Solving linear systems using multiplication and addition

To solve a linear system using both multiplication and addition, use the following steps:

1. Arrange the equations with like terms in columns.
2. Look at the coefficients of $x$ and $y$ in both equations. Multiply one or both equations by a specific number that gives new coefficients for $x$ (or $y$) that are opposites.
3. Add the equations and solve for the variable that remains.
4. Substitute the value found in step 3 into one of the original equations and solve for the other variable.
5. Check the solution in both of the original equations.

Solve

\[
\begin{align*}
3x - y &= 2 \\
4(3x - y) &= 4(2) \\
12x - 4y &= 8 \\
-2x + 4y &= 2 \\
\underline{10x} &= 10 \\
x &= 1
\end{align*}
\]

Multiply the first equation by 4.
Add.
Solve for $x$, the remaining variable.

Substitute the $x$-value into one of original equations and solve for $y$.

Check

\[
\begin{align*}
3(1) - y &= 2 \\
-2(1) + 4(1) &= 2 \\
3 - 1 &= 2 \\
-2 + 4 &= 2 \\
2 &= 2 & \text{True}
\end{align*}
\]

Thus, the solution of the linear system is $(1, 1)$.

State the best number to be multiplied by one equation to eliminate one of the variables using addition.

1. $x - 2y = 4$
   $-3x + 5y = -3$
2. $-3x - 6y = 11$
   $2x + y = 4$
3. $7x + 2y = 5$
   $-x - 6y = 8$
4. $6x - 5y = 8$
   $-2x - y = 3$

Solve each system using multiplication and addition.

5. $4x + y = 5$
   $2x + 5y = 7$
6. $7x + 2y = -4$
   $3x + 6y = 24$
7. $-2x + 5y = 8$
   $-5x + 3y = 1$
8. $-5x + 2y = 22$
   $-3x + 2y = 6$
9. $x + y = 0$
   $2x - 11y = 39$
10. $-3x + 4y = 1$
    $x + 2y = 3$
11. $2x + 3y = 12$
    $5x + 2y = 8$
Answers will vary. Possible answers:
1. \( y = -3x + 4; 2. x = 5y + 10; 3. y = -6x - 3; \)
4. \( x = -5y - 11; 5. (-4, -1); 6. (-2, -5); 7. (-7, 3); \)
8. \((4, -2); 9. (1, 1); 10. (4, -5); 11. (-3, 3); \)
12. \((-4, 3); 13. (3, 1)\)

Neither set of variables have opposite coefficients. To fix this, multiply the first equation by -3. Explanations will vary.

Multiply first equation by 3; 2. Multiply second equation by 6; 3. Multiply second equation by -5; 4. Multiply second equation by -5; 5. (1, 1); 6. (-2, 5); 7. (1, 2); 8. (-8, -9); 9. (3, -3); 10. (1, 1); 11. (0, 4) \n
1, a, one solution; 2, c, one solution; 3, d, infinitely many solutions (same line); 4, b, no solution (parallel lines); 5, no solution \n6. (1, -4), one solution

-5x + 4y = 3
3x - 2y = 4
x + y = -3

Josh = 2 years, Cheryl = 8 years; 2. 26, 22; 3. Joe’s = $7.25, Jim’s = $9.75; 4. fries = $7.50, hamburger = $1.00; 5. quarters = 15, nickels = 24