Simplifying Radicals:

a) \[ \sqrt{12} = \sqrt{4 \cdot 3} = 2 \sqrt{3} \]
b) \[ \sqrt{60} = \sqrt{4 \cdot 15} = 2 \sqrt{15} \]
c) \[ \sqrt{112} = \sqrt{4 \cdot 28} = 2 \sqrt{28} \]
d) \[ (2\sqrt{3})^2 = 4 \cdot 3 = 12 \]
e) \[ (7\sqrt{2})^2 = 49 \cdot 2 = 98 \]

Solve:

a) \[ x^2 + 4 = 9 \]
   \[ x^2 = 5 \]
   \[ x = \pm \sqrt{5} \]
   \[ x = \sqrt{5} \]

b) \[ 16 + a^2 = 80 \]
   \[ a^2 = 64 \]
   \[ a = 8 \]

9.1 PYTHAGOREAN THEOREM (right triangles)

If a triangle is a right triangle then \[ a^2 + b^2 = c^2 \]
a, b are the legs
* c is called the hypotenuse (side opposite the right angle)
* c is also the longest side

1. \[ h = \_ \]
2. \[ r = \_ \]
3. Find the area of the triangle. Area = _________

\[ A = \frac{1}{2} b \cdot h \]

\[ A = \frac{1}{2} (24) (18) \]

\[ A = 216 \ \text{in}^2 \]

4. What is the length of the diagonal shown in the rectangular box below? Length = _________

\[ 15^2 + 20^2 = d^2 \]

\[ 25^2 = d^2 \]

\[ d = 25 \text{ ft} \]

Give all answers rounded to the nearest 0.1 unit.

1. \( a = _____ \)

2. \( p \approx _____ \)

3. \( x = _____ \)

4. Area = 39 in\(^2\)

\[ h \approx _____ \]

5. Find the area.

6. Find the coordinates of \( C \) and the radius of circle \( A \).

7. Find the area.

8. \( RS = 3 \) cm. Find \( RV \).

9. Base area = 16\( \pi \) cm\(^2\) and slant height = 3 cm. What’s wrong with this picture?

10. Given \( \triangle PQR \), with \( m \angle P = 90^\circ \), \( PQ = 20 \) in., and \( PR = 15 \) in., find the area of \( \triangle PQR \), the length of the hypotenuse, and the altitude to the hypotenuse.
9.2 IS THE CONVERSE TRUE?

Yes, If the lengths of the three sides of a triangle satisfy the Pythagorean equation, then the triangle is a right triangle.

If \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.

The three sides that work in the Pythagorean formula are called **Pythagorean Triples**.

Examples of Pythagorean triples: (remember longest side is the hypotenuse)

\[
\begin{array}{ccc}
3-4-5 & 9-12-15 & 12-16-20 \\
\begin{align*}
2^2 + 4^2 &= 5^2 \\
5 &= 5 & \text{yes}
\end{align*} & \\
\begin{align*}
9^2 + 12^2 &= 15^2 \\
225 &= 225 & \text{yes}
\end{align*} & \\
\begin{align*}
a^2 + b^2 &= c^2 \\
225 &= 225 & \text{yes}
\end{align*}
\end{array}
\]

2. Is \( \triangle \text{PYT} \) a right triangle? (yes/no) = **NO**

\[
8^2 + 6^2 = 12^2 \\
100 = 144
\]

**Double, Triple,....**

The following are examples of **Pythagorean Primitives** (Three lengths of a right triangle with no common factors):

\[
\begin{array}{cccc}
3-4-5 & 8-15-17 & 5-12-13 & 7-24-25 \\
\end{array}
\]

Create two new right triangle side lengths for each of the primitives.

<table>
<thead>
<tr>
<th>Double</th>
<th>Triple</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4-5</td>
<td>9-12-15</td>
</tr>
<tr>
<td>8-15-17</td>
<td>24-45-5</td>
</tr>
</tbody>
</table>
1. Find the area of a right triangle with a hypotenuse that measures 17 cm and one leg that measures 15 cm.

2. Find the area of triangle ABC.

3. Find x. Explain your method.

4. Find the area of ABCD.

5. In Exercises 9–11, determine whether ABCD is a rectangle and justify your answer. If not enough information is given, write “cannot be determined.”

6. What’s wrong with this picture?

7. How high up on a building will a 15-foot ladder reach if the foot of the ladder is placed five feet from the building?
9.3 TWO SPECIAL RIGHT TRIANGLES

Number one: **Isosceles Right Triangle**

\[
\begin{align*}
\sqrt{2}a^2 &= c^2 \\
2a^2 &= c^2 \\
a\sqrt{2} &= c
\end{align*}
\]

![Isosceles Right Triangle Diagram]

1. \( c = \phantom{00} \)

2. \( q = \phantom{00} \sqrt{2} \)

Number two: **30-60-90 triangle**

Label the legs as:
Shortest, Middle, Longest

Fill in the Angles:

![30-60-90 Triangle Diagram]
Lets rewrite it without fractions:

\[ \sqrt{a^2} = \sqrt{3y^2} \]
\[ a = y\sqrt{3} \]

1. \( a = 5 \)

2. Find the area of the triangle below. Area = _______

\[ A = \frac{1}{2} \cdot b \cdot h \]
\[ A = \frac{1}{2}(8) \cdot 4\sqrt{3} \]
\[ A = 16\sqrt{3} \text{ in}^2 \]

2. Find \( b \) using the quadratic formula

\[ a^2 + b^2 = c^2 \]

b = _______

3. \( r = \frac{20\sqrt{3}}{} \)

\[ r = 20\sqrt{3} \]
4. What is the length of the hypotenuse of a 30-60 right triangle with a longer leg of length 16 m?

5. Find the area of an isosceles triangle with a base length of 12 cm and each of the congruent sides having lengths 10 cm.
   Area = 

6. Find the area of an equilateral triangle with sides measuring 6 meters.

Give your answers in exact form unless otherwise indicated. All measurements are in centimeters.

In Exercises 1–3, find the unknown lengths.

1. \( a = \quad \), \( b = \quad \)

2. \( a = \quad \), \( b = \quad \)

3. \( a = \quad \), \( b = \quad \)

4. Find the area of rectangle \( ABCD \).

5. Find the perimeter and area of \( KLMN \).

6. \( AC = \quad \), \( AB = \quad \), and area \( \triangle ABC = \quad \).

7. Find the area of an isosceles trapezoid if the bases have lengths 12 cm and 18 cm and the base angles have measure 60°.
Isosceles Right Triangle

\[
\begin{align*}
\text{a} & \quad \text{a} \\
45^\circ & \quad 45^\circ \\
\text{a} \sqrt{2} &
\end{align*}
\]

30-60-90 triangle

\[
\begin{align*}
30 & \quad 2x \\
60 & \quad x \\
\end{align*}
\]

In an Isosceles triangle, the **median** from the vertex angle to the base is also a **perpendicular bisector**, **angle bisector**, and an **altitude**. The base angles are also congruent.

1. The following is an equilateral triangle. Find \( x = \) ____ \( y = \) ____

Look at the centroid packet!

\[x = \frac{5\sqrt{3}}{3} \]
\[y = \frac{10\sqrt{3}}{3}\]
2. Find the area of the equilateral triangle using 30-60-90 triangles.

\[ A = \frac{1}{2} b \cdot h = \frac{1}{2} (6)(3\sqrt{3}) = 9\sqrt{3} \text{ cm}^2 \]

3. \( AB = 10 \text{ cm} \), Find the area of the equilateral triangle (altitude=median=angle bisector). Find the area of the circumscribed circle. (Hint: the perpendicular bisector of any chord goes through the center of the circle).

\[ A = \frac{1}{2} (10)(5\sqrt{3}) = 25\sqrt{3} \text{ cm}^2 \]
\[ A = \pi R^2 = \pi \left(\frac{10\sqrt{3}}{3}\right)^2 = \frac{100\pi}{3} \text{ cm}^2 \]

4. \( AB = 5\sqrt{3} \text{ cm} \), Find the area of the equilateral triangle, the area of the Circumscribed circle, and the area of the inscribed circle.

\[ r = \frac{2}{3} \left(\frac{15}{2}\right) = 5 \]
\[ A = \pi (5)^2 = 25\pi \text{ cm}^2 \]

\[ r = \frac{1}{3} \left(\frac{15}{2}\right) = \frac{5}{2} \]
\[ A = \frac{1}{2} \left(5\sqrt{3}\right) \left(\frac{15}{2}\right) = \frac{25\sqrt{3}}{4} \pi \text{ cm}^2 \]
9.4 WORD PROBLEMS

A 25-foot ladder is placed against a building. The bottom of the ladder is 7 feet from the building. If the top to the ladder slips down 4 feet, how many feet will the bottom slide out? No, it is not 4 feet. This is a two-step problem, so draw it with two right triangles.

\[ h^2 + b^2 = 25^2 \]
\[ 7^2 + h^2 = 625^2 \]
\[ h = 24 \]

\[ 20^2 + b^2 = 25^2 \]
\[ 400 + b^2 = 625 \]
\[ b = 15 \]

1. A 20 ft ladder reaches a window 18 ft high. How far is the foot of the ladder from the base of the building? How far must the foot of the ladder be moved to lower the top of the ladder by 2 ft?

2. Robin and Dovey have four pet pigeons that they train to race. They release the birds at Robin's house and then drive to Dovey's to collect them. To drive from Robin's to Dovey's, because of one-way streets, they go 3.1 km north, turn right and go 1.7 km east, turn left and go 2.3 km north, turn right and go 0.9 km east, turn left and go 1.2 km north, turn left and go 4.1 km west, and finally turn left and go 0.4 km south. How far do the pigeons have to fly to go directly from Robin's house to Dovey's house?

3. Hans needs to paint the 18 in.-wide trim around the roof eaves and gable ends of his house with 2 coats of paint. A quart can of paint covers 175 ft² and costs $9.75. A gallon can of paint costs $27.95. How much paint should Hans buy? Explain.

4. What are the dimensions of the largest 30°-60°-90° triangle that will fit inside a 45°-45°-90° triangle with leg length 14 in.? Sketch your solution.
9.5 DISTANCE IN COORDINATE GEOMETRY

Find the distance between the points (3, 1) and (6, 5) using a right triangle. Hint: 1. plot the points, 2. use the line between the two points as the hypotenuse.

\[ 3^2 + 4^2 = d^2 \]
\[ 9 + 16 = d^2 \]
\[ \sqrt{25} = \sqrt{d^2} \]
\[ 5 = d \]

Now we are going to find the formula

Distance formula: \[ D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \]

1. Find the distance between (1, 0) and (3, 2). Leave your answer in simple radical form.

\[ D = \sqrt{(3-1)^2 + (2-0)^2} \]
\[ = \sqrt{4 + 4} \]
\[ = \sqrt{8} \]
\[ = 2\sqrt{2} \]

2. Find the distance between (-1, 2) and (-3, 4). Leave your answer in simple radical form.

\[ 2^2 + 2^2 = D^2 \]
\[ \sqrt{8} = \sqrt{D^2} \]
\[ 2\sqrt{2} = D \]
3. Find the distance between (20, 5) and (4, -3). Leave your answer in simple radical form.

\[ d = \sqrt{(20 - 4)^2 + (5 + 3)^2} = \sqrt{16^2 + 8^2} = \sqrt{256 + 64} = \sqrt{320} = 8\sqrt{5} \]

4. Find the distance between (4, -3) and (-3, -2). Leave your answer in simple radical form.

5. Find the perimeter of triangle CDF with vertices C(2,4), D(8,12), and F(24,0).

6. If the distance from point (x, 7) to (3, 11) is 5, then find x.

\[ 5 = \sqrt{(x - 3)^2 + (11 - 7)^2} \]
\[ (5)^2 = \sqrt{(x - 3)^2 + (4)^2} \]
\[ 25 = (x - 3)^2 + 16 \]
\[ 9 = (x - 3)^2 \]
\[ x - 3 = \pm 3 \]
\[ x = 6, 0 \]
EQUATION OF A CIRCLE: 

(x, y) represents any point on the circle. What we are looking for is an equation for the circle. If the distance between the points (x, y) and (1, 2) is 3, then find x and y.

1. first mark the point (1, 2) and find the points that are 3 units away. What shape does it make?

2. Plug the given information into the Pythagorean theorem. This is the equation that answers the question.

\[ a^2 + b^2 = c^2 \]
\[ (x-1)^2 + (y-2)^2 = 3^2 \] Equation for the given info
\[ (x-h)^2 + (y-k)^2 = r^2 \] Equation of a circle where:
\[ (h,k) = \text{center} \quad r = \text{radius} \]

1. Find the equation of the circle with:
   a) center at (-5, 4) and a radius of 3
   \[ (x+5)^2 + (y-4)^2 = 9 \]
   \[ (x-5)^2 + (y-4)^2 = 3^2 \]
   \[ (x+5)^2 + (y-4)^2 = 3^2 \]
   b) center at (0, 0) and a radius of 10
   \[ (x-0)^2 + (y-0)^2 = 10^2 \]
   \[ x^2 + y^2 = 100 \]

2. A circle with center (-3, 5) passes through (-9, -3). Find the circumference. Leave your answer in terms of \( \pi \).

\[ r = \sqrt{8^2 + 6^2} \]
\[ r = \sqrt{64 + 36} \]
\[ r = 10 \]
\[ C = 2\pi r \]
\[ C = 2\pi (10) \]
\[ C = 20\pi \]
In Exercises 1–3, find the distance between each pair of points.

1. \((-5, -5), (1, 3)\)  
2. \((-11, -5), (5, 7)\)  
3. \((8, -2), (-7, 6)\)

In Exercises 4 and 5, use the distance formula and the slope of segments to identify the type of quadrilateral. Explain your reasoning.

4. \(A(-2, 1), B(3, -2), C(8, 1), D(3, 4)\)

5. \(T(-3, -3), U(4, 4), V(0, 6), W(-5, 1)\)

For Exercises 6 and 7, use \(\triangle ABC\) with coordinates \(A(4, 14), B(10, 6),\) and \(C(16, 14)\).

6. Determine whether \(\triangle ABC\) is scalene, isosceles, or equilateral. Find the perimeter of the triangle.

7. Find the midpoints \(M\) and \(N\) of \(AB\) and \(AC\), respectively. Find the slopes and lengths of \(MN\) and \(BC\). How do the slopes compare? How do the lengths compare?

8. Find the equation of the circle with center \((-1, 5)\) and radius 2.

9. Find the center and radius of the circle whose equation is \(x^2 + (y + 2)^2 = 25\).

10. \(P\) is the center of the circle. What’s wrong with this picture?
9.6 CIRCLES AND THE PYTHAGOREAN THEOREM

Two previous conjectures that create right triangles:

1. A tangent to a circle is perpendicular to the radius drawn to the point of tangency. (the two tangents are also congruent)

2. Angles inscribed in a semicircle are right angles.

3. Find the shaded area. Area = _________
In Exercises 1 and 2, find the area of the shaded region in each figure. All measurements are in centimeters. Write your answers in terms of π and rounded to the nearest 0.1 cm².

1. \( AO = 5 \), \( AC = 8 \).
\[
A = \pi \left( \frac{5}{2} \right)^2 \approx 19.625 \text{ cm}^2
\]
\( \text{Shaded} = 25\pi - 2 \times 25 \approx 54.5 \text{ cm}^2 \)

2. Tangent \( PT \), \( QM = 12 \), \( m\angle P = 30° \).
\[
A_{\triangle} = \frac{60}{360} \pi (12)^2 = 24\pi
\]
\( A_T - A_S = 72\pi - 24\pi \approx 47.8 \text{ cm}^2 \)

3. \( AP = 63 \text{ cm} \). Radius of circle \( O = 37 \text{ cm} \).
How far is \( A \) from the circumference of the circle?

4. Two perpendicular chords with lengths 12.2 cm and 8.8 cm have a common endpoint. What is the area of the circle?

5. \( ABCD \) is inscribed in a circle. \( \overline{AC} \) is a diameter. If \( AB = 9.6 \text{ cm} \), \( BC = 5.7 \text{ cm} \), and \( CD = 3.1 \text{ cm} \), find \( AD \).

6. Find \( ST \).

7. The coordinate of point \( M \) is \( \left( -\frac{\sqrt{3}}{2}, \frac{1}{2} \right) \).
Find the measure of \( \angle AOM \).