Elementary Statistics Math 112

Make up Exam 2 100pts total Prof. Seres

Show all calculations, give answers as reduced fractions or to 4 decimal places. Box your answers! Good Luck!

I. Identify the given random variable as being discrete or continuous. (1pnt each)

(a) The number of supermodels who eat Tommy’s burger Discrete
(b) The exact life span of Elvis Presley (…no he is not alive in Las Vegas ☹) Continuous
(c) The weight of feather Continuous

II. Let random variable $X$ represent the number of girls in a family of three children. Construct a table describing a probability distribution, and then find the mean and standard deviation. *Hint: there are a total of eight simple events in your sample space, list all the possible outcomes, and then find probability distribution.* Is it unusual for a family of three to have three girls? (10 points)

Sample space = {GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB}
Pr($X=3G$) = 1/8 = 0.125 No, it’s not unusual to have three girls.
Note: unusual means less then 0.05

Probability distributions of girls:
Pr($X=0G$) = 1/8
Pr($X=1G$) = 3/8
Pr($X=2G$) = 3/8
Pr($X=3G$) = 1/8

(a) Show that the probability distribution above is a probability distribution.

$P(X) = \sum_{i=0}^{3} p(x_i) = 1$

(b) Find the mean $\mu$ of $x$

$\mu = E(x) = \sum_{i=0}^{3} x_i p(x_i) = 1.5$

(c) Find standard deviation $\sigma$ of $x$

$\sigma = \sqrt{\sum_{i=0}^{3} [x_i^2 p(x_i)] - E(x)^2} \approx 0.866$

III. Let $X$ be the number of success in 7 independent trials, where the probability of success on each trial is $p=0.27$. (10 points)

(a) Calculate $P(X=4)$, showing the binomial formula by hand.

$P(x = 4) = \binom{7}{4} (0.27)^4 (1-0.27)^3 = \frac{7!}{4!(7-4)!} (0.27)^4 (0.73)^3 \approx 0.07236$

(b) Complete the probability distribution table for $X$, using formula or TI-83+ calculator.

(c) Find the mean of the distribution.

$E(X) = np = 7 (0.27) = 1.89$

(d) Find the standard deviation of the distribution.

$\sigma = \sqrt{npq} = \sqrt{7 \cdot (0.27)(0.73)} = 1.175$
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IV. True - False questions (1 point each)
1. A continuous random variable has a uniform distribution if its graph has a rectangular shape. ____T____
2. Standard normal score have a mean of one and standard deviation of zero. ____F____
3. The area to the right of \( z = 1.52 \) is 0.4357 ____F____
4. Z scores measure the number of SD above or below the mean, they can be positive or negative. ____T____
5. Area under the curb can be positive or negative. _____F____
6. The standard deviation of the sample mean or standard error of the mean is the standard deviation of the population from which the samples have been taken and is denoted by \( \sigma_x = \frac{\sigma}{\sqrt{n}} \). ____T____
7. The Chi-square distribution is a symmetric distribution ____F____
8. The t-distribution has a mean of 1 and standard deviation 0 ____F____
9. A point estimate is a single value (or point) used to approximate a population parameter ____T____
10. The Central Limit Theorem states that if all possible random samples of size \( n \) are taken from any population, the sampling distribution of the sample means becomes approximately normal when the sample size \( n \) is large enough. ____T____
11. Binomial distribution is a discrete distribution ____T____

V. Provide the appropriate response. (3 points)
Explain how a nonstandard normal distribution differs from the standard normal distribution. Describe the process for finding probabilities for nonstandard normal distributions.

Both distributions are bell shaped. In a standard distribution, the mean is 0, and the standard deviation is 1. In a nonstandard distribution, the mean and standard deviation can have other values. The process should include sketching the diagram, marking the x scores on the diagram, computing the corresponding z-score, shading the appropriate area of interest, finding the area corresponding to the z-score in the table.

VI. Data is normally distributed with mean 0 and standard deviation 1. Find from table or calculator. (12 points)
a. Find the area under the normal curve of \( P(z < -0.98) \) also graph and shade the area asked for.

\[
P(z < -0.98) = 0.1635
\]

b. \( P(-a < z < a) = 0.450 \) find \( a \), also graph and shade area asked for.

\[
\text{InvNorm (0.275) = 0.5977 = 0.6}
\]

\[a \approx 0.6\]
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c. Find \( P_{40} \), the 40th percentile.

\[
\text{InvNorm}(0.40) = -0.2534
\]

VII. The random variable is normally distributed, with mean 25 and standard deviation 8. (6 points)

Graph and shade area asked for.

\[
P(Y \geq 31) = P \left( \frac{x - \mu}{\sigma} \geq \frac{31 - 25}{8} \right) = P(Z \geq 0.75)
\]

\[
\text{normalcdf}(0.75, 99) \approx 0.2266
\]

VIII. In a southern state, 5% of all individuals who drive automobiles are not properly licensed. Use the normal approximation to of the binomial distribution to find the probability that among 200 randomly selected individuals, between seven and nine, inclusive, are not properly licensed. (8 pts). Hint: find \( P(6.5 < x < 9.5) \).

\[
p = 0.05 \quad n = 200
\]

\[
\mu = np = 200(0.05) = 10
\]

\[
\sigma = \sqrt{npq} = \sqrt{9.5} = 3.08
\]

\[
P(7 \leq x \leq 9) = P \left( \frac{6.5 - 10}{3.08} < \frac{x - \mu}{\sigma} < \frac{9.5 - 10}{3.08} \right) = P(-1.136 < Z < -0.162) \approx 0.3076
\]

IX. Multiple-Choice

1. Assume that you have repeatedly taken samples of size 5 from a population of 30. What can be said about the individual sample means? Circle the right answer. (2 points)

a. They will be the population mean
b. They will vary, but close to the population mean.
c. The mean of means will equal to zero
d. The mean will equal 5.

2. Circle the statistics that target the population parameter.

a. mean, median, variance
b. proportion, mean, variance
c. median, variance, proportion
d. standard deviation, mean, proportion

X. Assume that women’s heights are normally distributed with a mean given by \( \mu = 63.6\text{in} \) and \( \sigma = 2.5\text{in} \) (10 points)
a. If a woman is randomly selected find the probability that her height's between 63.5 in and 64.5 in.

\[ P(63.5 < X < 64.5) = P\left(\frac{63.5 - 63.6}{2.5} < \frac{x - \mu}{\sigma} < \frac{64.5 - 63.6}{2.5}\right) = \]

\[ = (-0.04 < Z < 0.36) = 0.1565 \]

b. If 9 women are randomly selected, find the probability that they have a mean height between 63.5 in and 64.5 in.

\[ P(63.5 < X < 63.6) = P\left(\frac{63.5 - 63.6}{2.5/\sqrt{9}} < \frac{x - \mu}{\sqrt{9/n}} < \frac{64.5 - 63.6}{2.5/\sqrt{9}}\right) = \]

\[ P(-0.12 < Z < 1.08) = 0.4077 \]

XI. Do one of the following, as appropriate: (a) Find the critical value \( Z_{\alpha/2} \), (b) find the critical value \( t_{\alpha/2} \), (c) state that neither the normal nor the t distribution applies. (3 points each)

a) 98%; \( n = 7; \sigma = 27; \) population appears to be normally distributed.

(a) Critical value \( Z_{\alpha/2} = 2.326 \)

b) 98%; \( n = 7; \sigma = 27; \) population appears to be normally distributed.

(b) Critical value for \( t_{\alpha/2} = 1.65 \)

c) 95%; \( n = 11; \sigma \) is known; population appears to be very skewed.

c) neither the normal nor the t distribution applies.

XII. A machine is programmed to put 737 grams of sugar in a container. Due to uncontrolled variation in the process there is variation in content from container to container. To estimate the mean amount of sugar per container, a sample of 50 boxes is selected and \( \bar{x} = 739.5 \) grams and \( s = 7.5 \) grams. Find a 95% confidence interval for \( \mu \) and interpret. (8 points)

\[ n = 50, \quad \bar{x} = 739.5, \quad s = 7.5 \]

\[ t_{0.025,49} = 2.009 \]

\[ E = 2.009 \left(\frac{7.5}{\sqrt{50}}\right) = 2.1308 \]

So, \( \bar{x} \pm E = 7.5 \pm 2.1308 \)

(737.4 < \( \mu \) < 741.6)

With 95% confidence the population mean amount of sugar per container is between 737.4 g and 741.6 g.
XII. When consumers apply for credit, their credit is rated using FISCO (Fair, Isaac and Company) scores. Credit rating from a sample of 16 applicants for car loan is \( \bar{x} = 660.3 \) with \( s^2 = 9196.3625 \) (FISCO units). Construct a 99% confidence interval for the standard deviation of FISCO scores for all applicants for credit. Assume FISCO scores are normally distributed. Interpret. (8 points)

\[
\begin{align*}
  n &= 16, \quad \bar{x} = 660.3, \quad s^2 = 9196.3625 \\
  X_{1,0.005,15}^2 &= X_{0.005,15}^2 = 32.801 \\
  X_{0.995,15}^2 &= X_{0.995,15}^2 = 4.601 \\
  \frac{(n-1)s^2}{X_{1,0.995,15}^2} < \sigma^2 < \frac{(n-1)s^2}{X_{1,0.005,15}^2} &\Rightarrow \frac{15(9196.3625)}{32.801} < \sigma^2 < \frac{15(9196.3625)}{4.601} \\
  \sqrt{4205.52} < \sigma^2 < \sqrt{29981.62} \\
  64.85 < \sigma < 173.15
\end{align*}
\]

With 99% confidence the population standard deviation of FISCO scores is between 64.85 and 173.15 FISCO units.

Extra Credit, choose one only! Must show work for full credit! (7 points)

1. A probability distribution has a standard deviation equal to 2.5 and \( \sum [x^2 \cdot p(x)] = 10.25 \). Find the mean for this distribution.

2. The amounts (in ounces) of juice in eight randomly selected juice bottles are:

| 15.1 | 15.6 | 15.6 | 15.9 | 15.9 | 15.2 | 15.1 | 15.2 |

Construct a 98 percent confidence interval for the mean amount of juice in all such bottles and interpret and interpret.

3. In a Gallup poll, 1025 randomly selected adults were adult's surveyed and 29% of them said that they used the internet for shopping at least a few times a year. Find a 99% confidence interval estimate of the percentage of adults who use the Internet for shopping.