Chapter 3: Variable Expressions and Equations

Expressions (contain no “=” sign):
An expression is one or numbers or variables having some mathematical operations done on them.

Numerical Expressions:
3 + 5
3(4)
6/2
5-1
4

Expressions can be evaluated or simplified: 3 + 5 can be simplified to 8
This just means, “Whatever you see, do”

Algebraic Expressions:
x + 5
3x ➞ in Algebra, it is implied that 3x means 3 times x. The “proper” way to write the product of a number and a variable is to always write the number to the left of the variable.
x times 5 = 5x
When numbers are multiplied by variables, they are given a special name, “coefficient”. 5 is the coefficient of 5x.

The quantities being added in an algebraic expression are called the terms.
If a term is a variable or a combination of variables multiplied by numbers, it is called a variable term. Numbers that are just added in the expression are called constant terms.

Example:

\[ 4x^2 - 2x - 3 \]

Hey! 2x and 3 aren’t being added, they are being subtracted! If we write this expression as \[ 4x^2 + -2x + -3 \], then we have all term being added, and the constant term is -3 and the variable terms are \[ 4x^2 \] and \[ -2x \].
LIKE TERMS

Like terms are terms with exactly the same variables raised to the exactly the same powers. Any constants in an expression are considered like terms. Terms that are not like terms are called unlike terms.

<table>
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<th>Like Terms</th>
<th>Unlike Terms</th>
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<tr>
<td>$2x$, $3x$, $-4x$</td>
<td>$2x$, $2x^2$</td>
</tr>
<tr>
<td>Same variables, each with a power of 1.</td>
<td>Different powers</td>
</tr>
<tr>
<td>$3$, $5$, $-1$</td>
<td>$3$, $3x$, $3x^2$</td>
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<tr>
<td>Constants</td>
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<td>$5x^2$, $-x^2$</td>
<td>$5x^2$, $5y^2$</td>
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<tr>
<td>Same variables and same powers</td>
<td>Different variables</td>
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Only Like Terms can be combined!

Combining like terms is to add or subtract like terms. Combine like terms containing variables by combining their coefficients and keeping the same variables with the same exponents.

Example: $3x^2 - 8x^2 = (3 - 8)x^2 = -5x^2$
Example: $5x + 2x = (5 + 2)x = 7x$
Example: $3x^2 + 5x + 2 + x^2 - x - 3$
= $3x^2 + x^2 + 5x - x + 2 - 3$
= $4x^2 + 4x - 1$

Algebraic expressions can only be simplified if they have “like terms,” which are terms with the same variable and same exponent.

$x + 5 + 2$ Only $5 + 2$ can be simplified.
$x + 5 + 2$ becomes $x + 7$

$3x + x + 2$ can be simplified to $4x + 2$
$x^2 + x$ cannot be simplified because they don’t have the same exponents and are therefore not like terms.
Subtraction Property of Equality –
Subtracting the same thing from both sides of an equation does not change the solution.

\[ x + 2 = 5 \]

Subtract 2 from both sides:

\[ x + 2 - 2 = 5 - 2 = 3 \]
This new equation will have the same solution as \( x + 2 = 5 \).

\[ x + 0 = 3 \]
\[ x = 3 \]

*Hey! We just “isolated” \( x \)!*

Check your answer by substituting \( x = 3 \) in the original equation.

\[ 3 + 2 = 5? \text{ Yes!} \]

Soooo….., **if you have an equation with a number added to a variable on one side, subtract that number from both sides to get the variable by itself, or to “isolate the variable.”**

But that example was too easy. Try this one:

\[ x + 15 = 67 \]

**Addition Property of Equality –**
Adding the same thing to both sides of an equation does not change the solution.

\[ x - 17 = 34 \]

Add 17 to both sides.

\[ x - 17 + 17 = 34 + 17 \]
This new equation will have the same solution as \( x - 17 = 34 \).

\[ x + 0 = 51 \]
\[ x = 51 \]

Check your answer by substituting \( x = 51 \) in the original equation.

\[ 51 - 17 = 34? \text{ Yes!} \]

Soooo….., **if you have an equation with a number subtracted from a variable on one side, add that number to both sides to get the variable by itself.**

**Example**

Last year a hairdresser lost 17 customers who moved away. If he now has 73 customers, how many did he have originally?

What are we being asked to find?

The number of customers the hairdresser had before he lost some. Let this equal \( x \).

What is given? He has 73 customers now. He lost 17.

Translation: “Lost” means subtraction.

What was 17 subtracted from? the original number of customers, or \( x \). The current number of customers should equal the original number minus the amount lost because of moving away.

\[ \text{current number} = \text{original number} - \text{amount lost} \]
\[ 73 = x - 17 \]

Now solve for \( x \). Add 17 to both sides to isolate \( x \).

\[ 73 + 17 = x - 17 + 17 \text{. This results in } 90 = x \]

*Check: Does \( 90 - 17 = 73? \text{ Yes!} \)*
**Division Property of Equality** –
Dividing the same thing on both sides of an equation does not change the solution.

\[
5x = 15
\]

\[
\frac{5x}{5} = \frac{15}{5}
\]

\[
x = 3
\]

*Check:* \(5(3) = 15? Yes!\)

**If you have an equation with a number multiplied by a variable, divide both sides of the equation by that number to get the variable by itself.**

*Examples of English words for multiplication: multiplied by 15 (15x), increased by a factor of six (6x), doubled (2x), tripled (3x), quadrupled (4x), increased fivefold (5x).*

**Multiplication Property of Equality** –
Multiplying the same thing on both sides of an equation does not change the solution.

**If your equation has a variable divided by a number, multiply both sides by that number to isolate the variable.**

*Examples of English words for division: halved \((x/2)\), a third of \((x/3)\), split in four \((x/4)\)*

Now you try p.72 #58

\[
\frac{x}{3} = 13
\]

\[3 \left( \frac{x}{3} \right) = 3(13)\]

\[
x = 39
\]

*Check:* \(\frac{39}{3} = 13? Yes!\)

Notice that \(x\) divided by 3 is written as a fraction, \(x/3\).

When multiplying a whole number by a fraction, think of the whole number as a fraction with denominator 1. For example \(3 = 3/1\), so

\[
3 \left( \frac{x}{3} \right) = \left( \frac{3}{1} \right) \left( \frac{x}{3} \right) = \frac{x}{1} = x
\]
Solving equations of the form \( ax + b = c \)
In order to isolate the variable \( x \), we must first combine like terms before isolating the variable.

Example 2: Solve:
\[ 5 = 9 - 2x \]
5 and 9 are both constants. They can be combined. To get 9 to the other side, subtract it from both sides.
\[ 5 = 9 - 2x \]
\[ -9 - 9 \]
\[ -4 = -2x \] The minus sign to the left of \( 2x \) does not disappear when the 9 disappears.
Divide both sides by \(-2\) to solve for \( x \)
\[ \frac{-4}{-2} = \frac{-2x}{-2} \]
\[ 2 = x \]
CHECK THE RESULT! \( 5 = 9 - 2(2) \)?
\[ 5 = 9 - 4 = 5 \] YES!

You try this one: Solve: \( 3x - 6 = 12 \)
Example 4

Solve

\[4x - 3 = 8x - 7\]

4x and 8x are like terms that can be combined.
-3 and -7 are like terms that can be combined.

Let's combine the constants first. Usually we like to put the variables on the left and the constants on the right, but either way works.

We'll add 3 to both sides to get the constants on the right.

\[4x - 3 + 3 = 8x - 7 + 3\]
\[4x = 8x - 4\]

Now we'll combine the variable terms.

Subtract 8x from both sides to get the variable terms on the left.

\[4x - 8x = 8x - 8x - 4\]
\[-4x = -4\]

Now we can divide both sides by -4.

\[x = 1\]

CHECK!

\[4(1) - 3 = 8(1) - 7\]

1 = 1 YES!
Strategy for Solving Algebraic Equations:
1. Use the distributive property to remove parentheses:
   \[ 3(x - 3) + 3 = 18 - 5x \]
   becomes \[ 3x - 9 + 3 = 18 - 5x \]
2. Combine like terms on either side of the equation.
   -9 and 3 can be added to get -6.
   \[ 3x - 6 = 18 - 5x \]
3. Use the addition or subtraction properties of equality to get the variables on one side of the \( = \) symbol and the constant terms on the other.
   3x and 5x are like terms. Add 5x to each side to get the variable terms on the left.
   \[ 3x - 6 = 18 - 5x + 5x \]
   \[ 8x - 6 = 18 \]
4. Continue to combine like terms whenever possible.
   6 and 18 are like terms. Since 6 is subtracted from 8x, add 6 to both sides to move it to the other side.
   \[ 8x - 6 = 18 + 6 \]
   \[ 8x = 24 \]
5. Undo the operations of multiplication and division to isolate the variable.
   Divide both sides by 8 to get x by itself.
   \[ 8x/8 = 24/8 \]
   \[ x = 3 \]
6. Check the results by substituting your found value for x into the original equation.
   \[ 3(x - 3) + 3 = 18 - 5x \]
   \[ 3(3 - 3) + 3 = 18 - 5(3) \]
   \[ 3(0) + 3 = 18 - 15 \]
   \[ 0 + 3 = 3 \]
   Yes. So \( x = 3 \) is the solution to the equation.
Now it’s your turn!

Try:

\[5 + 3(a+4) = 7a - (9-10a) + 4\]

Step 1: Use distributive property to remove parentheses

Step 2: Combine like terms on each side.

Step 3: Use addition property of equality to combine like terms between sides.

Step 4: Continue to combine like terms wherever possible.

Step 5: Once variable terms are combined and isolated, use multiplication property of equality (multiply both sides by the reciprocal of the coefficient) to completely isolate the variable.

State your conclusion: \[a = \_\_\_\_\_\_\_\_\_\_\_

Step 6: Check your solution in the original equation.
Example
Solve: \(6 + 3(x-2) = 21\)
Don’t do the 6+3 first! Order of operations says do what’s inside the parentheses first, then any exponents, then multiplication.
We don’t know what \(x-2\) is, and there are no exponents, so the next step is the multiplication of 3 and \((x-2)\).

\[
6 + 3x + 3(-2) = 21 \\
6 + 3x - 6 = 21 \\
\]
Combine like terms, 6 and -6 are both constants, therefore like terms.

\[
6 - 6 + 3x = 21 \\
3x = 21 \\
\]
Now divide both sides by 3 to get \(x\) by itself.

\[
x = 7 \\
\]
Check in original equation:

\[
6 + 3(7-2) = 21? \\
6 + 3(5) = 21 \\
6 + 15 = 21 \\
21 = 21 \ ? \ YES
\]
Example 8 on p. 195

Solve: $2x = 2 - (4x + 14)$

Now why is there parentheses around the $4x + 14$? Because the whole thing is to be subtracted from 2. Get in the habit of changing subtraction into adding the opposite.

$2x = 2 - (4x + 14)$

$2x = 2 + -1(4x + 14)$

Now use the distributive property to remove the parentheses.

$2x = 2 + -4x + -14$

Now combine like terms.

$2x = -4x + -14 + 2$
$2x = -4x + -12$

Now we have like variable terms on each side of the equal sign. Just add $4x$ to both sides to remove $4x$ from the right side.

$2x + 4x = -4x + 4x + -12$

$6x = -12$
$x = -2$

Make sure to surround that $-2$ with parentheses!

Check: $2(-2) = 2 - (4(-2) + 14)$?

$-4 = 2 - (-8 + 14)$
$-4 = 2 - (6)$
$-4 = -4$ YES