Math 150 Review 1

Perform each of the following analytically except where it states you may use a calculator.

1. A box with no top is to be made from a 30 by 50 inch cardboard by cutting equal size squares from each corner and folding up the sides. Let $x$ be the length of the side of the square to be cut from each corner. Answer the following:
   
a) What is the restriction on $x$?
   b) Find the value of $x$ that will maximize the volume of the box. What is the maximum volume? (calc.)
   c) Find the interval of $x$ at which the volume of the box will be greater than 2500 cubic inches. (calc.)

2. A rectangular bin with an open top and volume of 100 cubic feet is to be built. The length of its base must be twice the width and the bin must be at least 5 feet high. Material for the base of the bin costs $12 per square foot and material for the sides costs $8 per square foot. If it costs $1200 to build the bin, what are its dimensions? (calc.)

3. Let $f(x) = 2x^2 - 3x - 4$, evaluate and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$, where $h \neq 0$.

4. For the given functions $f$ and $g$, find the functions $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$, and find their domains. (calc.)
   $$f(x) = \sqrt{x^2 + 2x - 8} \quad ; \quad g(x) = \sqrt{15 + 2x - x^2}$$

5. Graph the function and evaluate it at the given values. (calc.)
   $$f(x) = \lfloor x + 2 \rfloor - 3 \quad \text{evaluate } f(-4.5), f(-3), f(-1.4), f(5), \text{ and } f(7.2)$$

6. Graph the function $f(x) = \begin{cases} 2x + 1 & \text{if } x \leq -2 \\ x^2 & \text{if } -2 < x < 1 \\ 3x - 1 & \text{if } x \geq 1 \end{cases}$, and evaluate $f(-3), f(-2), f(0), f(1), f(4)$ (calc.)
7. a) (be careful) Find an equation for the function \( g(x) \) that is formed by the following transformation.
   The graph of the function \( f(x) = |x| \) is shifted to the right 3 units, down 4 units, reflected about the x-axis, and then vertically stretched by a factor of 2.
   
   b) Find an equation for the function \( h(x) \) that is formed by the following transformation. The graph of the function \( f(x) = |x| \) is vertically stretched by a factor of 2, reflected about the x-axis, shifted to the right 3 units, and then shifted down 4 units.
   
   c) Are the functions for \( g(x) \) and \( h(x) \) the same? why or why not?

8. Given that \( f(x) = \frac{x+5}{x+1} \) and \( g(x) = \frac{x}{x-2} \), find the function \((f \circ g)(x)\) and state its domain.

9. Find the inverse of the function \( f(x) = \frac{2x-3}{5x+1} \), and show that \((f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x\)

10. Solve.
   a) \( x^{2/3} - 6x^{1/3} - 16 = 0 \)  
   b) \( 3x^{-2} - x^{-1} - 2 = 0 \)  
   c) \( 4m^{1/2} = 2 + m \)

11. A radiator contains 16 quarts of fluid, 44% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 65% antifreeze?

12. (calc.) Find a linear model for the men’s 100-meter Olympic games freestyle data given in the following table, where \( x \) is years since 1968 and \( y \) is winning time in seconds (round answer to three decimal places.).
   
   a) What is the approximate rate of change in the winning times?
   
   b) What is the approximate rate of change in the winning times every Olympic game?
   
   c) Use your model to predict the winning time of the men’s 100-meter freestyle in 2024.

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13. Rewrite the function \( f(x) = -x^2 - 4x + 12 \) in the form \( f(x) = a(x-h)^2 + k \). Find the vertex, x-intercept, y-intercept, equation of axis of symmetry, and graph the parabola.
14. Find a number \( k \) such that the given equation has exactly one real solution.
\[
kx^2 + 24x + 9 = 0
\]

15. A bat, the flying kind, has a rocket strapped to its back. The bat is was launched straight up. After 5 seconds, it is 500 ft above the ground. After 10 seconds it is 750 ft above the ground.
   a) Assuming the rocket is propelling the bat at a constant speed, create a function that represents the bats height, \( h \), after \( t \) seconds.
   b) Verbally interpret the slope and \( y \)-intercept.
   c) How high will the bat be after 20 seconds?

16. A projectile is fired at an inclination of 45° to the horizontal. The height of the projectile is given by the function \( h(x) = -\frac{x^2}{500} + x \), where \( x \) is the horizontal distance from the firing point.
   a) Find the maximum height of the projectile.
   b) How far from the firing point will the projectile strike the ground?
   c) When the height of the projectile is 100 feet above the ground, how far has it traveled horizontally? (calc.)

17. Solve the equation and its related inequalities.
   a) \( \sqrt{2x-3} - \sqrt{x+7} = 2 \)  \( \quad \) b) \( \sqrt{2x-3} - \sqrt{x+7} \leq 2 \) (calc.)  \( \quad \) c) \( \sqrt{2x-3} - \sqrt{x+7} \geq 2 \) (calc.)

18. Solve the equation and its related inequalities.
   a) \( \left(x^2 - 3x\right)^{\frac{1}{4}} = 10^{\frac{1}{4}} \) \( \quad \) b) \( \left(x^2 - 3x\right)^{\frac{1}{4}} > 10^{\frac{1}{4}} \) (calc.) \( \quad \) c) \( \left(x^2 - 3x\right)^{\frac{1}{4}} < 10^{\frac{1}{4}} \) (calc.)

19. Solve the quadratic equation and its related inequalities.
   a) \( 15x^2 + x - 6 = 0 \) \( \quad \) b) \( 15x^2 + x - 6 > 0 \) \( \quad \) c) \( 15x^2 + x - 6 \leq 0 \)

20. Solve the following absolute inequalities:
   a) \( |2x - 10| > -3 \) \( \quad \) b) \( |2x - 10| \leq 0 \) \( \quad \) c) \( |2x - 10| \geq 0 \)
   d) \( |2x - 10| > 0 \) \( \quad \) e) \( |2x - 10| < -3 \)

21. Rationalize the denominator. \[ \frac{1}{2 - \sqrt{-9}} \]
Answers:

1. a) $0 < x < 15^\circ$  b) $x = 6.0685^\circ$, $V_{\text{max}} = 4104.41 \text{ in}^3$  c) $2.12^\circ < x < 10.94^\circ$

2. Dimensions are $2.22^\" \times 4.44^\" \times 10.16^\"$

3. $D.Q. = 4x - 3 + 2h$

4. $(fg)(x) = \sqrt{x^2 + 2x - 8\sqrt{15 + 2x - x^2}}$, Domain = $[2, 5]$, \( \left( \frac{f}{g} \right)(x) = \frac{\sqrt{x^2 + 2x - 8}}{\sqrt{15 + 2x - x^2}}, \) Domain = $[2, 5]$

5. $f(-4.5) = -6, f(-3) = -4, f(-1.4) = -3, f(5) = 4, f(7.2) = 6$

6. $f(-3) = -5, f(-2) = -3, f(0) = 0, f(1) = 2, f(4) = 11$

7. a) $g(x) = -2|x - 3| - 4$  b) $g(x) = -2|x - 3| - 4$  c) no, if we reflect across the $x$-axis and then shift vertically, the resulting function is not same as shifting vertically and then reflecting across the $x$-axis.

8. $(f \circ g)(x) = \frac{3x - 5}{x - 1}$; Domain = $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

9. $f^{-1}(x) = \frac{x + 3}{2 - 5x}$, don't forget to show work for domain.

10. a) -8, 512  b) 1, -3/2  c) $(2 + \sqrt{2})^2, (2 - \sqrt{2})^2$ or $6 \pm 4\sqrt{2}$

11. Six quarts

12. a) The winning time is decreasing 0.103 seconds every year.  b) The winning time is decreasing 0.412 seconds every 4 years.  c) $y = -0.103x + 51.509$, $y = \text{time in 2024} = 45.716$ seconds

13. $f(x) = -(x + 2)^2 + 16$  vertex = $(-2, 16)$, Axis of symmetry: $x = -2$, $y_{\text{max}} = 12$. $x_{\text{mid}} \{ -6, 2 \}$

14. $k = 16$

15. a) $h(t) = 50t + 250$,  b) Slope = The bat is traveling at 50 ft per second, Y-intercept = The bat was launched from a height of 250 feet.  c) 1250 ft.  16. a) $h_{\text{max}} = 125^\prime$, b) $x = 500^\prime$, c) $h = \{ 138.2^\prime, 361.8^\prime \}$

17. a) $x = 42$,  b) $[1.5, 42]$,  c) $[42, \infty)$

18. a) $x = \{-2.5\}$,  b) $(-\infty, -2) \cup (5, \infty)$,  c) $(-2.0) \cup [3, 5]$
19. a) \( x = \left\{ -\frac{2}{3}, \frac{3}{5} \right\} \), b) \( (-\infty, -\frac{2}{3}) \cup \left( \frac{3}{5}, \infty \right) \), c) \( \left[ -\frac{2}{3}, \frac{3}{5} \right] \)

20. a) \( \mathbb{R} \) b) \( x = 5 \) c) \( \mathbb{R} \) d) \( (-\infty, 5) \cup (5, \infty) \) e) no solution (\( \emptyset \))

21. \( \frac{2}{13} + \frac{3}{13} i \)