4.1 Interior angles of a triangle.

Example:

\[ a + 70 + 35 = 180 \degree \]

So \( a = 75 \degree \)

1–3. Find the missing measurements.

1. \( a = \______ \)

2. \( b = \______ \)

3. \( a = \______ \)

Triangle Sum Conjecture: The sum of the measures of the angles in every triangle is 180 degrees.

Here is why:

4. In a right triangle the two acute angles are \______ angles.

Draw a triangle and think about the answer.
4.2 Properties of an Isosceles Triangle. Two sides of the same length

Use your ruler, compass and protractor to confirm the following and mark the picture accordingly.

a) Two sides are equal in measure.

b) Two angles are congruent.

Properties of an Equilateral Triangle. All sides of the same length

Use your ruler, compass and protractor to confirm the following and mark the picture accordingly.

a) Three sides are equal in measure.

b) Three angles are congruent

1. \( d = \), \( e = \), \( f = \)

2. \( j = \), \( k = \), \( l = \)

3. The perimeter of triangle \( NBC \) is 555m. Find the length of side \( NB \).
4. Construct an isosceles triangle and then an equilateral triangle with only a compass and straight edge. Use the back of the previous page.

5. Construct an isosceles triangle with a vertex angle that measures 30 degrees.
4.3 Triangle Inequalities

Using your compass and a straight edge construct a triangle using the following lines:

__________________________

__________________________

__________________________

Using your compass and a straight edge construct a triangle using the following lines (if not possible then state why):

__________________________

__________________________

__________________________
Measure the lengths of the sides of each triangle

**Triangle Inequality Conjecture** The sum of the lengths of any two sides of a triangle is__________ than the length of the third side.

*An extension of the Triangle Inequality Conjecture: It is only possible to draw a triangle with three sides, if the sum of the shortest sides is ______________ than the longest side.

4-6. Is it possible to draw a triangle with sides of the given lengths? Answer yes or no.
4. 3, 4, 5 _________
5. 1, 7, 8 _________
6. 3, 5, 9 _________

**1. Side-Angle Inequality Conjecture** In a triangle, if one side is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

2–3. Arrange the letters’ values in order from greatest to least.
2. ________
3. ________
4. ________
2. **Triangle Exterior Angle Conjecture** The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

Why is this true?

7. \( x + y = \)

8. \( a = \)

4.4 and 4.5 **TRIANGLE CONGRUENCE CONJECTURES**.

**Ways to show two Triangles are Congruent:**

- Side-Side-Side
- Side-Side-Angle
- Side-Angle-Angle
- Angle-Angle-Angle
- Angle-Side-Angle
- Side-Angle-Side
Construct a triangle, more than one if possible, using the following:

a) \[ \begin{array}{c}
\text{A} \\
\hline
\text{B} \\
\hline
\text{C} \\
\end{array} \]

b) \[ \begin{array}{c}
\text{A} \\
\hline
\text{B} \\
\hline
\text{C} \\
\end{array} \]

c) \[ \begin{array}{c}
\text{A} \\
\hline
\text{B} \\
\hline
\text{C} \\
\end{array} \]

Draw a triangle with the following interior angles 45, 45, 90.

Why did SSA not work?

Why did AAA not work?
Mark the following triangles given the stated congruent triangle conjecture.

$\triangle ABC \cong \triangle DEF$

<table>
<thead>
<tr>
<th>SSS</th>
<th>SAA</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="SSS Diagram" /></td>
<td><img src="image2" alt="SAA Diagram" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASA</th>
<th>SAS</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="ASA Diagram" /></td>
<td><img src="image4" alt="SAS Diagram" /></td>
</tr>
</tbody>
</table>

Determine which triangles are congruent and why?

Using your previous conjectures mark congruent angles and sides if they exist and then find the corresponding congruent triangle. State the reason the two triangles are congruent.

a. $\triangle ADB \cong \triangle \text{_______}$
b. $\triangle STU \cong \triangle \text{_______}$
c. $\triangle EFX \cong \triangle \text{_______}$
1. \( \triangle OHW \cong \triangle \underline{\text{}} \) 
Which conjecture supports the congruence statement?(if not enough info, write not enough info) 

\[\triangle ABC \cong \triangle \underline{\text{}}\] 

2. \( \text{sAB} \parallel \text{sCD} \) (hint: transversal?), \( \text{sAD} \cong \text{sBC} \), \( \triangle ABC \cong \triangle \underline{\text{}} \)
There is not enough info here, why? 

3. \( \text{sAC} \cong \text{sBD} \), \( \text{sAD} \cong \text{sBC} \), \( \triangle ADB \cong \triangle \underline{\text{}} \)
Which conjecture supports the congruence statement? 

\[\triangle ADB \cong \triangle \underline{\text{}}\]
5. \( s_{QD} \cong s_{AD}, \ s_{QU} \cong s_{AU} \) If you construct segment \( DU \), you can show \( \Delta QDU \cong \Delta ADU \). Which conjecture tells you they are congruent? ______

![Diagram of triangle QDU with segment DU](image)

6. \( s_{AB} \cong s_{CD}, \ \angle CDB \cong \angle ABD \) Which conjecture tells you that \( \Delta ABD \cong \Delta CDB \)? ______

![Diagram of quadrilateral ABCD with diagonal AC](image)
In Exercises 1–3, name the conjecture that leads to each congruence.

1. \( \triangle PAT \cong \triangle IMT \)

\[ \begin{array}{c}
\text{A} \\
\text{T} \\
\text{P} \\
\text{M} \\
\text{I} \\
\end{array} \]

2. \( \triangle SID \cong \triangle JAN \)

\[ \begin{array}{c}
\text{S} \\
\text{D} \\
\text{A} \\
\text{N} \\
\text{T} \\
\end{array} \]

3. \( \overline{TS} \) bisects \( \overline{MA} \), \( \overline{MT} \cong \overline{AT} \), and \( \triangle MST \cong \triangle AST \)

In Exercises 4–9, name a triangle congruent to the given triangle and state the congruence conjecture. If you cannot show any triangles to be congruent from the information given, write “cannot be determined” and redraw the triangles so that they are clearly not congruent.

4. \( M \) is the midpoint of \( \overline{AB} \) and \( \overline{PQ} \).

\( \triangle APM \cong \triangle \) \[ \begin{array}{c}
\text{P} \\
\text{M} \\
\text{A} \\
\text{Q} \\
\end{array} \]

5. \( KITE \) is a kite with \( KI = TI \).

\( \triangle KIE \cong \triangle \) \[ \begin{array}{c}
\text{I} \\
\text{E} \\
\text{K} \\
\end{array} \]

6. \( \triangle ABC \cong \triangle \) \[ \begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{X} \\
\text{Y} \\
\end{array} \]

7. \( \triangle MON \cong \triangle \) \[ \begin{array}{c}
\text{M} \\
\text{O} \\
\text{N} \\
\text{T} \\
\text{Q} \\
\end{array} \]

8. \( \triangle SQR \cong \triangle \) \[ \begin{array}{c}
\text{Q} \\
\text{S} \\
\text{R} \\
\text{U} \\
\text{T} \\
\end{array} \]

9. \( \triangle TOP \cong \triangle \) \[ \begin{array}{c}
\text{O} \\
\text{P} \\
\text{T} \\
\text{D} \\
\end{array} \]

10. Which conjecture supports each congruence statement?

\( \triangle HFB \cong \triangle DFB \) \[ \begin{array}{c}
\text{H} \\
\text{F} \\
\text{B} \\
\end{array} \]

\( \triangle ABH \cong \triangle CBD \) \[ \begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\end{array} \]

\( \triangle HGF \cong \triangle DEF \) \[ \begin{array}{c}
\text{H} \\
\text{G} \\
\text{F} \\
\end{array} \]
4.6 CPCTC

1. \( \triangle ABC \cong \triangle \) _______.
   
   Which conjecture supports the congruence statement? ________

Because of ___________ both triangles are congruent. Which means

\( sAC \cong sDB, \quad \angle CBD \cong \angle BCA, \quad \text{and} \quad \angle CAB \cong \angle BDC \)

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CPCTC ----- Corresponding Parts of Congruent Triangles are Congruent

2. \( \triangle ABC \cong \triangle \) _______.
   
   Which conjecture supports the congruence statement? ________

Because of ___________ both triangles are congruent. Which means

\( sAB \cong sED, \quad sBC \cong sDC, \quad \angle B \cong \angle D, \quad \text{and} \quad \angle DCE \cong \angle BCA \)

---

3. \( sAB \perp sCD, \quad sAB \cong sCD, \quad EC \perp sCD, \quad \angle A \cong \angle D, \quad \triangle ABC \cong \triangle \) _______.
   
   Which conjecture supports the congruence statement? (remember to split the triangles) ________

Because of ___________ both triangles are congruent. Which sides and angles are congruent?
1. Give the shorthand name for each of the four triangle congruence conjectures.

In Exercises 2–5, use the figure at right to explain why each congruence is true. WXYZ is a parallelogram.

2. \( \angle WXZ \cong \angle YZX \)
3. \( \angle WZX \cong \angle YXZ \)

4. \( \triangle WZX \cong \triangle YXZ \)
5. \( \angle W \cong \angle Y \)

For Exercises 6 and 7, mark the figures with the given information. To demonstrate whether the segments or the angles indicated are congruent, determine that two triangles are congruent. Then state which conjecture proves them congruent.

6. M is the midpoint of WX and YZ. Is \( \overline{YW} \cong \overline{ZX} \)? Why?

7. \( \triangle ABC \) is isosceles and \( \overline{CD} \) is the bisector of the vertex angle. Is \( \overline{AD} \cong \overline{BD} \)? Why?

In Exercises 8 and 9, use the figure at right to write a paragraph proof for each statement.

8. \( \overline{DE} \cong \overline{CF} \)
9. \( \overline{EC} \cong \overline{FD} \)

10. TRAP is an isosceles trapezoid with \( TP = RA \) and \( \angle PTR \cong \angle ART \). Write a paragraph proof explaining why \( \overline{TA} \cong \overline{RP} \).
4.7 FLOWCHART THINKING

1. Provide each missing reason or statement in the flow-chart proof.

Given: \( sHR \cong sSE \)
\( \angle R \cong \angle S \)

Show: \( sHU \cong sEU \)

Flow-chart Proof:

\( \overline{HR} \cong \overline{SE} \)
Given

\( \angle R \cong \angle S \)
Given

\( \angle HUR \cong \angle EUS \)
Given

\( \triangle HRU \cong \triangle ESU \)

3. \_

\( \overline{HU} \cong \overline{EU} \)

4. \_

5. From the picture fill in the given and

Prove: \( \angle O \cong \angle K \)

\( \_
\)

\( \_
\)

\( \_
\)
6. Find the two congruent triangles and draw them next to each other, then write a proof.

Given: \( \triangle PRS \cong \triangle PQS \)
\( \triangle TPS \cong \triangle TPS \)

Prove: \( \angle PRT \cong \angle PQS \)

7. From the picture fill in the given and

Prove: \( \triangle OW \cong \triangle YW \)

8. Given: \( \angle D \cong \angle C \)
\( \triangle SDE \cong \triangle SEC \)

Prove: \( \triangle AEB \cong \triangle BAC \)
Complete the flowchart for each proof.

1. Given: $PQ \parallel SR$ and $PQ \equiv SR$
   
   Show: $SP \equiv QR$
   
   Flowchart Proof
   
   [Flowchart diagram]

2. Given: Kite $KITE$ with $KE \equiv KT$
   
   Show: $KT$ bisects $\angle EKI$ and $\angle ETI$
   
   Flowchart Proof
   
   [Flowchart diagram]

3. Given: $ABCD$ is a parallelogram
   
   Show: $\angle A \equiv \angle C$
   
   Flowchart Proof
   
   [Flowchart diagram]
4.8 More Properties of an Isosceles Triangle. Two sides of the same length

Use your ruler, compass and protractor to confirm the following and mark the picture accordingly.

a) Two sides are equal in measure.
b) Two angles are congruent
c) The bisector of the vertex angle is the *altitude* and the *median* to the base

1. The following triangles are isosceles.

a) A=

b) B=

c) C=
In Exercises 1–3, use the figure at right.

1. \( \overline{CD} \) is a median, perimeter \( \triangle ABC = 60 \), and \( AC = 22 \). \( AD = \) _____

2. \( \overline{CD} \) is an angle bisector, and \( m \angle A = 54^\circ \). \( m \angle ACD = \) _____

3. \( \overline{CD} \) is an altitude, perimeter \( \triangle ABC = 42 \), \( m \angle ACD = 38^\circ \), and \( AD = 8 \).
   \( m \angle B = \) _____, \( CB = \) _____

4. \( \triangle EQU \) is equilateral.
   \( m \angle E = \) _____

5. \( \triangle ANG \) is equiangular and perimeter \( \triangle ANG = 51 \).
   \( AN = \) _____

6. \( \triangle ABC \) is equilateral, \( \triangle ACD \) is isosceles with base \( \overline{AC} \), perimeter \( \triangle ABC = 66 \), and perimeter \( \triangle ACD = 82 \).
   Perimeter \( ABCD = \) _____

7. Complete a flowchart proof for this conjecture: In an isosceles triangle, the altitude from the vertex angle is the median to the base.
   **Given:** Isosceles \( \triangle ABC \) with \( \overline{AC} \equiv \overline{BC} \) and altitude \( \overline{CD} \)
   **Show:** \( \overline{CD} \) is a median

   **Flowchart Proof**

   - \( \overline{CD} \) is an altitude
   - \( \angle ADC \) and \( \angle BDC \) are right angles
   - Definition of angles
   - \( \angle ADC \equiv \angle BDC \)
   - \( \overline{AC} \equiv \overline{BC} \)
   - Given

8. Write a flowchart proof for this conjecture: In an isosceles triangle, the median to the base is also the angle bisector of the vertex angle.
   **Given:** Isosceles \( \triangle ABC \) with \( \overline{AC} \equiv \overline{BC} \) and median \( \overline{CD} \)
   **Show:** \( \overline{CD} \) bisects \( \angle ACB \)