LINEAR FUNCTION

Slope = \( m = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}} \)

1. a) Graph the points (4,2) and (-3,0)

b) Find the slope using the formula

Find the slope by counting

SLOPE-INTERCEPT FORM

Find the slope and y-intercept for the following

\[ y = mx + b \]
GRAPHING AND X AND Y-INTERCEPTS

Graph the following and label the x and y-intercepts

a) \( y = \frac{3}{2}x + 3 \)  
   b) \( 2x - 3y = 6 \)  
   c) \( x = 2 \)  
   d) \( y = -4 \)

WRITING AN EQUATION OF A LINE

\[ y = mx + b \]

1st find the slope  
2nd find the y-intercept

1. Given a point (3,4) on the line and a slope of -3. Find the equation of the line.

Or

Try: Given a point (-1,5) on the line and a slope of 2. Find the equation of the line.

2. Given two points on the line (3,2) and (4,3). Find the equation of the line.

Try: Given two points on the line (-1, 5) and (2, -4). Find the equation of the line.
LINEAR APPLICATIONS

1) Jimmy “The Hands” Bowman charges customer $5 monthly plus $1 per minute for backrubs. Form a linear equation:

2) Matt’s door service charges customer $100 monthly plus 30 per hour. Form a linear equation:

3) The number of hot dogs I have eaten after a given number of days, \( t \), is given by the formula \( A = 5t + 10 \)

At what rate is the number of hot dogs I have eaten changing?

How many hot dogs did I initially eat?

4) The price in dollars of one share of Apple Computer stock over the span of 108 days from August 15 to December 1, 2006, can be modeled by the function \( P(x) = 0.23x + 66.45, \ 0 \leq x \leq 108 \), where \( x \) is days after August 15th.

a) At what rate was the price changing during that time period? Was the price going up or down?

b) What was the price on August 15th (0 days)? on December 1st (108 days)? Do your results from parts A and B agree?

5) A geologist is alerted to a seismic disturbance at sea that causes a tsunami headed toward the coast of Japan. At that time, the tsunami is moving at the rate of 235 miles per hour, and is 700 miles away. Find the linear function describing the distance \( d \) of the tsunami from Japan \( h \) hours later. If it would take 2.5 hours to evacuate the coastal communities, will they be able to accomplish this before the tsunami reaches Japan?

6) After four months of use, Biffs Spiffy Spam-Free computer had dropped to $1100 in value. After ten months, the value had declined to $620. Assuming the value of the computer is linear with respect to time, write an equation that expresses the value of the computer, \( V \), in terms of time, \( t \).
LINEAR EQUATIONS AND MODELS

SOLVING LINEAR EQUATIONS - Algebraic

1) Simplify:  i) Distribute/Multiply  ii) Combine like term  iii) Remove fractions -- Multiply by the LCD

2) Isolate the x  (pick an x side)

3) Divide

\[
\begin{align*}
5w - (7w - 4) - 2 &= 5 - (3w + 2) \\
\frac{2}{3x} + \frac{1}{2} &= \frac{4}{x} + \frac{4}{3} \\
\frac{2x}{x+4} &= 7 - \frac{6}{x+4}
\end{align*}
\]

SOLVING FOR A GIVEN VARIABLE (FORMULAS)

\[
\begin{align*}
a) \quad P &= 2L + 2W, \text{ for } W \\
b) \quad A &= \frac{1}{2}h(B + b), \text{ for } b \\
c) \quad \frac{1}{a} &= \frac{1}{b} + \frac{1}{c}, \text{ for } c \\
d) \quad r + ar &= 2 - 3y, \text{ for } r
\end{align*}
\]
i) Determine if the following are Conditional, an Identity, or a Contradiction equation.

ii) If the equation is Conditional, then write the answer/s

iii) If the equation is an identity, then write all real numbers.

iv) If the equation is a Contradiction, then write no solution.

v) Look at the equations graphically and compare.

a) $4(3x - 3) = 2(6x + 16)$

b) $\frac{x}{2} - 2 = \frac{x}{5} + \frac{2}{5}$

c) $\frac{2x}{x-3} = 2 + \frac{6}{x-3}$
**Linear Applications**

1) Suppose you are at a river resort and rent a motor boat for 5 hours starting at 7 am. You are told that the boat will travel at 10 miles per hour in calm water. You head north. If the current of the stream is 2 miles per hour from the north, then how far can you travel and return in the 5 hour time.

<table>
<thead>
<tr>
<th>Against current</th>
<th>With current</th>
</tr>
</thead>
<tbody>
<tr>
<td>X=______________</td>
<td>Y=______________</td>
</tr>
</tbody>
</table>

\[ D = R \times T \]

2) A chemist mixes distilled water with a 90% solution of sulfuric acid to produce a 50% solution. If 5 liters of distilled water is used, how much 50% solution is produced?

3) A radiator contains 16 quarts of fluid, 44% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 65% antifreeze?
Linear regression

<table>
<thead>
<tr>
<th>Price $/bushel</th>
<th>Supply billion bushels</th>
<th>Demand billion bushels</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.15</td>
<td>1.55</td>
<td>2.6</td>
</tr>
<tr>
<td>5.79</td>
<td>1.86</td>
<td>2.4</td>
</tr>
<tr>
<td>5.88</td>
<td>1.94</td>
<td>2.18</td>
</tr>
<tr>
<td>6.07</td>
<td>2.08</td>
<td>2.05</td>
</tr>
<tr>
<td>6.15</td>
<td>2.15</td>
<td>1.95</td>
</tr>
<tr>
<td>6.25</td>
<td>2.27</td>
<td>1.85</td>
</tr>
<tr>
<td>6.65</td>
<td>2.53</td>
<td>2.67</td>
</tr>
</tbody>
</table>

1) List:  
STAT, Edit, enter the data.  
2nd QUIT

2) \( y=ax+b \)

STAT, CALC, 4, \( L_1, L_2 \)

Find a linear equation for the supply in terms of the price:

Find a linear equation for the demand in terms of the price:

Find the equilibrium price for Soybeans.
Section 2.3

**GRAPHING QUADRATICS**

Warm up: Fill in the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>( f(x) = x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( f(3) = (3)^2 = 9 )</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = (1)^2 = 1 )</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

Graph and label the axis of symmetry.

\[ f(x) = x^2 \]

What is happening to the graph of \( f(x) = x^2 \) in the following?

\[ f(x) = 2x^2 \]

\[ f(x) = 3x^2 \]

\[ f(x) = \frac{1}{2}x^2 \]

\[ f(x) = a(x - h)^2 + k \]

\( a = \) \( h = \) \( k = \)

Vertex = (h,k)  axis of symmetry is x=h

Graph the following, label the vertex, and axis of symmetry.

\[ f(x) = 5x^2 \]

\[ f(x) = 3x^2 - 3 \]

\[ f(x) = 1 + (x - 2)^2 \]

\[ f(x) = 1 - 2(x - 2)^2 \]
Graph \( f(x) = 2(x - 1)^2 - 5 \). Label 3+ points.

a) Compared to \( y = x^2 \), we can conclude \( f(x) \):
   
   i) has vertex: ( , )

   ii) has shifted _____ units Left Right

   iii) has shifted _____ units UP DOWN

   iv) is Stretched Normal Compressed

   v) opens: UP DOWN

b) ii) Domain: iv) Range: v) The minimum value = _____ when \( x = \) _____.

c) \( x \)-intercept d) \( y \)-intercept
**Vertex Formula**

\[
\left(-\frac{b}{2a}, y\right)
\]

\[
P(x) = x^2 + 2x - 3
\]

\[
P(x) = x^2 - 6x
\]

\[
P(x) = 2x^2 - 1x - 3
\]

\[
P(x) = 3x^2 - 12x - 1
\]

Vertex=  

Vertex=  

Vertex=  

Vertex=  
1) Suppose the marketing department of Samsung has found that, when a certain model of cellular telephone is sold at a price of $p$ dollars, the daily revenue $R$ (in dollars) as a function of the price $p$ is $R(p) = -5p^2 + 600p$

a) For what price will the daily revenue be maximized?

c) What is the maximum daily revenue?

2) **Falling Object Without Air Resistance.**

$h(t) = -16t^2 + h_0$

$h_0 = \text{the initial height, } t = \text{time in seconds, } h(t) = \text{height after } t\text{ seconds}$

A forest ranger drops a coffee cup off a fire watchtower. If the cup hits the ground 1.5 seconds later, how high is the tower?

3) A projectile is fired at an inclination of 45° to the horizontal. The height of the projectile is given by the function $h(x) = -\frac{x^2}{500} + x$, where $x$ is the horizontal distance from the firing point.

a) Find the maximum height of the projectile.

b) How far from the firing point will the projectile strike the ground?

c) When the height of the projectile is 100 feet above the ground, how far has it traveled horizontally?
**COMPLEX NUMBERS**

\[ i = \]

1) \( \sqrt{-16} \)  
2) \( -\sqrt{-64} \)  
3) \( 2\sqrt{-9} \)  
4) \( -5\sqrt{-8} \)

**COMPLEX NUMBERS**

\[ \rightarrow a + bi \]

Identify the following complex numbers as real, imaginary, or pure imaginary

\(-6 + 4i\)  
\(\frac{4 + 3i}{2}\)  
\(3 + 0i\)  
\(0 + 2i\)

Adding complex numbers:

\[ (3 + 2i) + (4 - 3i) \]

Multiplying complex numbers:

\[ i^2 = \]  
so \( 3i^2 = \)  
and \( (5i^2) = \)

\(\sqrt{-4}\sqrt{-9}\)  
\(2i(3 - 4i)\)  
\((3 - 2i)(1 - 4i)\)  
\((2 - 3i)^2\)
Dividing complex numbers:  
Conjugate of $3 + 2i$ is $3 - 2i$  
Conjugate of $-3 - i$ is $-3 + i$

Give the conjugate of the following and Simplify:

\[
\frac{3}{2 - 3i} \quad \quad \quad \frac{1 - 3i}{4i}
\]

\[
\frac{7i}{8 - 5i} \quad \quad \quad \frac{4}{i}
\]

Try the following:
\[
\frac{1}{2 - \sqrt{-9}} \quad \quad \quad (2 - \sqrt{-1})(5 + \sqrt{9})
\]
SOLVING USING FACTORING

\[ x^2 + 6x = -5 \quad \]  \[ (3x + 2)(x - 3) = 7x - 1 \]

QUADRATIC FORMULA

\[ ax^2 + bx + c = 0 \quad \rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x + 4 = 0 \]

Solve. \[ 5x(x - 1) - 7 = 4x(x - 2) \]
OTHER PROBLEMS THAT INVOLVE QUADRATICS.

1) \( x - \frac{24}{x} = -2 \)

2) \( \frac{x}{x + 4} = \frac{37}{x^2 - 16} + 2 \)

3) \( a^2 + c = c^2 \) for \( c \)

DISCRIMINANT \[ \sqrt{b^2 - 4ac} \]

<table>
<thead>
<tr>
<th>( \sqrt{0} = 0 )</th>
<th>( \sqrt{\text{positive}} = 2 \text{ real} )</th>
<th>( \sqrt{\text{negative}} = 2 \text{ imaginary} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>One real solution</td>
<td>( x^2 + 8x + 16 = 0 )</td>
<td>( 4x^2 = 6x + 3 )</td>
</tr>
</tbody>
</table>

Find a number \( k \) such that the given equation has exactly one real solution.

\[ kx^2 + 24x + 9 = 0 \]
APPLICATIONS OF QUADRATIC EQUATIONS

1) An architect is designing a small A-frame cottage for a resort area. A cross-sections of the cottage is an isosceles triangle with a base of 5 meters and an altitude of 4 meters. The front wall of the cottage must accommodate a sliding door positioned as shown in the figure.

![Diagram of A-frame cottage with sliding door]

a) Express the area $A(w)$ of the door as a function of the width $w$ and state the domain of this function.

$A(w) = \frac{1}{2}bh$ where $b = w$ and $h = 4$ m.

The domain of $A(w)$ is $[0, 5]$ m, since the width cannot exceed the base of the triangle.

b) A provision of the building code requires that doorways must have an area of at least 4.2 square meters. Find the width of the doorways that satisfy this provision.

Set $A(w) = 4.2$ and solve for $w$.

$c) What width will give us the maximum area?
Section 2.6
ADDITIONAL EQUATION - SOLVING TECHNIQUES

RADICALS

True or false

a) \((4x)^2 = 16x^2\)
   Why?

b) \((4 + x)^2 = 16 + x^2\)
   Why?

c) If \(x = 9\), then \(\sqrt{x} = -3\)
   Why?

Check your answers

6 = \sqrt{5x + 1}

-4 - \sqrt{x + 2} = -5

ISOLATE the radical and square both sides

Check your answers

Check your answer
Try these:

\[ 10 = 4 + \sqrt{5x+4} \quad \frac{3}{2}x + 1 = \sqrt{1-x} \]

SPECIAL CASES

\[ x = \sqrt{5x^2 + 9} \]

\[ \sqrt{3x+4} - \sqrt{2x-4} = 2 \]

\[ \sqrt{3x+6} - \sqrt{x+4} = \sqrt{2} \]

Check your answers:
U-SUBSTITUTION

\[ x^{2/3} - 3x^{1/3} - 10 = 0 \quad 6x^{-2} - 5x^{-1} - 6 = 0 \quad 4m^{-2} = 2 + m^{-4} \]

APPLICATIONS (part II)

Get into groups and try the following:

A water trough is constructed by bending a 6-foot by 8-foot rectangular sheet of metal down the middle and attaching triangular ends (see the figure). If the volume of the trough is 9 cubic feet, find the width/s correct to two decimal places.

![Water Trough Diagram]

a) Possible domain for \( w \):

b) Volume = (area of the base)(height)

Height:

Area of the base:
## SOLVING INEQUALITIES

<table>
<thead>
<tr>
<th>Linear</th>
<th>Compound</th>
<th>Compound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3x + 1 \leq 4$</td>
<td>$-6 \leq 2z + 4 &lt; 16$</td>
<td>$-4 \leq -2z + 4 &lt; 10$</td>
</tr>
</tbody>
</table>

### Absolute Value

**Less than and Greater than**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>x</td>
</tr>
</tbody>
</table>

Special Cases:

Use your calculator to look at the following and think about the answer. If there is no solution, then say no solution. If there are infinitely many solutions, then use the proper interval notation.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>x - 5</td>
</tr>
<tr>
<td>$</td>
<td>x - 5</td>
</tr>
<tr>
<td>$</td>
<td>5x + 1</td>
</tr>
</tbody>
</table>
Putting it all together.

Solve the equation algebraically, find the domain, and its related inequalities using your calculator.

1) a) $15x^2 + x - 6 = 0$
   b) $15x^2 + x - 6 > 0$
   c) $15x^2 + x - 6 \leq 0$

   Domain:_______________________________________________

2) a) $(x^2 - 3x)^{1/4} = 10^{1/4}$
   b) $(x^2 - 3x)^{1/4} > 10^{1/4}$
   c) $(x^2 - 3x)^{1/4} < 10^{1/4}$

   Domain:_______________________________________________

3) a) $\sqrt{2x-3} - \sqrt{x+7} = 2$
   b) $\sqrt{2x-3} - \sqrt{x+7} \leq 2$
   c) $\sqrt{2x-3} - \sqrt{x+7} \geq 2$

   Domain:_______________________________________________