Multiplying fractions

To multiply fractions, simply multiply the numerators and multiply the denominators. Thus, the product of any two fractions is equal to the product of their numerators over the product of their denominators. If the fractions have common factors in the numerators and denominators, they can be simplified before they are multiplied.

1. \( \frac{1}{4} \times \frac{5}{6} \)
   \[ \frac{1 \times 5}{4 \times 6} = \frac{5}{24} \]
   Multiply numerators and multiply denominators.
   Final answer in simplest form

2. \( \frac{4}{5} \times \frac{7}{12} \)
   \[ \frac{4 \times 7}{5 \times 12} = \frac{28}{60} \]
   Simplify
   \[ \frac{4}{5 \times 3} = \frac{4}{15} \]
   Multiply numerators and multiply denominators.
   Final answer in simplest form

3. \( \frac{9}{21} \times \frac{14}{27} \)
   \[ \frac{9 \times 14}{21 \times 27} = \frac{126}{567} \]
   Simplify
   \[ \frac{1 \times 3}{2 \times 7} = \frac{1}{7} \]
   Multiply numerators and multiply denominators.
   Final answer in simplest form

Multiply each expression. Express each answer in lowest terms.

1. \( \frac{3}{8} \times \frac{3}{7} \)
2. \( \frac{3}{10} \times 2 \)
3. \( \frac{1}{12} \times 5 \)
4. \( \frac{2}{5} \times \frac{8}{11} \)
5. \( \frac{3}{10} \times \frac{2}{13} \)
6. \( \frac{2}{7} \times \frac{13}{14} \)
7. \( 2 \times \frac{3}{8} \)
8. \( \frac{1}{2} \times \frac{9}{10} \)
9. \( \frac{4}{11} \times \frac{3}{4} \)
10. \( \frac{5}{9} \times \frac{5}{9} \)
11. \( \frac{3}{5} \times \frac{1}{8} \)
12. \( \frac{1}{7} \times \frac{1}{6} \)
13. Find the product of \( \frac{a}{3} \times \frac{4}{5} \).
14. Find \( \frac{1}{4} \) of 3 feet.
15. A rope is \( \frac{7}{12} \) yards long. How long is \( \frac{1}{3} \) of the rope?
**Mixed numbers**

A mixed number is the sum of a whole number and a fraction. It is good to use a mixed number when there is an improper fraction. An improper fraction is a fraction whose numerator is larger than its denominator. 

$\frac{2}{4}$ is an improper fraction.

To change an improper fraction to a mixed number, follow these steps:

1. Divide the numerator by the denominator.
   
   $4 \div 4 = 2 \text{ R } 1$

2. Write the whole number answer in step 1 as the whole number and the remainder as the numerator of the fraction with the denominator the same as that in the original improper fraction.

   Thus, the mixed number is $2 \frac{1}{4}$.

To change a mixed number to an improper fraction, follow these steps:

Change $3 \frac{1}{5}$ to an improper fraction.

1. Multiply the denominator of the fraction times the whole number.

   $3 \times 5 = 15$

2. Add the whole number found in step 1 to the numerator of the fraction in the original mixed number. This is the numerator of the improper fraction.

   $15 + 1 = 16$

3. Write the numerator found in step 2 over the denominator of the original fraction in the mixed number.

   Thus, $\frac{16}{5}$ is the improper fraction.

Express each mixed number as an improper fraction.

1. $3 \frac{1}{4}$

2. $6 \frac{5}{6}$

3. $2 \frac{2}{5}$

4. $1 \frac{3}{4}$

5. $4 \frac{8}{11}$

6. $5 \frac{1}{7}$

7. $8 \frac{1}{2}$

8. $5 \frac{2}{3}$

Express each improper fraction as a mixed number.

9. $\frac{5}{3}$

10. $\frac{8}{7}$

11. $\frac{17}{7}$

12. $\frac{16}{9}$

13. $\frac{10}{3}$

14. $\frac{26}{5}$

15. $\frac{30}{11}$

16. $\frac{45}{4}$

Multiply. Express answer in lowest terms.

17. $\frac{3}{5} \times \frac{7}{2}$

18. $\frac{5}{4} \times 6$ (Note: first rewrite 6 as 6 $\div 1$.)
**Multiplying mixed numbers**

To multiply mixed numbers, change the numbers to improper fractions.

1. Multiply $2 \frac{3}{4} \times 5 \frac{2}{3}$
   
   \[
   2 \frac{3}{4} = \frac{8 + 3}{4} = \frac{11}{4}
   \]
   \[
   5 \frac{2}{3} = \frac{15 + 2}{3} = \frac{17}{3}
   \]
   \[
   \frac{11}{4} \times \frac{17}{3} = \frac{187}{12}
   \]
   \[
   \frac{187}{12} = 15 \frac{7}{12}
   \]

   Change mixed numbers to improper fractions.

2. Multiply $1 \frac{1}{2} \times \frac{1}{2}$
   
   \[
   1 \frac{1}{2} = \frac{2 \times 1}{2} = \frac{3}{2}
   \]
   \[
   \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}
   \]

   Change $1 \frac{1}{2}$ to a mixed number.

Multiply fractions and express answer in simplest form.

Name the improper fractions that would replace each whole number or mixed number to be able to multiply the expression.

1. $4 \times 5 \frac{1}{2}$
2. $1 \frac{1}{6} \times 1 \frac{1}{7}$
3. $3 \times 5 \frac{2}{9}$
4. $3 \frac{1}{4} \times 3 \frac{1}{5}$
5. $4 \times 4 \frac{3}{4}$
6. $1 \frac{3}{7} \times 4 \frac{1}{10}$
7. $2 \frac{1}{2} \times 2 \frac{1}{3}$
8. $6 \frac{1}{4} \times 7 \frac{3}{5}$
9. $3 \frac{1}{3} \times 7$

Multiply each expression. Express each answer in lowest terms.

10. $\frac{7}{3} \times 4 \frac{1}{2}$
11. $2 \frac{3}{4} \times 1 \frac{1}{2}$
12. $5 \frac{1}{5} \times 3 \frac{2}{3}$
13. $\frac{3}{5} \times 6$
14. $8 \frac{1}{4} \times 2 \frac{1}{3}$
15. $4 \times 5 \frac{1}{3}$

Use the formula for the area of a rectangle, $A = \ell \times w$, where $\ell$ is length and $w$ is width, to find the area of each rectangle below.

16. $\ell = 2 \frac{1}{2}$ in., $w = 4 \frac{1}{4}$ in.
17. $\ell = 8 \frac{3}{5}$ ft., $w = 3$ ft.
Dividing fractions and mixed numbers

To divide any two fractions, multiply by the reciprocal of the divisor. Reciprocal numbers are two numbers whose product is 1. To find the reciprocal of a fraction, invert the numerator and denominator.

1. Divide \( \frac{3}{2} \) by \( \frac{1}{2} \)

\[
\frac{3}{2} \div \frac{1}{2} = \frac{3}{2} \times \frac{2}{1}
\]

Invert the divisor and change to multiplication.

\[= \frac{3 \times 2}{2 \times 1} = \frac{6}{2} = 3\]

Multiply.

2. Divide \( 2\frac{1}{4} \) by \( 4\frac{1}{3} \)

\[
2\frac{1}{4} = \frac{9}{4} \quad \text{Change mixed numbers to improper fractions.}
\]

\[4\frac{1}{3} = \frac{13}{3} \]

Write problem using improper fractions.

\[= \frac{9}{4} \div \frac{13}{3} \quad * \quad \text{Change to multiplication by inverting second fraction.}
\]

\[= \frac{27}{52} \quad \text{Multiply.}
\]

1. What are reciprocals? Give an example.

Name the reciprocal of each number.

2. \( \frac{4}{7} \)  
3. \( 5 \)  
4. \( \frac{3}{10} \)  
5. \( \frac{1}{2} \)

Write each division expression as a multiplication expression. Then solve. Write answers in lowest terms.

6. \( \frac{1}{4} \div 4 \)  
7. \( 1\frac{1}{6} \div 2\frac{1}{2} \)  
8. \( \frac{1}{5} \div \frac{1}{5} \)

9. \( \frac{3}{5} \div \frac{7}{10} \)  
10. \( 2\frac{4}{7} \div \frac{1}{3} \)  
11. \( 3\frac{1}{3} \div 5\frac{1}{2} \)

Solve each equation for \( x \). Write answers in lowest terms.

12. \( 6x = 2\frac{1}{2} \)  
13. \( 2x = 3\frac{1}{4} \)  
14. \( \frac{3}{4}x = 5\frac{1}{2} \)