Directions:
- Please use a pencil, write neatly and succinct.
- Show all calculations when ever possible. If you use calculator write down what function you are using.
- Round z-values and standard deviations to two decimal places, rest four decimal places.
- Graph, label, and shade appropriate area when every possible.
- Box your answers!!

I. Identify the given variable as being discrete or continuous. (1 point each)
1. The weight of your book._Continuous
2. The breaking time of a car._Continuous
3. The number of students in our class._Discrete

II. Write true or false next to each question in the space provided. (1 point each)
1. The probability of an event that is certain to happen is 1. _____T____
2. The probability of an impossible event is 0.___T____
3. Probability on rare occasions can be negative.____F____
4. The following values cannot be a probability \( \{ 2^{0}, \sqrt{0.9}, 2.13 \times 10^{-2} \} \) ____F____
5. The majority of X, means more then 25% of X_____F_____ 
6. Pr( A or B) suggests the addition rule._____T________
7. Pr( A and B) suggests the multiplication rule.____T________
8. Disjoint events are not independent._____T_______
9. Mutually exclusive events are independent. __F_______
10. If an event is the complement of another event, then those two events are disjoint.____T____
11. For a probability distribution, the sum of all probabilities must be strictly less then 1.__F___
12. Binomial distribution is a continuous distribution.____F_______
13. The normal distribution is a continuous distribution._____T_______
14. Recording the gender of 25 newborns is a binomial distribution.____T____
15. The values of a uniform distribution are spread evenly over the range of values.____T____
16. Standard normal distribution has mean 0 and variance 1.____T____
17. It is ok if the total area under the curve is less then 1.____F____
III. The table below describes the smoking habits of a group of asthma suffers. (12 points)

<table>
<thead>
<tr>
<th></th>
<th>Nonsmoker</th>
<th>Occasional Smoker</th>
<th>Regular Smoker</th>
<th>Heavy Smoker</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>384</td>
<td>33</td>
<td>64</td>
<td>49</td>
<td>530</td>
</tr>
<tr>
<td>Women</td>
<td>349</td>
<td>44</td>
<td>72</td>
<td>28</td>
<td>493</td>
</tr>
<tr>
<td>Sum</td>
<td>733</td>
<td>77</td>
<td>136</td>
<td>77</td>
<td>1023</td>
</tr>
</tbody>
</table>

Show work clearly with notation and formula. If you don’t use formula please be clear in your logic.

1. If one person is randomly selected, find the probability that the person is a woman.

   \[
   \Pr(X = \text{Women}) = \frac{493}{1023} \approx 0.4819
   \]

2. If one person is randomly selected, find the probability that the person is a heavy smoker or nonsmoker.

   \[
   \Pr(\text{HS} \cup \text{N}) = \Pr(\text{HS}) + \Pr(\text{N}) - \Pr(\text{HS} \cap \text{N})
   \]

   \[
   = \frac{77}{1023} + \frac{733}{1023} - \frac{0}{1023} = \frac{810}{1023} \approx 0.7918
   \]

3. If one person is randomly selected, find the probability that the person is a man or a heavy smoker.

   \[
   \Pr(\text{M} \cup \text{HS}) = \Pr(\text{M}) + \Pr(\text{HS}) - \Pr(\text{M} \cap \text{HS})
   \]

   \[
   = \frac{530}{1023} + \frac{77}{1023} - \frac{49}{1023} = \frac{558}{1023} \approx 0.5454
   \]

4. If one person is randomly selected, find the probability that the person is an occasional smoker given that the person is a woman.

   \[
   \Pr(\text{OS} \mid \text{W}) = \frac{\Pr(\text{OS} \cap \text{W})}{\Pr(\text{W})} = \frac{\frac{44}{1023}}{\frac{493}{1023}} = \frac{44}{493} \approx 0.0892
   \]
IV. Let the random variable \( X \) represent the number of boys in a family of three children. (12 points)

1. Construct a table describing the probability distribution of boys, then check if it is a probability distribution. Hint: First construct the sample space then find probabilities. There are eight simple events in your sample space.

Sample space = \{ GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB \}

Probability distributions of girls:

\[
\begin{align*}
Pr(X=0B) &= \frac{1}{8} \\
Pr(X=1B) &= \frac{3}{8} \\
Pr(X=2B) &= \frac{3}{8} \\
Pr(X=3B) &= \frac{1}{8}
\end{align*}
\]

Yes, it is probability distributions because probability distribution adds up to 1.

2. What is the probability of the complement of two boys?

\[
Pr(2B) = 1 - Pr(2B) = 1 - \frac{3}{8} = \frac{5}{8} = 0.625
\]

3. Find the mean and standard deviation of the probability distributions.

\[
\begin{align*}
\mu &= E(x) = \sum_{i=0}^{3} x_{i}p(x_{i}) = 1.5 \\
\sigma &= \sqrt{\sum_{i=0}^{3} [x_{i}^2 p(x_{i})] - E(x)^2} \approx 0.866
\end{align*}
\]

V. In a clinical test of a cold medicine 6.73% of the subjects treated experienced runny nose. Seven subjects are randomly selected. Find the probability that at least one person experiences runny nose. Show all work step by step with notation using the formula of the binominal distribution. Hint: Complement. (8 points)

\[
X = Subjects\ who\ experienced\ runny\ nose
\]

\[
n=7, \quad p=0.0673, \quad q=1-0.0673=0.9327
\]

\[
Pr(X \geq 1) = 1 - P(X = 0)
\]

\[
= 1 - \binom{7}{0} (0.0673)^0 (0.9327)^7
\]

\[
= 1 - \frac{7!}{0!(7-0)!} (0.0673)^0 (0.9327)^7
\]

\[
= 1 - (1 \times 1 \times 0.6140) \approx 0.386
\]
VI. Explain the difference between a nonstandard normal distribution and the standard normal distribution, draw graph. (4 points)

Standard normal distribution has mean 0 and standard deviation is 1. Nonstandard normal distribution has means other number then 0 and standard deviation different from 1. We use Z transformation to go from nonstandard normal to standard normal.

VII. Z is a standard normal variable, find the probability. (3 points each)

a) The probability that Z is no more than -0.95.

\[ Pr(z \leq -0.95) = 0.1711 \]

b) The probability that Z is between -1.72 and 0.47.

\[ Pr(-1.72 < z < 0.47) = 0.6381 \]

VII. Assume the readings of thermometers are normally distributed with a mean of 0°C and a standard deviation of 1.00°C. A thermometer is randomly selected and tested. (8 points)

If 2.5% of the thermometers are rejected because they have reading too high and another 2.5% are rejected because they have reading too low, find the two readings that are cutoff values separating the rejected thermometers from the others.

\[ \text{Invnorm}(0.025) = -1.96 \]
VII. (8 points) Assume that X is normally distributed, with mean 25 and standard deviation 7. Find $P(X > 30)$.

$$P(X > 30) = P\left(\frac{X - \mu}{\sigma} > \frac{30 - 25}{7}\right) = P(z > 0.7173) = \text{normalcdf}(0.7173, 99) \approx 0.2375$$

VIII. The weights of fish in a certain lake are normally distributed with a mean of 15 lb and standard deviation of 6 lb. (12 points)

a) If one fish is randomly selected, what is the probability that fish will be between 12.6 and 18.9 lb?

$$P(12.6 < X < 18.9) = P\left(\frac{12.6 - 15}{6} < \frac{X - \mu}{\sigma} < \frac{18.9 - 15}{6}\right) = P(-0.4 < z < 0.65) = \text{normalcdf}(-0.4, 0.65) \approx 0.3976$$

b) If 4 fish are randomly selected, what is the probability that the mean weight will be between 12.6 and 18.6 lb?

$$P(12.6 < \bar{X} < 18.9) = P\left(\frac{12.6 - 15}{\frac{6}{\sqrt{4}}} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{18.6 - 15}{\frac{6}{\sqrt{4}}}\right) = P(-0.8 < z < 1.2) \approx 0.6731$$

IX. In a southern state 4.5% of all individuals who drive automobiles are not properly licensed. Use the normal approximation to the binomial to find the probability that among 174 randomly selected individuals, no more than 9 are not properly licensed. (10 points)

$$p = 0.045$$
$$n = 174$$
$$\mu = np = 0.045 \times 174 = 7.83$$
$$\sigma = \sqrt{npq} = \sqrt{0.045 \times 174 \times 0.955} \approx 2.7345$$

$$P(X \leq 9) = P(X \leq 9.5) = P\left(\frac{X - \mu}{\sigma} < \frac{9.5 - 7.83}{2.7345}\right) = P(z < 0.6107) = \text{normalcdf}(-99, 0.6107) \approx 0.7293$$
a) A contractor is considering a sale that promises a profit of $71,000 with a probability 63.07% or a loss or $27,000 with a probability of 36.93%. What is the expected profit?

b) Assume that the weight loss for the first month of a diet program varies between 5 pounds and 11 pounds, and is spread evenly over the range of possibilities, so that there is a uniform distribution. Find the probability of losing at least 7 pounds.