Parabolas

\[ y = x^2 \]

Vertex = (0, 0)

\[ x = 2y^2 \]

\[ x = \frac{1}{4} y^2 \]

\[ x = -3y^2 \]

If the equation is \( y = \), then the graph opens in the \( \_ \_ \_ \_ \_ \) direction.

If the equation is \( x = \), then the graph opens in the \( \_ \_ \_ \_ \_ \) direction.

Parabola--- The set of all points in a plane equidistant from a fixed point and a line in the plane.

Focus -- the fixed point. \( y^2 = 4ax \) or \( x^2 = 4ay \)

If \( x^2 = 4ay \), then the focus is \( c \) units up from the vertex. (0, 0)

If \( y^2 = 4ax \), then the focus is \( c \) units to the right of the vertex. (\( a \), 0)

Directrix -- the line

If \( x^2 = 4ay \), then the directrix is \( y = -c \)

If \( y^2 = 4ax \), then the directrix is \( x = -a \).
Find the Focus and the equation for the directrix for the following and graph the parabola, focus, and directrix.

\[ y^2 = 4ax \quad \text{or} \quad x^2 = 4ay \]

<table>
<thead>
<tr>
<th>Equation for graphing:</th>
<th>focus:</th>
<th>directrix:</th>
<th>Equation for graphing:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ y^2 = 4x ]</td>
<td>(0, 0)</td>
<td>( x = -1 )</td>
<td>[ \frac{1}{4}y^2 = x ]</td>
</tr>
<tr>
<td>[ \frac{4c}{y} = x ]</td>
<td>( c = 1 )</td>
<td>( y = -2 )</td>
<td>[ \frac{1}{8}x^2 = y ]</td>
</tr>
<tr>
<td>[ y^2 = 8y ]</td>
<td>(0, 2)</td>
<td>( y = -2 )</td>
<td>[ \frac{1}{8}x^2 = y ]</td>
</tr>
<tr>
<td>[ \frac{4c}{y} = 8 ]</td>
<td>( c = 2 )</td>
<td>( x = \frac{1}{2} )</td>
<td>[ -\frac{1}{2}x^2 = y ]</td>
</tr>
<tr>
<td>[ y^2 = -6x ]</td>
<td>(0, 0)</td>
<td>( x = \frac{1}{2} )</td>
<td>[ -\frac{1}{2}x^2 = y ]</td>
</tr>
<tr>
<td>[ \frac{4c}{y} = -6 ]</td>
<td>( c = -\frac{3}{2} )</td>
<td>( y = -2 )</td>
<td></td>
</tr>
<tr>
<td>[ \frac{1}{8}x^2 = y ]</td>
<td>( c = -\frac{3}{2} )</td>
<td>( x = \frac{1}{2} )</td>
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<tr>
<td>[ -8y = x^2 ]</td>
<td>(0, -2)</td>
<td>( y = 2 )</td>
<td>[ y = -\frac{1}{8}x^2 ]</td>
</tr>
<tr>
<td>[ \frac{4c}{y} = -8 ]</td>
<td>( c = -2 )</td>
<td>( x = -\frac{1}{16} )</td>
<td>[ x = -\frac{1}{16}y^2 ]</td>
</tr>
<tr>
<td>[ \frac{1}{4}x = y^2 ]</td>
<td>(1/16, 0)</td>
<td>( x = -\frac{1}{16} )</td>
<td>[ x = \frac{1}{4}y^2 ]</td>
</tr>
<tr>
<td>[ 4c = \frac{1}{4} ]</td>
<td>( c = \frac{1}{16} )</td>
<td>( x = 1 )</td>
<td>[ x = \frac{1}{4}y^2 ]</td>
</tr>
<tr>
<td>[ -4x = y^2 ]</td>
<td>(-1, 0)</td>
<td>( x = 1 )</td>
<td>[ x = -\frac{1}{4}y^2 ]</td>
</tr>
</tbody>
</table>
Find the equation of a parabola with vertex at the origin, axis of symmetry the x or y axis, and:

- a) Directrix y=4
- b) Directrix x= -9
- c) Focus (0,5)
- d) Focus (-2,0)
- e) x-axis symmetry and contains the point (4,8)
- f) y-axis symmetry and contains the point (-5,10)
- g) A parabolic bridge is used with my son’s Thomas the Trains set. The arch of the bridge is 2 inches high and spans 6 inches. Find the equation of the arch. Hint: you need to find 4a. The origin is placed at the vertex of the parabola $x^2 = 4ay$

Find the points of intersection in the first quadrant for each pair of parabolas to three decimal places using your calculator.

$x^2 = 3y, \quad y^2 = 3x$
CIRCLE

EQUATION OF A CIRCLE: \((x,y)\) represents any point on the circle. What we are looking for is an equation for the circle.

If the distance between the points \((x, y)\) and \((0, 0)\) is 3, then find \(x\) and \(y\).

\[
x^2 + y^2 = 3^2
\]

\((0,0) = \text{center}\quad r = \text{radius}\)

1. Find the equation of the circle with:
   a) a radius of 4 and graph it.
   b) a radius of 10 and graph it.

\[
x^2 + y^2 = 4
\]

Radius = \(\sqrt{4} = 2\)

\[
x^2 + y^2 = 16
\]

\((0,0)\quad r = 4\)

\[
x^2 + y^2 = 100
\]

The square root under the \(x^2\) give you the units in the x-direction from the center.

The square root under the \(y^2\) give you the units in the y-direction from the center.
In exercises 1 – 8, graph the given ellipse.

1) \( \frac{x^2}{25} + \frac{y^2}{9} = 1 \)  
2) \( \frac{x^2}{16} + \frac{y^2}{4} = 1 \)  
3) \( \frac{x^2}{4} + \frac{y^2}{25} = 1 \)  
4) \( \frac{x^2}{16} + \frac{y^2}{36} = 1 \)

5) \( \frac{100x^2}{100} + \frac{9y^2}{900} = 1 \)  
6) \( 9x^2 + 25y^2 = 225 \)

7) \( 4x^2 + 100y^2 = 400 \)  
8) \( 25x^2 + 4y^2 = 100 \)
**Definition:** An ellipse is the set of all points in a plane such that the sum of their distances from two fixed points is constant. The two fixed points are called FOCI (the plural of the word FOCUS).

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

Length of one part of the string is \(a+c\)
Length of the other part of the string is \(a-c\)
So the length of the string is \(2a\)

Foci is \(c\) and \(-c\) units form the center along the major axis.

If \(X\) is the major axis, then
\[
b^2 + c^2 = a^2
\]
Where \(c = \sqrt{a^2 - b^2}\) if \(a > b\)

If \(y\) is the major axis, then
\[
a^2 + c^2 = b^2
\]
Where \(c = \sqrt{b^2 - a^2}\) if \(b > a\)

Graph the ellipse, then find the vertices, major axis, and the foci of the given ellipse.

\[
16x^2 + 25y^2 = 400
\]

\[
\frac{x^2}{25} + \frac{y^2}{16} = 1
\]

Vertices: \((5,0)\) \((5,0)\)

Major axis: \(X-\text{axis}\)

Foci: \(c = \sqrt{25-16} = \sqrt{9} = 3\)
\((-3,0)\) \((3,0)\)
Find the equation of the ellipse given the following information.

a) major axis on x-axis, major axis length = 10, minor axis length = 6

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

\[ 2a = 10 \quad 2b = 6 \]

\[ a = 5 \quad b = 3 \]

\[ \frac{x^2}{25} + \frac{y^2}{9} = 1 \]

b) major axis on y-axis, major axis length = 8, minor axis length = 4

\[ \frac{x^2}{4} + \frac{y^2}{16} = 1 \]

\[ 2b = 8 \quad 2a = 4 \]

\[ b = 4 \quad a = 2 \]

\[(0,12) \quad (0,10)\]

c) major axis on y-axis, major axis length = 24, distance of foci from center is 10

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

\[ b = 12 \quad c = 10 \]

\[ c = \sqrt{b^2 - a^2} \]

\[ c = b^2 - a^2 \]

\[ 100 = 144 - a^2 \]

\[ a^2 = 44 \]

\[ a^2 = 144 - 100 \]

\[ \frac{x^2}{44} + \frac{y^2}{100} = 1 \]

d) Find the equation where each of the points have a distance from (0,1) that is one-half of the distance from the line y=4. We cannot assume it is an ellipse.

\[ \frac{x^2}{3} + \frac{3}{4} y^2 = 3 \]

\[ y^2 + 4x^2 + 3y^2 = \frac{12}{12} \]

\[ \frac{x^2}{3} + \frac{y^2}{4} = 1 \]
**Definition:** A hyperbola is the set of all points in a plane such that the absolute difference of their distances from two fixed points is constant. The two fixed points are called FOCI.

---

**Standard Equation of a Horizontal Hyperbola**

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1
\]

- **x-intercepts:** Vertices ⇒ Vertices at \((-a,0)\) and \((a,0)\)
- **y-intercepts:** None
- **Foci:** \((-c,0)\) and \((c,0)\) where \(c = \sqrt{a^2 + b^2}\)
- **Equations of asymptote lines:** \(y = \pm \frac{b}{a}x\)

**Standard Equation of a Vertical Hyperbola**

\[
\frac{y^2}{c^2} - \frac{x^2}{a^2} = 1
\]

- **x-intercepts:** None
- **y-intercepts:** Vertices ⇒ Vertices at \((0,-b)\) and \((0,b)\)
- **Foci:** \((0,-c)\) and \((0,c)\) where \(c = \sqrt{a^2 + b^2}\)
- **Equations of asymptote lines:** \(y = \pm \frac{b}{a}x\)

*****The Vertices and Foci are on the same axis.

*****A box's corners are used to create the asymptotes

*****The Foci are \(c\) units from the origin.

*****The letter that comes first (positive coefficient) is the axis that the vertices and foci.

*****The letter that comes first (positive coefficient) is the direction the graph opens.

\[
\frac{(x)^2}{4} - \frac{(y)^2}{9} = 1
\]

**Graph and find the foci and the equations of the asymptotes.**

\[
y = \pm \frac{3}{2}
\]

\[
c = \sqrt{4 + 9} = \sqrt{13} \approx 3.6
\]
Graph and find the foci and the equations of the asymptotes.

1) \( \frac{x^2}{16} - \frac{y^2}{9} = 1 \)  
2) \( \frac{x^2}{64} - \frac{y^2}{36} = 1 \)  
3) \( \frac{y^2}{16} - \frac{x^2}{9} = 1 \)  
4) \( \frac{y^2}{64} - \frac{x^2}{36} = 1 \)

5) \( 100x^2 - 9y^2 = 900 \)  
6) \( 9x^2 - 25y^2 = 225 \)

7) \( 100y^2 - 4x^2 = 400 \)  
8) \( 4y^2 - 4x^2 = 100 \)

\[ \frac{y^2}{25} - \frac{x^2}{25} = 1 \]

\( b = 5, \quad a = 5 \)

\[ c = \sqrt{25 + 25} = \sqrt{50} \approx 7.07 \]

\[ y = \pm 1 \]

\[ y = \pm \frac{b}{a} \]
MIDPOINT = \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \)

M is the midpoint of A and B
1. A=(4,3), B=(-3,0), M= \( \left( \frac{4+(-3)}{2}, \frac{3+0}{2} \right) \) \( \left( \frac{1}{2}, \frac{3}{2} \right) \) \( (0.5, 1.5) \)

2. A=(4,-3), B=(4,2), M= \( \left( \frac{4+4}{2}, \frac{-3+2}{2} \right) \) \( (4, -0.5) \)

M is the midpoint of A and B
1. A=(0,0), M=(4,3), B=

B is at (________, ________).
EQUATION OF A CIRCLE: (x,y) represents any point on the circle. What we are looking for is an equation for the circle.

If the distance between the points (x , y) and (1, 2) is 3, then find x and y.

\[ d = \sqrt{(x-1)^2 + (y-2)^2} \]

\[ 3 = \sqrt{(x-1)^2 + (y-2)^2} \]

\[ 3^2 = (x-1)^2 + (y-2)^2 \]

\[ (x-h)^2 + (y-k)^2 = r^2 \]

(h,k) = center \hspace{1cm} r = radius

1. Find the equation of the circle with :
   a) center at (-5,4) and a radius of 3
   b) center at (0,0) and a radius of 10

Find the center and the radius of the given circle. Graph the circle.

\[ (x-1)^2 + (y-3)^2 = 9 \]

Center: \hspace{1cm} Radius:

\[ (x+3)^2 + (y-4)^2 = 4 \]

Center: \hspace{1cm} Radius:
In exercises 17 – 24, find the center and the radius of the given circle.

17) \( x^2 + y^2 + 2x - 6y - 6 = 0 \)  
18) \( x^2 + y^2 + 6x - 10y - 2 = 0 \)

19) \( x^2 + y^2 + 8x + 4y - 5 = 0 \)  
20) \( x^2 + y^2 - 4x + 2y - 4 = 0 \)

21) \( x^2 + y^2 - 6x + 4y - 3 = 0 \)  
22) \( x^2 + y^2 - 4x - 8y - 5 = 0 \)

23) \( x^2 + y^2 + 10x + 2y = 10 \)  
24) \( x^2 + y^2 + 2x - 6y = 6 \)

\[
\begin{align*}
\text{Center} &= (-1, 3) & \text{Radius} &= 4 \\
x^2 + y^2 + 2x - 6y &= 6 \\
(x + 1)^2 + (y - 3)^2 &= 16
\end{align*}
\]
TRANSLATIONS (Just the find the equations, type of curve, and graph) (omit 65-77)

Section 11.4

Graph.

\[
\frac{(x + 3)^2}{25} + \frac{(y - 4)^2}{9} = 1
\]

Center = \((-3, 4)\)

Type of graph:

\[
C = \sqrt{25 - 9} = \sqrt{16} = 4
\]

\[
(-3, 4) \Rightarrow (-3 + 4, 4), (-3 - 4, 4)
\]

\[
(y - 3) = \pm \frac{\sqrt{5}}{3}(x + 2)
\]

\[
(3, -4)
\]

\[
\frac{(x - 3)^2}{9} + \frac{(y + 4)^2}{4} = 1
\]

\[
(x - 3)^2 = 4(y + 4)
\]

\[
4 \frac{c = 4}{c = 1}
\]

\[
\frac{c = \sqrt{25 + 9}}{c = \sqrt{34}} \approx 5.8
\]

\[
(-3 + \sqrt{34}, 4) \text{ , \ } (-3 - \sqrt{34}, 4)
\]

\[
\text{vertex } (3, -4)
\]
Write the equation in standard form.

Circle

\[
x^2 + y^2 + 2x - 6y - 6 = 0
\]
\[
x^2 + 2x + y^2 - 6y = 6
\]
\[
(x^2 + 2x + \{1\}) + (y^2 - 6y + \{9\}) = 6 + \{1\} + \{9\}
\]
\[
(x + \underline{1})^2 + (y - \underline{3})^2 = 16
\]

Ellipse

\[
9x^2 + 18x + 4y^2 - 8y - 23 = 0
\]
\[
(9x^2 + 18x) + (4y^2 - 8y) = 23
\]
\[
9(x^2 + 2x + \{1\}) + 4(y^2 - 2y + \{1\}) = 23 + 9\{1\} + 4\{1\}
\]
\[
\frac{9(x + \underline{1})^2}{36} + \frac{4(y - \underline{1})^2}{36} = \frac{36}{36}
\]
\[
\frac{(x+1)^2}{4} + \frac{(y-1)^2}{9} = 1
\]

Hyperbola

\[
9x^2 + 18x - 7y^2 - 28y - 28 = 0
\]
\[
(9x^2 + 18x) + (-7y^2 - 28y) = 28
\]
\[
9(x^2 + 2x + \{1\}) - 7(y^2 + 4y + \{4\}) = 28 + 9\{1\} - 7\{4\}
\]
\[
\frac{9(x + \underline{1})^2}{9} - \frac{7(y - \underline{2})^2}{9} = \frac{9}{9}
\]
\[
\frac{(x+1)^2}{1} - \frac{(y-2)^2}{9} = 1
\]
Parabola

\[ y^2 + 12x + 4y - 32 = 0 \]
\[ y^2 + 4y = -12x + 32 \]
\[ (y^2 + 4y + 4) = -12x + 32 + 4 \]
\[ (y + 2)^2 = -12(x - 3) \]

Try: \[-9x^2 + 16y^2 - 72x - 96y - 144 = 0\]

\[
\begin{aligned}
16y^2 - 96y &\quad -9x^2 - 72x \\
16(y^2 - 6y + 9) &\quad 9(x^2 + 8x + 16) \\
16(y - 3)^2 &\quad 9(x + 4)^2 \\
16(y - 3)^2 &\quad 144 \\
\frac{9}{16} &\quad = 1
\end{aligned}
\]

Finding the equation of the asymptotes in a hyperbola.

\[
\begin{aligned}
\frac{(x+3)^2}{25} - \frac{(y-4)^2}{9} &= 1 \\
(y - k) &= \pm \frac{b}{a} (x - h) \\
(y - 4) &= \pm \frac{3}{5}(x + 3)
\end{aligned}
\]

Try:

\[
\begin{aligned}
\frac{(y + 7)^2}{25} - \frac{(x - 4)^2}{36} &= 1 \\
(y + 7) &= \pm \frac{5}{6}(x - 4)
\end{aligned}
\]
Conics Review

1. Tell whether the graph of the ellipse has a major axis parallel with the x or y-axis.
   
a) Vertices at (4,3) and (4, 5)
   
   b) Foci at (-2,7) and (5, 7)

2. Tell whether the graph of the hyperbola opens in the x or y-direction.
   
a) Vertices at (5,3) and (5, 7)
   
   b) Foci at (2,-5) and (4, -5)

3. Find the location of the foci for the ellipse, if c is 5, the center is (1,10), and one of the vertices is (1, 20).

   \[(1,10) + 5 \]  
   \[(1, 15) \]

4. a) Find the center of the ellipse with foci at (-2,7) and (5, 7).

   \[
   \text{midpoint} = \left( \frac{-2 + 5}{2}, \frac{7 + 7}{2} \right) = \left( \frac{3}{2}, 7 \right)
   \]

   b) Find the center of the graph of the hyperbola with foci (5,3) and (5, 7).

   \[
   \left( \frac{5 + 5}{2}, \frac{3 + 7}{2} \right) = \left( \frac{5}{2}, 5 \right)
   \]

5. Find the equation and graph the ellipse with Vertices at (5,0) and (5, 10), and a focus at (5,9)

\[
\frac{(x - 5)^2}{9} + \frac{(y - 5)^2}{25} = 1
\]

<table>
<thead>
<tr>
<th>Major</th>
<th>Center</th>
<th>Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5, 10 )</td>
<td>(5, 10)</td>
<td>(5, 9 )</td>
</tr>
<tr>
<td>(5, 5 )</td>
<td>(5, 5)</td>
<td>(5, 5 )</td>
</tr>
<tr>
<td>b = 5</td>
<td>c = \sqrt{25 - 9} = 4</td>
<td>e = 4</td>
</tr>
</tbody>
</table>

\[
\frac{c^2}{a^2} = \frac{9}{16} = \frac{1}{2} \quad a^2 = 25 - 16 \quad a = 3
\]

16
6. If the end points of the minor axis of an ellipse are at $(2, -3)$ and $(8, -3)$, and one of the two foci is at the point $(5,1)$; find the equation of the ellipse. Graph the ellipse. 

\[
\frac{(x - 5)^2}{9} + \frac{(y + 3)^2}{25} = 1
\]

<table>
<thead>
<tr>
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<th>Center</th>
<th>Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(8, -3)$</td>
<td>$(\frac{2+8}{2}, \frac{-3+3}{2})$</td>
<td>$(5, 1)$</td>
</tr>
<tr>
<td>$(5, -3)$</td>
<td>$(5, -3)$</td>
<td>$(5, -3)$</td>
</tr>
</tbody>
</table>

\[a = 3\quad c = 4\]

7. Find the equation of the described parabola. Graph the parabola. Focus at the $(2,1)$ and directrix $y = -3$

\[a = 2\]

8. Find the equation of the hyperbola with vertices is at $(1,4)$ and $(1,-2)$, and one of the two foci is at $(1,-6)$. Write the equations of the asymptote lines and Graph the hyperbola.

\[
\frac{(y-1)^2}{9} - \frac{(x-1)^2}{40} = 1
\]

<table>
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<th>Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 4)$</td>
<td>$(\frac{1+1}{2}, \frac{4+2}{2})$</td>
<td>$(1, -6)$</td>
</tr>
<tr>
<td>$(1, 1)$</td>
<td>$(1, 1)$</td>
<td>$(1, 1)$</td>
</tr>
</tbody>
</table>

\[b = 3\quad c = 7\]

\[
\frac{c^2 - a^2 - b^2}{4} = 49 = a^2 + b^2\quad a = \sqrt{40} \approx 6.32
\]
More of the same:

1. Find the equation and graph the ellipse with Vertices at (-10,2) and (10, 2), and a focus at (6,2).

\[
\frac{(x + 10)^2}{100} + \frac{(y - 2)^2}{64} = 1
\]

<table>
<thead>
<tr>
<th>Major</th>
<th>Center</th>
<th>Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10, 2)</td>
<td>((-10 + 10, \frac{2 + 2}{2}))</td>
<td>(6, 2)</td>
</tr>
<tr>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
</tr>
</tbody>
</table>

\[a = 10\]

\[c = 6\]

\[c^2 = a^2 - b^2\]

\[6^2 = 10^2 - b^2\]

\[b^2 = 100 - 36\]

\[b^2 = 64\]

2. If the end points of the minor axis of an ellipse are at (3,-2) and (3,6), and one of the two foci is at the point (6,2); find the equation of the ellipse. Graph the ellipse.

\[
\frac{(x - 3)^2}{25} + \frac{(y - 2)^2}{16} = 1
\]

<table>
<thead>
<tr>
<th>Minor</th>
<th>Center</th>
<th>Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, 6)</td>
<td>(\left(\frac{3 + 3}{2}, \frac{-2 + 6}{2}\right))</td>
<td>(6, 2)</td>
</tr>
<tr>
<td>(3, 2)</td>
<td>(3, 2)</td>
<td>(3, 2)</td>
</tr>
</tbody>
</table>

\[b = 4\]

\[c = 3\]

\[c^2 = a^2 - b^2\]

\[9 = a^2 - 16\]

\[25 = a^2\]
3. Find the equation of the described parabola. **Graph the parabola.** Focus at the (5,3) and directrix \( x = 1 \)

\[
\text{Vertex} \quad x = 1 \\
(5,3) \text{ and } (1,3) \\
\left( \frac{5+1}{2}, \frac{3+3}{2} \right) \\
(3,3) \\
\text{Focus} \quad (5,3) \\
a = 2 \\
(y-3)^2 = 4a(x-3) \\
(y-3)^2 = 8(x-3)
\]

4. Find the equation of the hyperbola with vertices is at (1,0) and (-5,0), and one of the two foci is at (8,0). Write the equations of the asymptote lines and **Graph the hyperbola.**

\[
\text{Opens in= } \\
( -5 , 0 ) \\
( -2 , 0 ) \\
\text{Center} \quad \left( \frac{-5+0}{2}, \frac{0+0}{2} \right) \\
\left( -2 , 0 \right) \\
a = 3 \\
c = 10 \\
c^2 = a^2 + b^2 \\
100 = a^2 + b^2 \\
91 = b^2 \\
91 \approx b
\]