**Ratios and rates**

A ratio is a comparison by division of two numbers with the same units. Ratios are often expressed as fractions in simplest form or as decimals. A ratio can also be expressed with a colon (1:3, or one to three). For example, express the following as a fraction in simplest form and as a decimal:

3 children out of 4 children have a sibling.

\[
\frac{3}{4} = \frac{1}{3} \quad \text{Fraction in simplest form}
\]

\[
\frac{3}{9} = \frac{1}{3} = 0.33 \quad \text{Decimal with a line over the 3 to show repeating}
\]

Thus, \(\frac{1}{3}\) or 0.33, of the children have a sibling.

A rate is a comparison of two measurements with different units of measure and is considered to be a special kind of ratio. For example, the ratio of 1 dozen golf balls for $6 can be written: \(\frac{1 \text{ doz. golf balls}}{56} \). This rate compares golf balls and their price.

To find the price per golf ball, first simplify the rate so the denominator is $1. A rate with a denominator of 1 is called a unit rate.

What is the cost of one golf ball using the above information?

\[
\begin{align*}
\frac{1 \text{ doz. golf balls}}{56} & \quad \text{Set up the initial rate.} \\
\frac{12 \text{ golf balls}}{56} & \quad \text{Simplify 1 doz. = 12 golf balls.} \\
\frac{12 \div 6}{56 \div 6} = \frac{2}{1} & \quad \text{To get the unit rate, divide numerator and denominator by 6. Notice 1 is now in the denominator.} \\
\frac{2 \text{ golf balls}}{51} & \quad \text{Replace in problem adding the units.}
\end{align*}
\]

Since 2 golf balls cost $1, 1 golf ball costs $0.50.

1. What is the difference between a rate and a ratio? Give an example of each.

2. Write a ratio that compares the boys to girls in your class.

Express each ratio or rate as a fraction in simplest form.

3. 12 out of 144

4. 7 out of 9 apples

5. 51 : 17

6. 72 to 9

Express each ratio as a unit rate.

7. 210.8 miles on 12.4 gallons

8. 2.5 in. of rain in 10 hours

9. $10.20 for 15 lbs.

10. $62.50 for 25 tickets
Proportions

A proportion is an equation that names two equivalent ratios.
\[
\frac{3}{4} = \frac{12}{16}
\]
This is a proportion.

In this proportion, 3 and 16 are called the extremes, and 4 and 12 are called the means. 3 \times 16 and 4 \times 12 are called the cross products. In a proportion, the cross products are always equal, meaning the product of the extremes is equal to the product of the means. For example, 3 \times 16 = 48 and 4 \times 12 = 48. This is a great test to show a true proportion. Look at the following examples:

Use cross products to determine if each pair of ratios forms a proportion.

1. \[
\frac{5}{6}, \frac{10}{14}
\]
\[
\frac{5}{6} = \frac{10}{14}
\]
\[
5 \times 14 = 6 \times 10
\]
\[
70 = 60
\]
Thus, \[
\frac{5}{6} = \frac{10}{14}
\] is not a proportion.

2. \[
\frac{1}{4}, \frac{8}{32}
\]
\[
\frac{1}{4} = \frac{8}{32}
\]
\[
1 \times 32 = 4 \times 8
\]
\[
32 = 32
\]
Thus, \[
\frac{1}{4} = \frac{8}{32}
\] forms a proportion.

Solve the proportion \[
\frac{n}{3} = \frac{6}{4}
\]

\[
\frac{n}{3} = \frac{6}{4}
\]
\[
n \cdot 4 = 3 \cdot 6
\]
Multiply cross products.
\[
\frac{9n}{4} = \frac{18}{4}
\]
Divide both sides by 4.
\[
n = 2
\]
Thus, the solution is 2.

1. What is the relationship of ratios and proportions? Give an example of a proportion.

2. State the steps to use to solve the proportion \[
\frac{2}{x} = \frac{20}{30}
\] and solve. Identify the means and the extremes.

Use cross products to tell whether each sentence is true. Write T (true) or F (false).

3. \[
\frac{3}{2} = \frac{63}{42}
\]
4. \[
\frac{2}{5} = \frac{14}{35}
\]
5. \[
\frac{10}{16} = \frac{5}{9}
\]
6. \[
\frac{7}{5} = \frac{36}{20}
\]

Solve each proportion.

7. \[
\frac{3}{4} = \frac{n}{10}
\]
8. \[
\frac{46}{92} = \frac{n}{100}
\]
9. \[
\frac{2}{3} = \frac{24}{n}
\]
10. \[
\frac{n}{2} = \frac{56}{112}
\]

Set up a proportion to use to solve for each variable and solve.

11. 9 gallons for $27
\[
x\text{ gallons for } $9.60
\]
12. 25 candles per 5 boxes
\[
150 \text{ candles per } x \text{ boxes}
\]
Using proportions to solve problems

The most important point to remember when using proportions to help solve problems is to be sure to have the same unit in both numerators, as well as the same unit in both denominators. Once the proportion is set up, simply solve as usual. Look at the following examples:

1. Out of 10 girls, 4 were chosen to go to the state competition. At this rate, how many girls would be chosen out of 50?

\[
\frac{4}{10} = \frac{x}{50} \quad \text{girls in competition} \\
\text{total girls}
\]

Set up initial proportion. Be sure units match in the numerators and the units match in the denominators.

\[
4 \times 50 = 10 \times x
\]

Multiply cross products.

\[
\frac{200}{10} = \frac{10x}{10}
\]

Divide both sides by 10.

\[
x = 20
\]

Thus, 20 girls would be chosen out of 50.

2. If a 4-lb. turkey takes 180 minutes to cook, how long would a 6-lb. turkey take to cook?

\[
\frac{4}{180} = \frac{6}{x} \quad \text{lb. minutes}
\]

Set up initial proportion.

\[
4 \times x = 6 \times 180
\]

Multiply cross products.

\[
\frac{4x}{4} = \frac{1080}{4}
\]

Divide both sides by 4.

\[
x = 120
\]

Thus, it would take a 6-lb. turkey 120 minutes to cook.

Set up a proportion to represent each problem and solve.

1. There are 220 calories in 4 ounces of beef. How many calories are there in 5 ounces?

2. If John can buy 8 liters of soft drinks at the store for $6.40, how much does it cost him to buy 12 liters?

3. Sherri bought a package of pens that contained 15 pens. How many packages should she buy if she needs 240 pens?

4. Steve won his election by a margin of 7 to 2. His opponent had 3,492 votes. How many votes did Steve have?

5. A car traveled 325 miles in 5 hours. How far did the car travel in 9 hours?

6. A recipe asks for 1 ½ cups of chocolate chips for 60 cookies. How many cups would be needed for 36 cookies?
Page 70
1. Solve a quadratic equation.
2. Define the function.
3. Find the roots.
4. Graph a linear equation.
5. Find the x-intercepts.
6. Determine the slope.
7. Solve a system of equations.

Page 71
1. Solve a system of equations.
2. Graph a quadratic equation.
3. Find the vertex.
4. Determine the axis of symmetry.
5. Find the focus.
6. Find the directrix.

Page 72
1. Graph a circle.
2. Find the center.
3. Determine the radius.
4. Graph a parabola.
5. Find the vertex.
6. Determine the axis of symmetry.
7. Graph an ellipse.
8. Find the center.
9. Determine the major and minor axes.
10. Graph a hyperbola.
11. Find the center.
12. Determine the transverse and conjugate axes.

Page 73
1. Graph a function.
2. Find the domain and range.
3. Determine the end behavior.
4. Find the intercepts.
5. Graph a piecewise function.
6. Find the intervals of increase and decrease.
7. Find the critical points.
8. Graph a composite function.
9. Find the composition.
10. Graph a transformation.
11. Find the transformations.

Page 74
1. Solve a system of equations.
2. Graph a linear inequality.
3. Find the shaded region.
4. Determine the boundary line.
5. Find the intercepts.
6. Graph a quadratic inequality.
7. Find the boundary line.
8. Find the intercepts.
9. Graph a system of inequalities.
10. Find the shaded region.

Page 75
1. Solve a proportion.
2. Find the ratio.
3. Determine the unit rate.
4. Graph a variation.
5. Find the constant of variation.
6. Determine the direct variation.
7. Graph an inverse variation.
8. Find the constant of variation.
9. Determine the inverse variation.
10. Graph a joint variation.
11. Find the constant of variation.

Page 76
1. Solve a linear equation.
2. Graph a linear equation.
3. Find the slope.
4. Determine the x-intercept.
5. Find the y-intercept.
6. Graph a system of equations.
7. Find the solution set.
8. Graph a system of inequalities.

Page 77
1. Solve a proportion.
2. Graph a proportion.
3. Find the constant of variation.
4. Determine the direct variation.
5. Graph an inverse variation.
6. Find the constant of variation.
7. Determine the inverse variation.
8. Graph a joint variation.
9. Find the constant of variation.
10. Determine the joint variation.

Page 78
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