method) or Snowflake Method (See next page).

Example: $2x^2 + 13x + 15$

(the $a*c$ method means multiply $2*15$ which is 30.
Find factors of 30 that add up to the middle term’s coefficient, which in this case is 13.
3*10=30 and 3+10 = 13.
Split the middle term into two parts, $13x$ becomes $10x + 3x$:

$$2x^2 + 10x + 3x + 15$$

and then factor by grouping.

$$2x(x+5) + 3(x+5) = (2x+3)(x+5)$$

Those methods don’t work? Maybe the polynomial is PRIME.

Example: $3x^2 + 8x + 5$

$ac = 3*5 = 15$, $b = 8$

Split $8x$ up into $3x + 5x$

$$3x^2 + 3x + 5x + 5$$

$$3x(x + 1) + 5(x + 1)$$

Factorization: $(x + 1)(3x + 5)$
AIM: Understanding how to factor quadratic functions when $a > 1$

**The SNOWFLAKE METHOD**

Setting up the snowflake for...

$$ax^2 + bx + c$$

Fill the missing spaces of the snowflake with the two numbers that when multiplied give you “$a \cdot c$” and when added together give you “$b$.”

**Let’s use the snowflake method to factor $3x^2 + 8x + 5$**

**Step 1:** Set up your snowflake.
**Step 2:** Fill in the missing spaces.
**Step 3:** Reduce the fractions on either side of the snowflake and put in proper form.

**ANSWER:**

$$(3x + 5)(x + 1)$$
Use the snowflake method to factor $4x^2 + 14x + 12$

**Snowflake method will not work if you don’t factor out the GCF!**

**Step 0:** Take out a factor of 2!

$$2(2x^2 + 7x + 6)$$

Step 1: Set up your snowflake.

Step 2: Fill in the missing spaces.

Step 3: Reduce the fractions on either side of the snowflake and put in proper form.

Don’t forget the GCF in the answer!

**ANSWER:**

$$2(x + 2)(2x + 3)$$
Use the snowflake method to factor $u^4 - 3u^2 - 4$

This polynomial is a trinomial but is not of degree 2. However, since the degree (highest power) is even, then we can use substitution and make it a polynomial if degree 2. Let $x = u^2$, then $x^2 = u^4$.

Our substituted polynomial becomes $x^2 - 3x - 4$

Step 1: Set up your snowflake.
Step 2: Fill in the missing spaces.
Step 3: Reduce the fractions on either side of the snowflake and put in proper form.
Step 4: Don't forget to put your original variable back in your factorization and factor more if necessary!

\[
(x - 4)(x + 1)
\]

Now remember what $x$ is:

\[
(u^2 - 4)(u^2 + 1)
\]

Can any of these be factored more? Yes.

Answer: $(u + 2)(u - 2)(u^2 + 1)$