Fractions are a form of division. When I ask what is $3/4$ I am asking “How big will each part be if I break 3 into 4 equal parts?” The answer is $\frac{3}{4}$. This is a fraction. A fraction is part of a whole. The **top part of the fraction is the numerator and the bottom part is the denominator.** Since $3 < 4$ this is a **proper** fraction because the numerator is less than the denominator. In an **improper** fraction, the numerator is greater than or equal to the denominator, such as $4/3$, or $3/3$. Proper fractions have values less than 1 and improper fractions have values greater than or equal to 1.
Writing a Fraction in Lowest Terms

Notice the same size box is split three different ways in equal parts: in 2 parts, 4 parts, or 8 parts.

By looking at the pictures, each box has the same *portion* (or fraction) filled, so these portions are called *equivalent fractions*.

**Fundamental Property of Fractions**

Multiplying or dividing the numerator AND the denominator of a fraction by the same nonzero number (not 0) does not change the value of the fraction. The fractions are *equivalent*.

\[
\frac{a}{b} = \frac{a \cdot x}{b} \quad \text{and} \quad \frac{a}{b} = \frac{a \div x}{b \div x}
\]
A Fraction in Lowest Terms
Means the numerator and denominator have no common factor other than 1. The denominator is the lowest it can be.

4/8 is not in lowest terms because 4 and 8 have common factors: 2 and 4. 2 goes into both the numerator, 4, and the denominator, 8, and 4 also goes into both 4 and 8.

2/4 is not in lowest terms because 2 and 4 have a common factor: 2. 2 goes into both the numerator, 2, and the denominator, 4.

½ is in lowest terms because there is no number (factor) that goes into both the numerator, 1, and the denominator, 2, other than 1.

Do self-check: Are the following fractions in lowest terms?
  a) 4/5   b) 6/18   c) 9/15   d) 17/46
Using Greatest Common Factor (GCF) to reduce a fraction to its lowest terms

The **Greatest Common Factor** is the *largest* number that goes into two numbers.

What is the GCF of 20 and 24? The largest number that goes into both 20 and 24 (the largest divisor of both 20 and 24)
Both numbers are divisible by 2. Is 2 the GCF?
20÷2 = 10, and 24÷2 = 12. But 10 and 12 have common factors, so 2 is not the GCF of 20 and 24.
A bigger number can go into both 20 and 24.
4 is the GCF of 20 and 24. 20 4 = 5, and 24÷4 = 6.
5 and 6 have no common factors, so 4 is the GCF.

*To reduce a fraction to its lowest terms, find the GCF of the numerator and the denominator, and divide both the numerator and denominator by the GCF.*

\[
\frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}
\]

so \(\frac{5}{6}\) is \(\frac{20}{24}\) in its lowest terms.
Example
If you write the prime factorization both the numerator and denominator, the fraction in lowest terms is easily found by cancelling out the common prime factors.

Write \( \frac{90}{126} \) in lowest terms.

\[
\frac{90}{126} = \frac{2 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 3 \cdot 7} = \frac{5}{7}
\]

You can only cancel out factors on the bottom (denominator) with corresponding factors on the top (numerator).
FRACTIONS

A fraction is just a division problem. \( \frac{\text{Numerator}}{\text{Denominator}} = \text{Numerator} \div \text{Denominator} \)

A fraction is in LOWEST TERMS, also known as REDUCED or SIMPLEST FORM when the numerator and denominator have no common factors.

\( \frac{6}{10} \) is not in lowest terms, \( \frac{3}{5} \) is in lowest terms

A fraction can be put in reduced form by dividing both the numerator and the denominator by the GCF. This can be done easily by putting both the numerator and denominator in prime-factored form and canceling the common factors between the numerator and denominator.

\[
\frac{6}{10} = \frac{2 \times 3}{2 \times 5} = \frac{3}{5}
\]

Mixed Number - A number, such as \( 6 \frac{3}{5} \), consisting of an integer and a fraction. A mixed number is just the sum of a whole number and a fraction.

Improper Fraction - A fraction in which the numerator is larger than the denominator. Converting from mixed number to improper fraction:

\[
6 \frac{3}{5} = \frac{6 \cdot 5 + 3}{5} = \frac{33}{5}
\]

Mixed numbers must be converted to improper fractions or decimals before doing ANY MULTIPLICATION OR DIVISION OPERATIONS on them.

A fraction can be converted into a decimal by dividing: \( \frac{\text{Numerator}}{\text{Denominator}} \)

\[
5)33 \quad \Rightarrow \quad 6 \frac{3}{5}
\]

\[-30 \]
\[3\]

When ADDING or SUBTRACTING FRACTIONS that must have the same denominator, then you just add the numerators and leave the denominator the same.

\[\frac{1}{5} + \frac{3}{5} = \frac{4}{5}\]

However, consider

\[\frac{1}{6} + \frac{3}{8}\]
Expressing a fraction in higher terms:

Later when we add fractions, we might have to express them in higher terms so that each fraction will have the same denominator.

Example:
Write \( \frac{5}{7} \) as an equivalent fraction with a denominator of 28.

\[
\frac{5 \times ?}{7 \times ?} = \frac{28}{28}
\]

\[
\frac{5 \times 4}{7 \times 4} = \frac{20}{28}
\]

Example
Write 4 as an equivalent fraction with a denominator of 6.
Is 4 a fraction? We can make it one by writing it as \( \frac{4}{1} \).

\[
\frac{4 \times ?}{1 \times ?} = \frac{6}{6}
\]

\[
\frac{4 \times 6}{1 \times 6} = \frac{24}{6}
\]
Multiplying fractions is easy: you multiply the top numbers and multiply the bottom numbers. For instance:
\[
\frac{2 \cdot 4}{3 \cdot 15} = \frac{2 \cdot 4}{3 \cdot 15} = \frac{8}{45}
\]

When possible, you reduce. In this case, however, nothing reduces, because 8 and 45 have no factors in common. If you're not sure, you can always do prime factorization:
\[
\frac{8}{45} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 5}
\]

Nothing cancels.

Often, though, something will cancel:

\[
\frac{\cancel{4} \cdot 49 \cdot \cancel{27}}{\cancel{9} \cdot \cancel{6} \cdot \cancel{28}} = \frac{\cancel{2} \cdot \cancel{2} \cdot \circ \cdot \circ \cdot \circ \cdot \circ}{\cancel{3} \cdot \cancel{3} \cdot \circ \cdot \circ \cdot \circ \cdot \circ} = \frac{7}{2}
\]

Also, you can simplify by cancelling common factors between numerators and denominators.

\[
\frac{1 \cdot \cancel{4} \cdot 49 \cdot \cancel{27}^{1}}{1 \cdot \cancel{9} \cdot \cancel{6} \cdot \cancel{28}^{1}} = \frac{7}{2}
\]

Dividing fractions is just about as easy: there's just one extra step. When you divide by a fraction, the first thing you do is "flip-n-multiply". That is, you take the second fraction, flip it upside-down, and multiply it by the first fraction. (The upside-down fraction is called the reciprocal). For instance:

\[
\frac{5}{6} \div \frac{5}{1} = \frac{5}{6} \cdot \frac{1}{5} = \frac{1}{6}
\]

\[
\frac{5}{6} \div \frac{5}{6} = \frac{\cancel{5}^{1}}{\cancel{6}^{1}} \cdot \frac{1}{\cancel{5}^{1}} = \frac{1}{6}
\]