CHAPTER  Sections 3.1-3.3  SOLVING LINEAR EQUATIONS

Equations:

*An equation is two expressions set equal to one another.*

Numerical Equations:

\[
\begin{align*}
4 &= 5 - 1 \\
8 &= 3 + 5 \\
12 &= 3(4) \\
3 &= \frac{6}{2}
\end{align*}
\]

Algebraic Equations:

\[
\begin{align*}
x &= 3(4) \\
y - 3 &= 4
\end{align*}
\]

Equations can be **true** or **false**.

True: \(3 = 2 + 1\)  
False: \(2 + 3 = 2(3)\)

Algebraic Equations can be true or false depending on the value of the variable.

\(y - 3 = 4\) is true if \(y = 7\) and false if \(y\) equals anything else.

Some algebraic equations are false no matter what: \(y = y + 2\) can never be true no matter what \(y\) is.

*Algebraic Equations can be solved.* A solution of an equation is the value of the variable that makes the equation true.

Example 1

Is -3 a solution of the equation \(4x + 16 = x^2 - 5\)?

Let’s substitute -3 for \(x\) in the equation (Don’t forget to surround it with parentheses!) and see if the equation is true.

\[
\begin{align*}
4(-3) + 16 &= (-3)^2 - 5 \\
-12 + 16 &= 9 - 5 \\
4 &= 4 \quad \text{TRUE}
\end{align*}
\]

Now you try this one.

Is 5 a solution of \(10x - x^2 = 3x - 10\)?

Two equations are **equivalent** equations when they have the same solution.

\(x + 1 = 4\) is equivalent to \(x + 2 = 5\)

In both cases, \(x\) must be 3 for the equation to be true.

How do you solve equations?

Get \(x\) by itself one side of the equation (isolate \(x\)) so you are left with \(x = \_\_\_\_.\) The right side of the equation will just have a numerical expression that can be simplified.

**SOLUTIONS OF EQUATIONS CAN AND SHOULD ALWAYS BE CHECKED!**

Just substitute the solution in the original equation and see if one side equals the other.
Solving Equations

**Subtraction Property of Equality**
Subtracting the same thing from both sides of an equation does not change the solution.

\[ x + 2 = 5 \]

Subtract 2 from both sides:

\[ x + 2 - 2 = 5 - 2 = 3 \]

This new equation will have the same solution as \( x + 2 = 5 \).

\[ x + 0 = 3 \]
\[ x = 3 \]

*Hey! We just “isolated” \( x \)!*

Check your answer by substituting \( x = 3 \) in the original equation.

\[ 3 + 2 = 5? \] Yes!

Soooo…. *if you have an equation with a number added to a variable on one side, subtract that number from both sides to get the variable by itself, or to “isolate the variable.”*

But that example was too easy. Try this one:

\[ x + 15 = 67 \]

**Addition Property of Equality**
Adding the same thing to both sides of an equation does not change the solution.

\[ x - 17 = 34 \]

Add 17 to both sides.

\[ x - 17 + 17 = 34 + 17 \]

This new equation will have the same solution as \( x - 17 = 34 \).

\[ x + 0 = 51 \]
\[ x = 51 \]

Check your answer by substituting \( x = 51 \) in the original equation.

\[ 51 - 17 = 34? \] Yes!

Soooo…. *if you have an equation with a number subtracted from a variable on one side, add that number to both sides to get the variable by itself.*

**Example**

Last year a hairdresser lost 17 customers who moved away. *If he now has 73 customers, how many did he have originally?*

What are we being asked to find?

The number of customers the hairdresser had before he lost some. Let this equal \( x \).

What is given? He has 73 customers now. He lost 17.

Translation: “Lost” means subtraction.

What was 17 subtracted from? the original number of customers, or \( x \). The current number of customers should equal the original number minus the amount lost because of moving away.

\[
\text{current number} = \text{original number} - \text{amount lost} \\
73 = x - 17
\]

Now solve for \( x \). Add 17 to both sides to isolate \( x \). \[ 73 + 17 = x - 17 + 17 \]
This results in \( 90 = x \)

*Check: Does \( 90 - 17 = 73 \)? Yes!*
Division Property of Equality –
Dividing the same thing on both sides of an equation does not change the solution.

\[
5x = 15
\]

\[
\frac{5x}{5} = \frac{15}{5}
\]

\[
x = 3
\]

Check: \(5(3) = 15\)? Yes!

If you have an equation with a number multiplied by a variable, divide both sides of the equation by that number to get the variable by itself.

Examples of English words for multiplication: multiplied by 15 \((15x)\), increased by a factor of six \((6x)\), doubled \((2x)\), tripled \((3x)\), quadrupled \((4x)\), increased fivefold \((5x)\).

Multiplication Property of Equality –
Multiplying the same thing on both sides of an equation does not change the solution.

If your equation has a variable divided by a number, multiply both sides by that number to isolate the variable.

Examples of English words for division: halved \((x/2)\), a third of \((x/3)\), split in four \((x/4)\)

Now you try p.72 #58

\[
\frac{x}{3} = 13
\]

\[
\left(\frac{x}{3}\right) \cdot 3 = 13 \cdot 3
\]

\[
x = 39
\]

Check: \(\frac{39}{3} = 13\)? Yes!

Notice that \(x\) divided by 3 is written as a fraction, \(\frac{x}{3}\).
When multiplying a whole number by a fraction, think of the whole number as a fraction with denominator 1. For example \(3 = \frac{3}{1}\), so

\[
3 \left( \frac{x}{3} \right) = \left( \frac{3}{1} \right) \left( \frac{x}{3} \right) = \frac{x}{1} = x
\]
Solving equations of the form $ax + b = c$

In order to isolate the variable $x$, we must first combine like terms before isolating the variable.

Example 2: Solve:

$5 = 9 - 2x$

5 and 9 are both constants. They can be combined.

To get 9 to the other side, subtract it from both sides.

$5 = 9 - 2x$

$-9$  $-9$

$-4 = -2x$  The minus sign to the left of $2x$ does not disappear when the $9$ disappears

Divide both sides by $-2$ to solve for $x$

$\frac{-4}{-2} = \frac{-2x}{-2}$

$2 = x$

CHECK THE RESULT!  $5 = 9 - 2(2)$?

$5 = 9 - 4 = 5$  YES!

You try this one: Solve: $3x - 6 = -5$
Example: When coefficient is a fraction…

$$\frac{2}{5}x - 3 = -7$$

3 and -7 are both constants, so we should combine them. To get 3 to the other side, use the addition property of equality. Since 3 is subtracted on the left side, add 3 to both sides to remove it from the left side and move to the right side.

$$\frac{2}{5}x - 3 + 3 = -7 + 3$$

$$\frac{2}{5}x = -4$$

Now we use the division property of equality to isolate x. Dividing both sides by 2/5 is the same as MULTIPLYING BY THE RECIPROCAL.

$$\left(\frac{5}{2}\right) \frac{2}{5}x = -4 \left(\frac{5}{2}\right)$$

$$x = -10$$

CHECK:

$$\frac{2}{5}(-10) - 3 = -7$$

$$-4 - 3 = -7$$

$$-7 = -7 \text{ YES!}$$
Example 4

Solve

\[4x - 3 = 8x - 7\]

4x and 8x are like terms that can be combined.
- 3 and - 7 are like terms that can be combined.

Let's combine the constants first. Usually we like to put the variables on the left and the constants on the right, but either way works.

We'll add 3 to both sides to get the constants on the right.

\[4x - 3 + 3 = 8x - 7 + 3\]
\[4x = 8x - 4\]

Now we'll combine the variable terms.

Subtract 8x from both sides to get the variable terms on the left.

\[4x - 8x = 8x - 8x - 4\]
\[-4x = -4\] Now we can divide both sides by -4.

\[x = 1\]

CHECK!

\[4(1) - 3 = 8(1) - 7\]
\[1 = 1 \text{ YES!}\]
Strategy for Solving Algebraic Equations:

1. Use the distributive property to remove parentheses:
   \[ 3(x - 3) + 3 = 18 - 5x \text{ becomes } 3x - 9 + 3 = 18 - 5x \]

2. Combine like terms on either side of the equation.
   -9 and 3 can be added to get -6.
   \[ 3x - 6 = 18 - 5x \]

3. Use the addition or subtraction properties of equality to get the variables on one side of the = symbol and the constant terms on the other.
   3x and 5x are like terms. Add 5x to each side to get the variable terms on the left.
   \[ 3x - 6 = 18 - 5x \]
   \[ + 5x \quad + 5x \]
   \[ 8x - 6 = 18 \]

4. Continue to combine like terms whenever possible.
   6 and 18 are like terms. Since 6 is subtracted from 8x, add 6 to both sides to move it to the other side.
   \[ 8x - 6 = 18 \]
   \[ + 6 \quad + 6 \]
   \[ 8x = 24 \]

5. Undo the operations of multiplication and division to isolate the variable.
   Divide both sides by 8 to get x by itself.
   \[ 8x/8 = 24/8 \]
   \[ x = 3 \]

6. Check the results by substituting your found value for x into the original equation.
   \[ 3(x - 3) + 3 = 18 - 5x \]
   \[ 3(3 - 3) + 3 = 18 - 5(3) \]
   \[ 3(0) + 3 = 18 - 15 \quad ? \]
   \[ 0 + 3 = 3 \quad \text{Yes. So } x = 3 \text{ is the solution to the equation.} \]
Now it’s your turn!

Try:

\[5 + 3(a+4) = 7a - (9-10a) + 4\]

Step 1: Use distributive property to remove parentheses

Step 2: Combine like terms on each side.

Step 3: Use addition property of equality to combine like terms between sides.

Step 4: Continue to combine like terms wherever possible.

Step 5: Once variable terms are combined and isolated, use multiplication property of equality (multiply both sides by the reciprocal of the coefficient) to completely isolate the variable.

State your conclusion: \( a = \) ____

Step 6: Check your solution in the original equation.
The Trick with Fractions!

Fractions are messy to deal with. When solving equations with fractions in them we can take advantage of the multiplication property of equality to get rid of them while keeping an equivalent equation.

What we do is multiply BOTH SIDES of the equation (that is everything on each side) by the LCD of all the fractions.

\[
\frac{2}{3} + \frac{1}{4}x = -\frac{1}{3}
\]

The denominators are 3 and 4. The LCD is 12

\[
(12)\frac{2}{3} + (12)\frac{1}{4}x = -\frac{1}{3}(12)
\]

\[
8 + 3x = -4
\]

Now we can combine like terms.

8 and - 4 are like terms.

Combine them by subtracting 8 from both sides.

\[
8 - 8 + 3x = -4 - 8
\]

\[
3x = -12
\]

Divide both sides by 3.

\[
\frac{3x}{3} = -\frac{12}{3}
\]

\[
x = -4
\]

CHECK!

\[
\frac{2}{3} + \frac{1}{4}(-4) = -\frac{1}{3}
\]

\[
\frac{2}{3} + (-1) = -\frac{1}{3} \quad \text{Change -1 so that it has a common denominator}
\]

\[
\frac{2}{3} + \left(\frac{-3}{3}\right) = -\frac{1}{3}
\]

\[
\frac{2+(-3)}{3} = -\frac{1}{3}
\]

\[
\frac{-1}{3} = -\frac{1}{3} \quad \text{Yes!}
\]
The Trick with Decimals!

Decimals are also are messy to deal with. When solving equations with fractions in them we can take advantage of the multiplication property of equality to get rid of them while keeping an equivalent equation.

What we do is multiply BOTH SIDES of the equation (that is everything on each side) by the power of 10 that corresponds to the number with the most decimal places.

\[
.05a + .2(a + 3) = 0.1
\]

The number with the most decimal places is .05 (2 places). This corresponds to \(10^2\), or 100. If we multiply both sides by 100, all the decimal numbers get changed to whole numbers.

\[
(100)(.05a + .2(a + 3)) = (0.1)(100)
\]

\[
5a + 20(a + 3) = 10 \quad \text{Now solve using your regular steps}
\]

\[
5a + 20a + 60 = 10
\]

\[
25a + 60 = 10
\]

\[
25a + 60 - 60 = 10 - 60
\]

\[
25a = -50
\]

\[
\frac{25a}{25} = \frac{-50}{25}
\]

\[
a = -2
\]

Check:
You Try 2 (p.128)

Solve:  \[ 0.08k - 0.2(k + 5) = -0.1 \]
Not all equations have a solution. Sometimes it is impossible to find a value for a variable that would make the equation true.

Example: Solve \( y = y + 2 \)

Is there any number that you can add 2 to it and still get the same number? No.

What happens when we try to solve it?

Combine like terms. There’s a variable on each side, so to eliminate the \( y \) on the right, you’d have to subtract \( y \) from both sides.

\[
y = y + 2 \\
-\ y \hspace{1cm} -\ y
\]

\[
0 = 0 + 2
\]

\[0 = 2 \quad ??\] You see, if you try to solve an equation that is “unsolvable” you will get a false statement. This means that no matter what value you have for \( y \), the equation will always be false.
Some equations have an infinite number of solutions.

Example:

$$3(x - 1) + 1 = 4x - (x + 2)$$

**Step 1: Use distributive property**

$$3x - 3 + 1 = 4x - x - 2$$

**Step 2: Combine like terms on each side**

$$3x - 2 = 3x - 2$$

**Step 3: Combine like terms between sides using addition property of equality.**

$$3x - 2 = 3x - 2$$

\[+2\quad +2\]

$$3x = 3x$$

\[-3x\quad -3x\]

$$0 = 0 \quad !!$$

*This statement is true no matter what value of x you choose.*

*Therefore the solution set is {all real numbers}*
Word Problems!

Step 1: Read the problem carefully. Identify what you are being asked to find. At the bottom of your paper, state the conclusion with a ______ for the unknown quantity.

Step 2: Choose a variable to represent the unknown quantity.

Step 3: Translate the problem from English into an equation using the chosen variable.
* Look for indicator words such as “added to, decreased by, the quotient of, etc…”
* Draw a picture if necessary to visualize the problem.
•Break up the problem into small parts.

Step 4: Solve the equation

Step 5: Check your answer in the original problem. Is it a reasonable answer? Does it makes sense in the context of the problem? If so, then use this answer in your final conclusion that you stated at the bottom of the page.
Example 7 p. 131

Five less than three times a number is the same as the number increased by seven. Find the number.

Step 1: What are we being asked to find? Find the number that meets the conditions in problem.

The number is ________.

Step 2: Choose a variable and say what it represents.

n = the number.

Step 3: Translate the problem into an algebraic equation. Substitute n everywhere the problem mentions “the number.”

Five less than three times a number is the same as the number increased by seven

\[ 3n - 5 = n + 7 \]

Step 4: Solve

\[
\begin{align*}
3n - 5 &= n + 7 \\
+5 & \quad +5 \\
3n &= n + 12 \\
-\ell & \quad -\ell \\
2n &= 12 \\
n &= 6
\end{align*}
\]

Step 5: Check use \( n = 6 \) in equation \( 3n - 5 = n + 7 \)

\[
\begin{align*}
3(6) - 5 &= 6 + 7 \\
18 - 5 &= 13 \\
13 &= 13 \text{ Yes!}
\end{align*}
\]

Restate in words.

Five less than three times six is the same as six increased by seven.

Does that make sense? Yes

YOU TRY 5 p.131

Three less than five times a number is the same as the number increased by thirteer
Find the number.

Step 1: What are we being asked to find? Find the number that meets the conditions in problem.

Step 2: Choose a variable and say what it represents.

n = the number.

Step 3: Translate the problem into an algebraic equation. Substitute n everywhere the problem mentions “the number.”

Three less than five times a number is the same as the number increased by thirteer

Step 4: Solve

Step 5: Check

CONCLUSION: THE NUMBER IS __.