Systems of Linear Equations

Section 8.1

Graph and find the solutions to the systems of equations:

1. \[
\begin{align*}
2x - 3y &= 9 \\
x &= 3
\end{align*}
\]

2. \[
\begin{align*}
y &= 3x + 1 \\
y &= 3x - 2
\end{align*}
\]

3. \[
\begin{align*}
y &= 2x - 4 \\
y &= 2x - 4
\end{align*}
\]

In exercises 1 – 4, solve the system of equations by GRAPHING.

1) \[
\begin{align*}
x + 4y &= 7 \\
2x - 3y &= 3
\end{align*}
\]

2) \[
\begin{align*}
2x - 5y &= 2 \\
x + y &= 8
\end{align*}
\]

3) \[
\begin{align*}
x + 2y &= 5 \\
2x + 5y &= 4
\end{align*}
\]

4) \[
\begin{align*}
3x + 7y &= -2 \\
2x - 3y &= 14
\end{align*}
\]
**SUBSTITUTION METHOD**

**X**

\[ x = -2y \]

**Y**

Substitute the \( x = -2y \)

\[ 3x + 2y = 4 \]

\[ 3(-2y) + 2y = 4 \]

\[ (\quad, \quad) \]

\[ 4x - 3y = -2 \]

\[ x + 2y = 5 \]

\[ (\quad, \quad) \]

\[ 4x - y = 7 \]

\[ -12x + 3y = -21 \]
ELIMINATION/ADDICTION METHOD

\[ 3x - 2y = 7 \]
\[ 2x + 4y = 2 \]

Multiply:
\[
\begin{align*}
(3x - 2y = 7) & \quad \rightarrow \\
(2x + 4y = 2) & \quad \rightarrow \\
\end{align*}
\]

\[ (\quad, \quad) \]

Substitute:
\[ 2x + 4y = 2 \]

\[ 4x - 3y = -2 \]
\[ x + 2y = 5 \]

Multiply:
\[ \rightarrow \]
\[ \rightarrow \]

\[ (\quad, \quad) \]

Substitute:
\[ 2x + 4y = 2 \]

\[ 4x - 3y = 5 \]
\[ -12x + 9y = -21 \]
In exercises 1 – 8, solve the system of equations using the elimination method.

1) \( x + 4y = 7 \) \hspace{1cm} 2) \( 2x - 5y = 2 \) \hspace{1cm} 3) \( x + 2y = 5 \) \hspace{1cm} 4) \( 3x + 7y = -2 \)

\( 2x - 3y = 3 \) \hspace{1cm} \( x + y = 8 \) \hspace{1cm} \( 2x + 5y = 4 \) \hspace{1cm} \( 2x - 3y = 14 \)

5) \( x = 3y - 4 \) \hspace{1cm} 6) \( x = 4y + 3 \) \hspace{1cm} 7) \( x = 3 - 2y \) \hspace{1cm} 8) \( x = 5 + 2y \)

\( 2x - y = 7 \) \hspace{1cm} \( x + y = 8 \) \hspace{1cm} \( y = x - 6 \) \hspace{1cm} \( y = 5x - 7 \)

Now try:

\[ 3x + 3y = y + 1 \]
\[ x + 3y = 9 - x \]
Applications
(just set up the equations for the following)

Mixture Problems

How many liters of a 14% alcohol solution must be mixed with 20L of a 50% solution to get a 30% solution?

\[ E_1: x + 20 = y \]
\[ E_2: 14x + 20(50) = 30y \rightarrow 14x + 1000 = 30y \]

How much pure dye must be added to 4 gal of 25% dye solution to increase the solution to 40%?

\[ E_1: x + 4 = y \]
\[ E_2: 100x + 100 = 40y \]

Uniform Motion (d=rt)

A crew of eight can row 20 kilometers per hour in still water. The crew rows upstream and then returns to its starting point in 15 minutes. If the river is flowing at 2 km/h, how far upstream did the crew row?

\[
\begin{array}{c|c|c}
\text{upstream} & \text{downstream} \\
\hline
\text{D} & \text{R} & \text{T} \\
\hline
D & 18 & 15 - t \\
\hline
D & 22 & t \\
\hline
\end{array}
\]

\[ D = 18(15 - t) \]
\[ D = 270 - 18t \]
\[ D = 22t \]
A plane carries enough fuel for 20 hours of flight at an airspeed of 150 mph. How far can it fly into a 30 mph headwind and still have enough fuel to return to its starting point? (This distance is called the point of no return)

\[
\begin{aligned}
D &= R \times T \\
D &= 120 \times t \\
D &= 180 \times (20 - t)
\end{aligned}
\]

\[
\begin{align*}
D &= 120t \\
D &= 3600 - 180t \\
D &= 120(12) + 1440 mph
\end{align*}
\]

Melissa Wright won $60,000 on a slot machine in Las Vegas. She invested part at 2% simple interest and the rest at 3%. In one year she earned a total of $1,600 in interest. How much was invested?

\[
\begin{align*}
\text{Invested} &: X + Y = 60,000 \\
\text{Interest} &: 0.02X + 0.03Y = 1600
\end{align*}
\]

Blake has a total of $4000 to invest in two accounts. One account earns 2% simple interest, and the other earns 5% simple interest. How much should be invested in both accounts to earn exactly $155 at the end of 1 year?

\[
\begin{align*}
\text{Invested} &: X + Y = 4000 \\
\text{Interest} &: 0.02X + 0.05Y = 155
\end{align*}
\]
At $1.40 per bushel, the daily supply for soybeans is 1,075 bushels and the daily demand is 580 bushels. When the price falls to $1.20 per bushel, the daily supply decreases to 575 bushels and the daily demand increases to 980 bushels. Assume that the supply and demand equations are linear.

A) use the supply data to form a supply equation.
Hint: (Price, supply) $\rightarrow (p, s)$

\[
m = \frac{1075 - 575}{1.4 - 1.2} = \frac{500}{0.2} = 2500
\]
\[
s = 2500p - 2425
\]

B) Use the demand data to form a demand equation.
Hint: (Price, demand) $\rightarrow (p, d)$

\[
m = \frac{580 - 980}{1.4 - 1.2} = \frac{-400}{0.2} = -2000
\]
\[
d = -2000p + 3380
\]

c) Find the equilibrium price and the quantity.

\[
2500p - 2425 = -2000p + 3380
\]
\[
4500p = 5805
\]
\[
p = \frac{5805}{4500}
\]
\[
d = -2000(1.29) + 3280 = 800
\]

A company produces Italian sausages and bratwursts at plants in Green Bay and Sheboygan. The hourly production rates at each plant are given in the table. How many hours should each plant be operated to exactly fill and order for 62,250 Italian sausages and 76,500 bratwursts?

<table>
<thead>
<tr>
<th>Plant</th>
<th>Italian sausage</th>
<th>Bratwurst</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Bay</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>Sheboygan</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

\[
800x + 500y = 62,250
\]
\[
800x + 1000y = 76,500
\]
Systems of three equations

Section 8.2

\[ 2x - 5y + 3z = -1 \]
\[ x + 4y - 2z = 9 \]
\[ x - 2y - 4z = -5 \]

Multiply:
\[
\begin{align*}
2x - 5y + 3z &= -1 \\
-2(x + 4y - 2z) &= 9 \\
-x - 2y - 4z &= -5
\end{align*}
\]
\[
\begin{align*}
-13y + 7z &= -19 \\
-26y + 14z &= -38 \\
-42y - 14z &= -98
\end{align*}
\]
\[
\begin{align*}
-68y &= -136 \\
&= 2
\end{align*}
\]

Back Substitute:
\[
\begin{align*}
x - 2y - 4z &= 5 \\
x - 2(2) - 4(1) &= -3 \\
x - 4 - 4 &= -5 \\
x - 8 &= -5 \\
x &= 3
\end{align*}
\]
\[
\begin{align*}
y &= 2
\end{align*}
\]
\[
\begin{align*}
z &\quad 6y + 2z = 14 \\
&\quad 12 + 2z = 14 \\
z &= 2
\end{align*}
\]
\[
(3, 2, 1)
\]
In exercises 9 – 24, solve the system of equations by any method.

9) \( x + y + z = 4 \)
   \( 2x - y + 2z = 5 \)
   \( x + 4y - 5z = 1 \)

10) \( 2x + 3y - 5z = 1 \)

11) \( x + y + z = 6 \)

12) \( 5x - 4y + z = -6 \)

13) \( 3x - 5y + 2z = 8 \)

14) \( x - 3y + 2z = 5 \)

15) \( x - 3y + 4z = 13 \)

16) \( 4x + 3z = -12 \)

17) \( x + 2y + 3z = -2 \)

18) \( x - 2y + 4z = 6 \)

19) \( 3x - y + 4z = -13 \)

20) \( 2x - y + 2z = -3 \)

21) \( \frac{1}{x} + \frac{2}{y} - \frac{3}{z} = 9 \)

22) \( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 4 \)

23) \( \frac{1}{x} + \frac{1}{y} - \frac{3}{z} = 6 \)

24) \( \frac{1}{a} + \frac{2}{b} + \frac{1}{c} = 1 \)

\[ x = \frac{1}{a}, \quad y = \frac{1}{b}, \quad z = \frac{1}{c} \]

\[ x + 2y + z = 1 \]

\[ 5x + 4y + z = -1 \]

\[ 3x + 5y + 2z = 0 \]
Infinitely many solutions or no solution?

\[ \begin{align*}
2x - 3y &= 4 \\
4x - 5z &= 10 \\
-12y + 10z &= -4
\end{align*} \]

\[ \begin{align*}
-12y + 10z &= -4 \\
2y - 5z &= 2
\end{align*} \]

\[ \begin{align*}
0 - 0 &= 0 \\
0 &= 0
\end{align*} \]

- Infinitely many solutions.

Special, but same principles:

\[ \begin{align*}
x + y + z &= 15 \\
x + 4y - z &= 11
\end{align*} \]

\[ \begin{align*}
3x - 2y + z &= 15 \\
x + 4y - z &= 11
\end{align*} \]

Try:

\[ \begin{align*}
x - y + z &= 1 \\
2x + y + z &= 6 \\
7x - y + 5z &= 15
\end{align*} \]
Application

Nina's nut shop produces a deluxe blend of almonds, cashews, and pecans. How many pounds of each nut should be used to produce 940 pounds of the deluxe blend at a cost of $4,300 if the blend is 50% cashews?

Almonds cost $5 per lb. Cashews cost $4 per lb. Pecans cost $5.40 per lb.

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th># of lbs</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Almonds</td>
<td>5</td>
<td>x</td>
<td>5x</td>
</tr>
<tr>
<td>Cashews</td>
<td>4</td>
<td>y</td>
<td>4y</td>
</tr>
<tr>
<td>Pecans</td>
<td>5.4</td>
<td>z</td>
<td>5.4z</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>940$4,300</td>
</tr>
</tbody>
</table>

\[x + y + z = 940\]
\[5x + 4y + 5.4z = 4300\]
\[y = 50\% (940)\]
\[y = 470\]

Try: Mega toys produces toys from three shops. Shop one requires $25 per 100 toys for employee wages, $20 per 100 toys for electrical costs, and $50 per 100 toys for materials. Shop two requires $35 per 100 toys for employee wages, $10 per 100 toys for electrical costs, and $40 per 100 toys for materials. Shop three requires $10 per 100 toys for employee wages, $20 per 100 toys for electrical costs, and $100 per 100 toys for materials. The total costs for employee wages, electrical costs, and materials in order are $16,500, $9000, $32,000. How many toys are made at each shop?

<table>
<thead>
<tr>
<th></th>
<th>Shop 1</th>
<th>Shop 2</th>
<th>Shop 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>wages</td>
<td>25</td>
<td>35</td>
<td>10</td>
<td>16500</td>
</tr>
<tr>
<td>costs</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>5000</td>
</tr>
<tr>
<td>materials</td>
<td>50</td>
<td>40</td>
<td>100</td>
<td>32000</td>
</tr>
</tbody>
</table>

\[25x + 35y + 10z = 16,500\]
\[20x + 10y + 20z = 5000\]
\[50x + 40y + 100z = 32,000\]
\[x = 200\]
\[y = 300\]
\[z = 100\]
1) Graph the solution region.
2) Is the solution region bounded or unbounded.
3) Find the coordinates of the corner points.

\[ y < 2x + 1 \text{ and } y \geq 3 \]

\[ \frac{y}{2} = \frac{2x + 1}{2} \quad \frac{3}{2} = \frac{2x}{2} \quad \frac{1}{2} = \frac{x}{1} \]

\[ (1, 3) \]

\[ 2x + y \leq 8 \]
\[ x + 3y \geq 12 \]
\[ x \geq 0 \]
\[ y \geq 0 \]

Corner points:
(0, 8), (0, 4), \( (\frac{12}{5}, \frac{16}{5}) \)
1) Graph the solution region.
2) Is the solution region bounded or unbounded.
3) Find the coordinates of the corner points.

\[ \begin{align*}
x + 3y &\leq 18 \\
-x + y &\geq 2
\end{align*} \]