Exam 1 Review:

1. Give an example of a quantity. What is a possible value for your example?

2. Give two quantities that one could have in mind when he/she says, “That's a big athlete!”

3. Consider this problem situation:
   
   The school cafeteria is ready to serve two kinds of sandwiches, tuna and ham, and pepperoni and vegetarian. There are 48 servings of pizza prepared. There are 4 sandwiches prepared than there are servings of pepperoni pizza. There are four sandwiches prepared than there are servings of vegetarian pizza. Altogether, how are prepared?

   A) List eight quantities involved in this problem.
   B) Sketch a diagram to show the relevant sums and differences in this situation.
   C) Solve the problem.

4. Considering the following problem situation:
   
   Two trains leave from different stations and travel toward each other on parallel tracks. They leave at the same time. The stations are 217 miles apart. One train travels at 65 mph, travels at 72 mph. How long after they leave their stations do they meet each other?

   List six quantities in the problem (note that you are not asked to solve this problem). If a value is given, write it next to the quantity. If no value is given, write an appropriate unit of measure.

5. The big dog weighs five times as much as the little dog. The little dog weighs 2/3 as much as the medium-sized dog. The medium-sized dog weighs 9 pounds more than the little dog. How much does the big dog weigh?

   A) List three quantities associated with this problem. If possible, give the associated value.
   B) Draw a diagram to represent the quantities in this problem.
   C) This diagram was provided by a 5th grader. Tell why it is not helpful.
   D) Solve the problem and explain your solution process.

6. The label on a can of chicken broth claims that its weight is 1.4 kg. Use your metric knowledge to tell how many milligrams this would be.

7. The larger the unit of measure used to express the value of a specific quantity, the larger its numeric value will be.

   A) True
   B) False
8. Complete the following:
   A) 2.3 km = ______ m
   B) 2 cm = ______ km
   C) 2.14 g = ______ kg

9. What metric prefix means one-hundredth?

10. If a Pascal is some unit of measure, use your knowledge of metric prefixes to complete:
    4 kiloPascals = ________ Pascals

11. What are some advantages to using the metric system of measurement?

12. Use strip diagrams to solve the following problems. (Hint: Use strip diagrams such as in
    the exercises for 1.4.)
    A) Jesse collects stamps. He now has 444 stamps. He has three times as many stamps from
       European countries as he does from Asian countries. How many of his stamps are from
       European countries?
    B) Silvia and Jesus are buying a new table and new chairs for their dining area. Chairs
       with arm rests are $45; those with no arms are $8.50 cheaper. The table is four times as
       much as a chair without arm rests. If they buy a table and six chairs, two with arms and
       four without, what is the total price they pay?
    C) Joe lives 8 miles from campus. Jim lives 2 miles further away from campus that Joe
       does. If each drives a car to campus, how many miles altogether do Joe and Jim drive to
       and from campus?
    D) A Grade 3-4 elementary school classroom has 29 students. There are seven more third
       graders than there are fourth graders. How many students are there in each grade?

13. Most present-day societies use the Hindu-Arabic numeration system.
    A) True
    B) False

14. How many tens are in 7654? How many whole tens are in 7654?

15. How many tenths are in 1.03? How many whole tenths?

16. In base ten, 215.687 is exactly _______ ones, exactly _________ tens, exactly
    _________ hundreds, and exactly _________ thousands; also, 3421 is exactly
    _________ tenths and exactly _________ hundredths.

17. (Roman numerals) IX = _______ ten and XI = _______ ten.

18. 23 has 230 hundredths in it.
    A) True
    B) False

19. I am a number with 21 tens, 14 ones, and 11 tenths. What number am I?
20. A soap factory packs 100 bars of soap in each box for shipment. If the factory makes 15,287 bars of soap, how many full boxes will they have for shipment? (mental math only) Explain.

21. Grady thinks that 0.36 is bigger than 0.4 because 36 is bigger than 4. Comment on Grady's reasoning using your understanding of place values with base ten.

22. In base $b$, there are $b - 1$ different digits.
   A) True
   B) False

23. These are the digits that are needed for a base seven place-value system: 0, 1, 2, 3, 4, 5, 6, 7.
   A) True
   B) False

24. In base $b$, $3 + 2b^3 + b$ would be written ________________.

25. $524\text{eight} = \underline{\hspace{2cm}}\text{ten}$

26. $287\text{ten} = \underline{\hspace{2cm}}\text{four}$

27. $1012\text{five} = \underline{\hspace{2cm}}\text{in base ten}$.

28. $32\text{ten} = \underline{\hspace{2cm}}\text{in base four}$.

29. $2.31\text{four} = \underline{\hspace{2cm}}\text{as a mixed number in base ten}$.

30. $6\frac{2}{3}$ in base ten = ___________ in base three.

31. $214.3\text{five} = \underline{\hspace{2cm}}\text{in base ten}$

32. $203.6\text{ten} = \underline{\hspace{2cm}}\text{five}$

33. $2003\text{five} = \underline{\hspace{2cm}}\text{ten}$

34. You are living and working on a planet that uses only base five. How many five-dollar bills can you get for $1234.20\text{five}$? Write your answer in base five since you are living on the planet. Write enough (numbers, words, etc.) to make your thinking clear.

35. In base ten, the two whole numbers immediately before 2001 are __________ and __________.

36. In base five, the two whole numbers immediately before 2001$_{five}$ are __________$_{five}$ and __________$_{five}$.

37. If you are counting in base five, what would be the next six numerals after 2314$_{five}$?
38. Sketch the base blocks that show \(1203_{\text{seven}}\), and give the English words for the base ten value of each different sized piece. Use dots for units, lines for longs, squares for flats, and cubes for cubes.

39. The best coins to use in thinking about the first three whole-number place values in base five would be the penny, the nickel, and the quarter.

A) True

B) False

40. \[
\begin{array}{c}
241_{\text{six}} \\
\quad + 135_{\text{six}}
\end{array}
\]

41. \[
\begin{array}{c}
127_{\text{nine}} \\
\quad - 58_{\text{nine}}
\end{array}
\]

42. \[
\begin{array}{c}
4.4_{\text{five}} \\
\quad + 3.3_{\text{five}}
\end{array}
\]

43. \[
\begin{array}{c}
0.24_{\text{seven}} \\
\quad - 0.15_{\text{seven}}
\end{array}
\]

44. \[
\begin{array}{c}
21_{\text{six}} \\
\quad + 35.2_{\text{six}}
\end{array}
\]

45. Use drawings of multibase blocks to illustrate \(32_{\text{five}} + 23_{\text{five}}\). Use Dots for units, lines for longs, squares for flats, and cubes for cubes.

46. Two basketball coaches, A and B, are talking.
   A says to B: “Your tallest player is 6 inches taller than my tallest player!”
   B says to A: “Yes, but your second-tallest player is 8 inches taller than my second tallest player.”
   A says to B: “Hmm. My second-tallest player is 4 inches shorter than my tallest player.”
   Make a drawing, and tell the difference in heights of Coach B’s two tallest players.

47. Marge bought several types of candy for Halloween: Milky Ways, Tootsie Rolls, Reese's Cups, and Hershey Bars. Milky Ways and Tootsie Rolls together were six more than the Reese's Cups. There were four fewer Reese's Cups than Hershey Bars. There were 12 Milky Ways and 28 Hershey Bars. How many Tootsie Rolls did Marge buy?
   List five quantities involved in this problem.
   Sketch a diagram to show the relevant sums and differences in this situation.
   Solve the problem.
48. The school cafeteria is ready to serve two kinds of sandwiches, roast beef and peanut butter, and two kinds of pizza, cheese and vegetarian. There are 60 servings of pizza prepared. There are eight fewer roast beef sandwiches prepared than there are servings of cheese pizza. There are six more peanut butter sandwiches prepared than there are servings of vegetarian pizza. Altogether, how many servings of sandwiches are prepared?
   A) List eight quantities involved in this problem.
   B) Sketch a diagram to show the relevant sums and differences in this situation.
   C) Solve the problem.

49. A local community college has two sections of Math 210 (Sections A and B), and two sections of Math 211 (Sections C and D). Together, Sections C and D have 46 students. Section A has six more students than Section D. Section B has two fewer students than Section C. How many students are there in Section A and Section B altogether?
   A) For each given value, write the quantity next to it.
   B) Sketch a diagram to show the relevant sums and differences in this situation.
   C) Solve the problem. Show all your work here.

50. A first grade teacher always reads subtraction statements such as “7 − 5 = 2” to his class as “seven take away five is two.” That is, he always reads the minus sign as “take away.” Comment on why this might not be a good practice.

51. Write a missing-addend problem using $35.95 and $19.50.

52. Suppose you are using toothpicks to act out the following story problem:
   Jack had eight candy bars. Bill had four.
   A) How many more candy bars did Jack have than Bill?
   B) How many toothpicks would you need to act the problem out? Explain your answer. What type of subtraction is this?

53. Rita is given this problem:
   Zetta has $39, but she needs $78 to buy a jacket she wants. How much more does she need?
   Rita's reply: “79 minus 40 is 39, so she needs $39.”
   Explain Rita's reasoning. What is your reaction to this method of doing the problem?

54. A) Make drawings of circular “pizzas” to illustrate 6 − 2, take-away view.
   B) A child is shown nine apples and six oranges and asked “How many more apples than oranges?” She says that apples and oranges are different things, and so she doesn’t understand the question. What might you do to help her?

55. Finish the story so that your question could be answered by the given calculation, and so that your story involves the view given.
   A) 6 − 2.5, missing addend. The two joggers decided to run at the beach...
   B) 6 − 2.5, comparison. The two joggers decided to run at the beach...

56. Give the rest of the “family of facts” for $k − 3 = p.$
57. A visitor to a first-grade classroom saw a teacher ask a child to solve this problem: *Jaime gets $5 a week for keeping the yard in good shape. He is saving his money for the county fair. After four weeks, how much has he saved?*

A) She thinks to herself: This is a multiplication problem, and first-graders have not yet been taught multiplication, so they can't answer this problem.” But after a few minutes, Li-Li say that the answer is 20. She explains how she did this problem and she did not do any formal multiplication, much to the visitor's surprise. What did she most likely do to find the answer?

B) This visitor also saw another problem the children worked: “8 miles of highway are being paved. If the workers pave 2 miles a day, how long will it take them to pave all 8 miles?” She thought: “This is a division problem and first graders have not yet learned to divide.” But then Belinda said that it would take four days. How do you suppose she explained this answer, without using division?

58. A second-grade boy is asked to subtract 64 – 55, written vertically. The child thinks about the problem and then writes 9. He explains his thinking by saying, “6 take away 5 is 1, I mean, 60 take away 50 is 10. 5 take away 4 is 1, and 10 take away 1 is 9.” Is he correct? Use his thinking to find 243 – 124.

59. Find 21 + 49 using an empty number line.

60. Find 509 – 239 using an empty number line.

61. Mitchell decides to get his car painted and to buy new hubcaps. He selects five colors he likes and three styles of hubcaps. Then he decides to paint the roof a different color than the body. He decides to let his wife make the final decision. How many choices does she have? Explain your answer.

62. A clothes designer comes up with women’s “mix and match” items with two styles of skirts, one pair of pants, three types of tops, and two styles of jackets. How many different outfits could be purchased, if each outfit has a skirt or pants, a top, and a jacket?

63. Give our label (e.g., take-away, etc.) for the situation, and write the equation for solving this problem.

A coffee shop has four kinds of pastries that you like. You always drink coffee, tea, or milk with your pastry. In how many ways could you place a pastry-plus-drink order?

64. Write a word problem for $37 ÷ 5$ for which the answer would be 2.

65. Write two word problems about cars so that the first problem shows the repeated subtraction meaning of division, while the second problem shows the partitive or sharing meaning of division.
66. Antonio asks, “When I multiply [for example, 49 × 23, shown below], why do I have to put in the 0 [points to the zero in 980]?”
What would you say to Antonio?

\[
\begin{array}{c}
49 \\
\times 23 \\
147 \\
980 \\
1127 \\
\end{array}
\]

67. Is this child's thinking all right? If it is, complete the second calculation using the child's method. If the thinking is not all right, explain why not.

(Child's work)  
Second calculation (or explanation if not ok)

\[
\begin{array}{c}
124 \\
\times 15 \\
1000 \\
200 \\
40 \\
500 \\
100 \\
\_ 20 \\
1860 \\
\end{array}
\]

68. Following is an example of a child's work. You are to study the work and then to judge the student's understanding.

Hiro was asked to divide 4240 by 6. His work is shown below.

Hiro’s work:

\[
\begin{array}{c}
\underline{7 6} \div 4 \\
6 \underline{4} \\
42 \\
\underline{42} \\
040 \\
\underline{36} \\
4 \\
\end{array}
\]

A) Is Hiro's work correct or incorrect?
B) If the work is incorrect, please explain how.
69. Consider the following work of a student:

\[
\begin{array}{c}
84 \\
\times \quad 45 \\
\hline \\
\quad 20 \\
\quad 400 \\
\quad 160 \\
\quad 320 \\
\hline \\
\quad 900 \\
\end{array}
\]

A) There is an error with the 20.
B) There is an error with the 400.
C) There is an error with the 160.
D) There is an error with the 320.
E) There is no error with this student's work.

70. In each pair, choose the larger. Explain your reasoning. Your justification should appeal to number and operation sense, not to computation. You cannot use a calculator or actually calculate the answer.

A) 1838 + 517 or 1836 + 514
B) 612 – 29 or 613 – 34
C) 0.578 or 0.002 + 0.0328

71. Find the sum of the digits for \(1111112^2\) using inductive reasoning.
**Answer Key**

1. The length of this room is a quantity. A possible value is 20 feet.
2. Height, weight, popularity…
3. It will be difficult for your students to avoid algebra or trial-and-error on this problem; decide whether you wish to prohibit the use of algebra. You might also consider omitting part C.
   A) E.g., number of kinds of sandwiches, number of kinds of pizza, number of servings of pizza prepared, difference in number of tuna sandwiches prepared versus number of servings of pepperoni pizza, difference in number of ham sandwiches prepared versus number of servings of vegetarian pizza, total number of sandwiches prepared, number of tuna sandwiches prepared, number of ham sandwiches prepared, number of servings of pepperoni pizza, number of servings of vegetarian pizza, difference in number of tuna sandwiches and number of ham sandwiches…
   B) There are other praiseworthy drawings possible, but the following suggests the solution (for the total number of sandwiches) pretty easily.

   ![Diagram](image)

   C) There are 52 sandwiches prepared in all \((8 + 48 - 4)\).
4. Samples (quantity, value or unit if value unknown)
   Distance between stations, 217 miles
   Speed of one train, 65 miles per hours
   Speed of other train, 72 miles per hour
   Total speed of the two trains, miles per hour
   Time until trains meet, hours (or minutes)
   Distance first train has traveled when they meet, miles
   Distance second train has traveled when they meet, miles
5. A) E.g., weight of large dog, weight of medium dog, weight of small dog, large dog's weight in terms of small dog's weight; small dog's weight in terms of medium dog's weight
   B) Large dog: 5 times weight of small dog (S)
   Small dog: 1S (which is 2/3 M)
   Medium dog: (M is 3/2 of S)
   C) The diagram does not tell anything about their sizes other than which was larger and smaller than the medium dog.
   D) From the diagram for part B, if the medium dog is 9 pounds more than the small dog, then the medium dog weighs 27 pounds and the small dog weighs 18 pounds. The large dog weighs five times as much as the small dog, so it is \(5 \times 18 = 90\) pounds.
6. 1,400,000 mg
7. B) False. The larger the unit, the smaller the numerical value will be for describing the same measurement.
8. A) 2300  
    B) 0.00002  
    C) 0.00214  
9. Centi  
10. 4000  
11. The metric system allows easy conversion of units because units differ by powers of ten. It is used in science for this reason, in all countries. Most countries use it for all measures.
12. 
   A) European  
   Asian  
   444 in all, so 111 Asian and 333 European  
   B) Chairs with arm rests  
   Chairs without arm rests  
   Table  
   \[
   2 \times \$45 + 4 \times (\$45 - \$8.50) + 4 \times (\$45 - \$8.50) = \$382
   \]
   C) Joe  
   Jim  
   8 miles  
   2 miles  
   Jim drives 10 miles one way, so 20 miles both ways, and Joe drives 16 miles both ways, so together they drive 36 miles per day.  
   D) 18 in third grade, 11 in fourth grade  
   3rd grade  
   4th grade  
   29 students in all  
   (The two strips of equal size must represent 22 students.)
13. A  
14. 765.4, 765  
15. 10.3, 10  
16. In base ten, 215.687 is exactly 215.687 ones, exactly 21.5687 tens, exactly 2.15687 hundreds, and exactly 0.215687 thousands; also, 3421 is exactly \( \frac{34210}{100} \) tenths and exactly \( \frac{342100}{1000} \) hundredths.  
17. 9, 11  
18. B) False. It has 2300 hundredths in it.  
19. 225.1  
20. 152, because there are 152 hundreds in 15,287.  
21. Grady is reasoning as though the numbers were whole numbers. Grady does not recognize that 4 tenths will be bigger than 3 tenths and only 6 hundredths.  
22. B) False. There are \( b \) digits: 0, 1, 2, 3, ..., \( b - 1 \).  
23. B) False. 7 is not a digit used in a base seven place-value system.  
24. 2013\(_b\)
25. $340_{\text{ten}}$
26. $10133_{\text{four}}$
27. $132_{\text{ten}}$
28. $200_{\text{four}}$
29. $2 \frac{1}{16}_{\text{ten}}$
30. $20.2_{\text{three}}$
31. $59.6_{\text{ten}}$
32. $1303.3_{\text{five}}$
33. $253_{\text{ten}}$
34. 123. As in base ten, $1230_{\text{five}} = 123$ fives.

So, in base five, $1234.2_{\text{five}}$ is: 123 fives + something small.
35. 1999 and 2000 (either order)
36. 1444 and 2000 (either order)
37. 2320, 2321, 2322, 2323, 2324, and 2330
38. With large dot representing a small cube as the unit

![Diagram of cubes](image)

(small cube = 1; no longs)

large cube = $7^3 = 343$, or three hundred forty-three
flat = $7^2 = 49$, or forty-nine
small “cube” = 1, or one

39. A
40. $420_{\text{six}}$
41. $58_{\text{nine}}$
42. $13.2_{\text{five}}$
43. $0.06_{\text{seven}}$
44. $100.2_{\text{six}}$
45. The small square is being used as the unit.

```
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

The five small squares can be traded for a five (long) leaving 0 ones. There are now six longs. Five would be traded for a flat of 25, leaving one five (long). The answer is therefore $110_{\text{five}}$.

46. There are several possible arrangements. Below is one that helps to see that the difference asked for (BT vs. B2T) is 18 inches. Students may assign an arbitrary number to the height of B’s tallest player rather than rely on their drawing. Point out that they have unnecessarily (probably) ignored their drawing in arriving at their answer.
47. The five quantities are usually easy: E.g., the number for each type of candy, and some of the explicit comparisons mentioned. Here is a diagram, with the deduced numbers of bars in parentheses, giving 18 TRs (start with the HBs, then determine the RCs, then the TR + MW total, and finally the TRs).

48. Again, depending on whether you have used the earlier, similar problem, many of your students will use algebra or trial and error on this problem. We suggest, for now, prohibiting algebra. You might also consider omitting part C. But the problem can be solved with the use of a drawing, as seen below.

A) Number of kinds of sandwiches, number of kinds of pizza, number of servings of pizza prepared, difference in number of roast beef sandwiches prepared versus number of servings of cheese pizza, difference in number of peanut butter sandwiches prepared versus number of servings of vegetarian pizza, total number of sandwiches prepared, number of roast beef sandwiches prepared, number of peanut butter sandwiches prepared, number of servings of cheese pizza, number of servings of vegetarian pizza, difference in number of roast beef sandwiches and number of peanut butter sandwiches, total number of servings of pizza and sandwiches…

B) There are other praiseworthy drawings possible, but the following suggests the solution (for the total number of pizza servings and sandwiches) pretty easily.

C) The number of sandwiches is (60 + 6) – 8 = 58.

49. A) 46 students, total number of students in C and D
   6 students, difference in numbers of students in A and D
   2 students, difference in numbers of students in B and C
B) (sample drawing)

C) \((46 - 2) + 6 = 50\) students for Sections A and B together

50. Reading “–” only as “take away” ignores the fact that other situations—an additive comparison and missing addend—might also involve subtraction.

51. Various possibilities. Each should involve an addition situation describable by \(19.50 + n = 35.95\) (or \(n + 19.50 = 35.95\)).

52. A) Four
   B) 12, because there are the two separate amounts; this situation involves an additive comparison.

53. Rita has increased the minuend and subtrahend by the same amount, so the difference stays the same. (Think of a comparison subtraction drawing, even though this is a missing-addend setting.)

54. A) The drawing should show six circles, with two being removed by arrows or otherwise marked out in some way.
   B) (One possible way....) Line up the apples and then the oranges in a 1-to-1 ratio. Ask how many apples don't have an orange partner and then ask whether there are more apples than oranges, and how many more.

55. A) They usually run 6 miles. How much farther do they have to run, if they have already run 2.5 miles?
   B) One runs 6 miles and the other runs 2.5 miles. How much farther does the first jogger run than the second jogger does?

56. Any order: \(3 + p = k\) \(p + 3 = k\) \(k - p = 3\)

57. A) She probably used repeated addition: 5 and 5 is 10, and 5 more is 15, and 5 more is 20.
   B) She probably subtracted 2 from 8 four times, until she reached 0, then counted the number of times she subtracted 2—4 times.

58. He is correct. \(200 - 100\) is 100. \(40 - 20\) is 20. \(4 - 3\) is 1. \(120 - 1\) is 119.

59. Here is one.

Other methods might make sense as well (e.g., \(21 + 9 = 30\); \(30 + 40 = 70\)).

60.
61. \((5 \times 4) \times 3 = 60\), assuming color compatibility.

62. \((2 + 1) \times 3 \times 2 = 18\)

63. Fundamental counting principle. \(4 \times 3 = n\)

64. Thirty-seven children want to play a game that involves teams of five players. How many children won't be on a team?

65. (Repeated subtraction, or measurement) The big bag has 48 plastic cars, to be put into bags holding six cars each. How many bags of cars will there be? (Partitive, or sharing) The big bag has 48 plastic cars, to be split fairly among six youngsters. How many cars will each youngster get?

66. The 980 comes from \(20 \times 49\), the 2 is the number of tens in 20, so \(2 \times 49\) is a number of tens. The 2 is in the tens column, so we must start in the tens column.

67. Yes it is okay. Second calculation (in columns): 
\[1000 + 300 + 20 + 400 + 120 + 8 = 1848.\]

68. A) Hiro's work is incorrect.
   B) In considering the 04 (the number of tens left), Hiro forgot to note in the quotient that there are 0 tens for \(40 \div 6\).

69. D

70. A) 1838 + 517 because each addend is larger than the corresponding one in 1836 + 514
   B) 612 – 29 because 612 – 29 is the same as 613 – 30, OR 613 is only 1 more than 612, but subtracting 34 rather than 30 more than overcomes that.
   C) 0.578 because the sum of the addends in the second sum will not reach 0.5.

71. 64